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MEASURING THE TENSILE STRENGTH THROUGH SIZE EFFECT CURVES

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Abstract
This paper shows that size effect curves for peak load are in most cases inconclusive when determining the tensile strength f_t of quasi-brittle materials modeled with the cohesive crack model. Two different softening functions such as linear and trapezoidal give the same very accurate results for notched and unnotched three point bending beams but using different values of f_t . The same trend is shown for splitting tests on square prisms. To determine the appropriate softening curve one needs additional experimental information such as the load- displacement curve.
Key words: Cohesive Crack Model, Tensile Strength, Softening Curve, Linear Softening, Trapezoidal softening.

1 Introduction

Fracture of concrete, rocks, ceramics and other quasi-brittle materials can be adequately modeled by means of the cohesive crack model, first introduced by Hillerborg, Modeer and Petersson (1976) and further developed by the authors. A review of the basic properties of this model was done in Elices and Planas (1989) and Elices, Planas and Guinea (1993).

The softening function governs the behaviour of a cohesive material and it is supposed by hypothesis to be a material property, geometry –and size– independent. When a cohesive crack opens, the stress transferred between its faces relates with the crack opening at this point according to the so-called softening curve. For monotonic mode I opening, the stress σ is normal to the crack faces and is a unique function of the crack opening w, $\sigma=f(w)$, as shown in Fig. 1. The stress at the tip of the cohesive crack is equal to the tensile strength of the material, f_t, and progressively reduces as the crack opening increases. When the crack opening reaches the critical crack opening w_c, the cohesive stress drops to zero and a *true* (stress free) crack propagates.

In many practical cases and for normal sized structures the peak load is reached well before any point in the cohesive crack experiences complete softening, and thus only the initial portion of the softening curve is relevant. This situation is sketched in Fig. 2, where it is shown that at peak load no point in the specimen has softened further than the shadowed zone in Fig. 2b. This fact makes indistinguishable the results obtained with the true softening curve from others obtained with a softening function with the same initial behaviour, such as the dashed line shown in the figure. This is so for notched specimens of laboratory sizes (Guinea, Planas and Elices, 1994a) and for unnotched specimens of all sizes (Planas, Guinea and Elices, 1995).

Since the peak load behaviour is determined by the tensile strength f_t and the initial part of the softening curve, the inverse question arises: Can the tensile strength and the initial part be uniquely resolved from a set of peak loads for various specimen sizes? As will be shown later, this it is not possible for all geometries without resort to matching additional experimental results other than peak loads, such as the complete load-displacement curve.

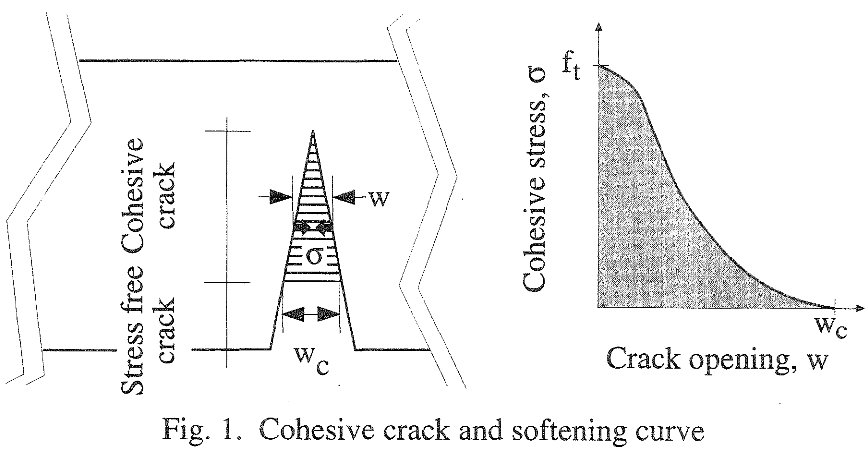

Fig. 1. Cohesive crack and softening curve

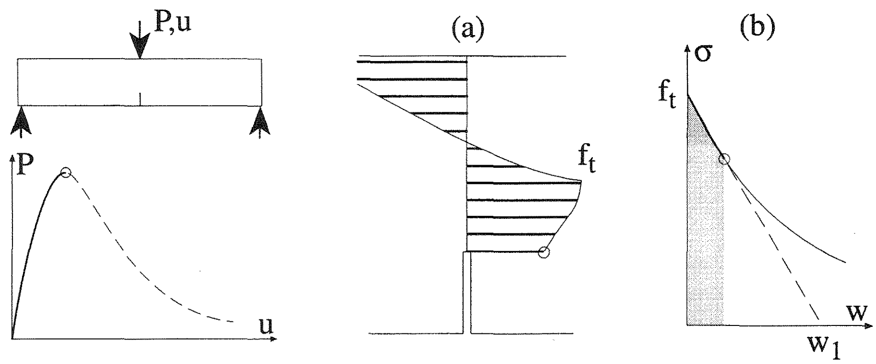

Fig. 2. Peak load situation for a notched beam. (a) Stress profile. (b) Softening function. The stress at the notch is marked with a circle.

Basically there are two types of initial softening behaviour, in correspondence with the presence or not of a horizontal plateau sometimes reported in the literature (Hillerborg, 1991, Bao and Zok, 1993). Fig. 3 shows the two simplest approximations to these softenings: the linear softening for curves with initial slope, and the trapezoidal softening for curves which exhibit a horizontal plateau.

In the following section, peak loads are computed using linear and trapezoidal softening curves for two common laboratory geometries: the three-point bending beam (notched and unnotched) and the Brazilian square prism. The work shows that both the linear and trapezoidal softenings can predict accurately the same size effect curve for the three point bend specimens but with different values of the tensile strength. The analysis performed with the Brazilian square prism geometry shows that in this case the predicted values of f_t are less sensitive to the softening curve than for the bending specimens. The paper closes with some comments and conclusions.

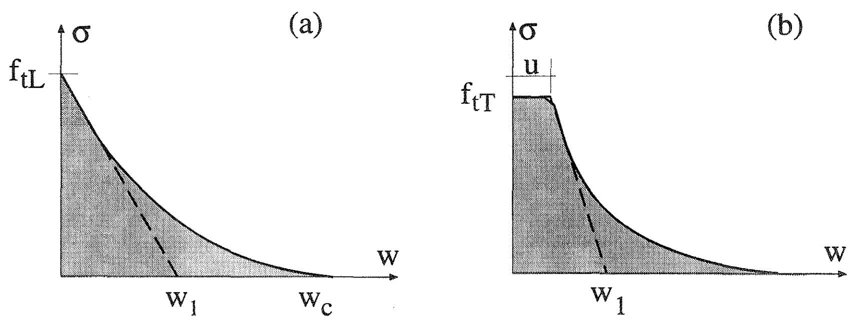

Fig. 3. Initial approximations to softening curves. (a) Linear softening. (b) Trapezoidal softening.

2 Tensile strength from size effect curves

For clarity, two series of fracture data for a particular concrete have been used to determine the tensile strength and the shape of the initial part of the softening curve.

The concrete was made in our laboratory with a Type III Portland cement and maximum aggregate size of 5 mm. The water cement ratio was 0.5 and the grading met the requirements of ASTM C33. Other details of casting and curing conditions can be found elsewhere (Rocco, 1996).

Two geometries were tested: three point bending beams (with and without notch, Fig. 4a) and Brazilian square prism specimens (Fig. 4b).

2.1 Three point bend specimens

The nominal strength for this geometry σ_{Nu} is defined from the peak load P_u as:

$$\sigma_{Nu} = \frac{3P_u\, s}{2BD^2} \tag{1}$$

where s is the loading span, D the beam depth and B the thickness.

Fig. 5 shows the nominal strength for half-notched (a/D=0.5) and unnotched (a/D=0) beams as a function of the specimen depth. For all the specimens the loading span was four times the beam depth, and the thickness B was kept constant, equal to 50 mm.

To determine the size effect curve, four sets of geometrically similar specimens were tested, corresponding to depths D = 17, 35 , 75 and 150 mm, respectively.

Tests were performed according the RILEM recommendation (RILEM, 1985), with some additional refinements suggested by the authors (Guinea, Planas and Elices,1992; Planas, Elices and Guinea, 1992; Elices, Guinea and Planas, 1992).

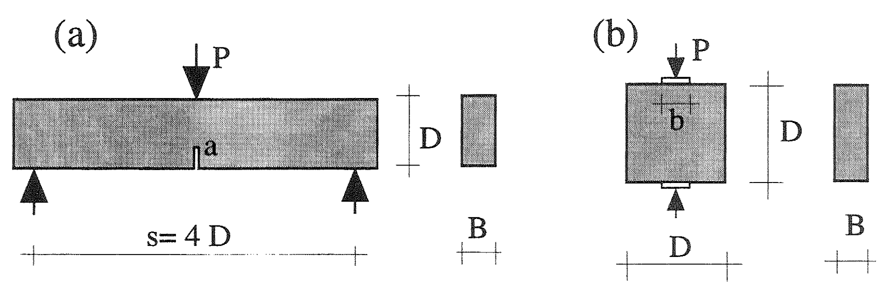

Fig. 4. Test geometries.

Three softening curves were used to fit the experimental data, a linear softening function (Fig. 3a) and two trapezoidal softenings (Fig. 3b) with relative plateau widths u/w_1=0.1 and 0.5, respectively. For each softening curve, the tensile strength, f_t, and the characteristic length, l_{ch}, were adjusted to obtain the best fit to the two series of data (notched and unnotched). The characteristic length is defined as (Hillerborg, 1976):

$$l_{ch} = \frac{E\,G_F}{f_t^2} \tag{2}$$

where E is the elastic modulus, f_t the tensile strength and G_F the specific fracture energy which can be computed as the area under the softening curve. For a linear softening (Fig. 3a), l_{ch} is equal to $Ew_1/2f_t$ and for the trapezoidal (Fig. 3b) is given by $E(u+w_1)/2f_t$. The value of E was obtained from the initial compliance of the curve of load vs. crack mouth opening displacement (CMOD) for the notched specimens. The result was E=31GPa.

The results are shown in Fig. 5. The three softenings fit reasonably well the notched as well the unnotched beams but with different values of the tensile strength, f_t, which varies up to 20% from one softening to another. As a consequence, size effect curves, even for unnotched beams, seem to be inconclusive to infer the right value of f_t and the true shape of the softening curve. From another point of view, the results in Fig. 5 show that a linear softening is adequate enough to model the size effect of three point bending beams whatever the softening function.

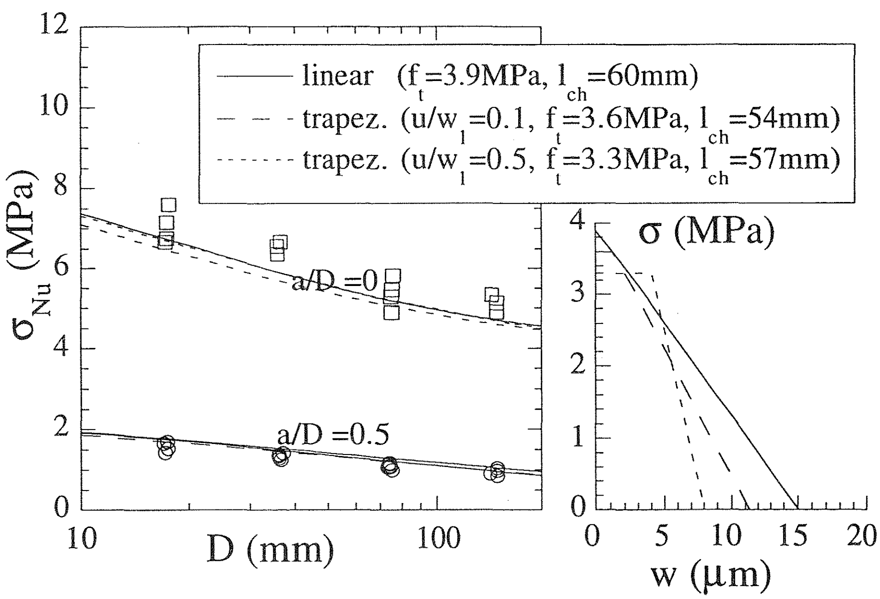

Fig.5. Size effect curves for three point bend specimens and best fit for linear and trapezoidal softening functions

2.2 Brazilian square specimens

The Brazilian square prism is a popular geometry for evaluating the tensile strength of cement based materials. The geometry is depicted in Fig.4b. The load is usually applied through a narrow strip of plywood of width b, usually between 8-16% of specimen depth, D (BS 1881-117, 1983).

For this specimen, the nominal strength σ_{Nu} is defined from the peak load P_u as:

$$\sigma_{Nu} = \frac{2P_u}{\pi BD} \tag{3}$$

where D is the specimen depth and B the thickness.

Fig. 6 shows the results of two series of tests with different loading zone, b/D=0.08 and 0.16. For b/D=0.08, prisms with D=37, 75 and 150 mm were tested. For b/D=0.16, the depths were D=17,37,75,150 and 300 mm. The concrete mix was the same as that for the three point bend tests described in section 2.1.

To model the size effect curves, the linear and the u/w_1=0.1 and u/w_1=0.5 trapezoidal softening functions were again applied. The parameters f_t and l_{ch} were obtained using a least squares fitting algorithm to adjust simultaneously the two set of experimental points (b/D=0.08 and 0.16). Fig. 6 plots the results. In this case, the differences between the softening functions are embedded in the experimental scatter, and no conclusions as to which is the most appropriate softening can be drawn. Nevertheless, the Brazilian splitting geometry seems to be less sensitive to the softening functions compared to the predicted values of the tensile strength, which differ now less than 6% (from 3.5 to 3.7 MPa).

3 The general fitting procedure

To differentiate between the softening curves it is necessary to resort to other experimental results apart from peak loads, as demonstrated in the foregoing sections. As an example, Fig. 7 shows the load-CMOD curves predicted with the softening functions in Fig. 5 for a notched beam of a/D=0.5 and D=150 mm. The shadowed zone corresponds to the experimental results. As seen in this figure, the three softenings fit well only the peak load region, giving a poor prediction for the whole curve.

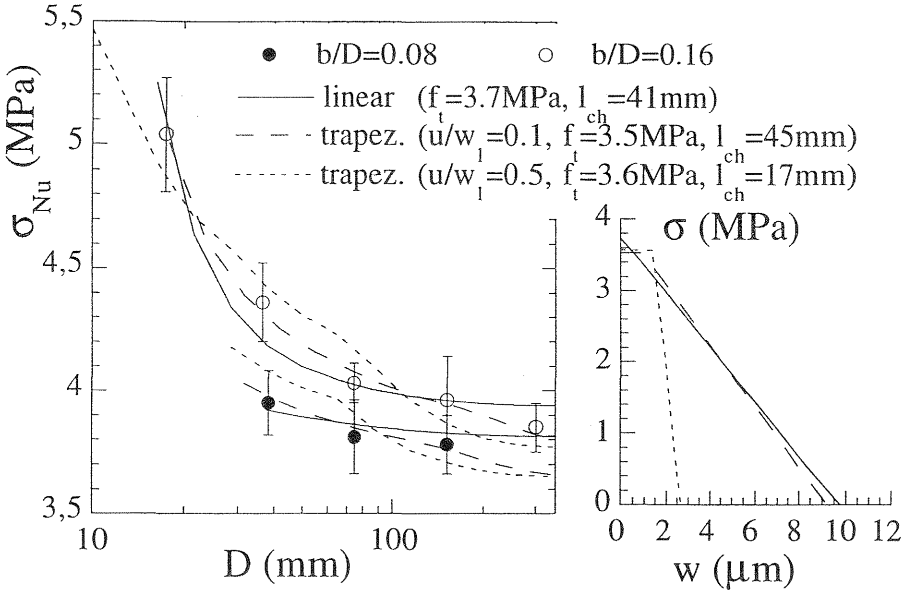

Fig.6. Size effect curves for Brazilian square specimens and best fit for linear and trapezoidal softening functions. Mean values (circled points) and 95% confidence interval are shown for each size.

Following a method developed by the authors (Guinea, Planas and Elices, 1994b), a bilinear softening curve for this microconcrete has been determined. The method makes use of the peak loads as well as the complete load-displacement curves obtained in three point bend tests. The bilinear softening function is plotted in Fig. 8 together with the softening functions of Fig. 5, for comparison purposes. The first segment of the bilinear curve matches the linear function to properly fit the size effect curves for the three point bend specimens, which are known to be dependent on the initial portion of the softening. The second part of the bilinear curve has been adjusted as described in the reference above, to match the post-peak behaviour.

The prediction with the bilinear softening function is shown in Fig. 7. The bilinear curve gives a realistic description of all the load-CMOD curves. Note that since the bilinear curve has the same initial part as the linear softening function, predictions with both curves are identical up to peak load. Then the second part of the bilinear softening comes into play and the two predictions diverge.

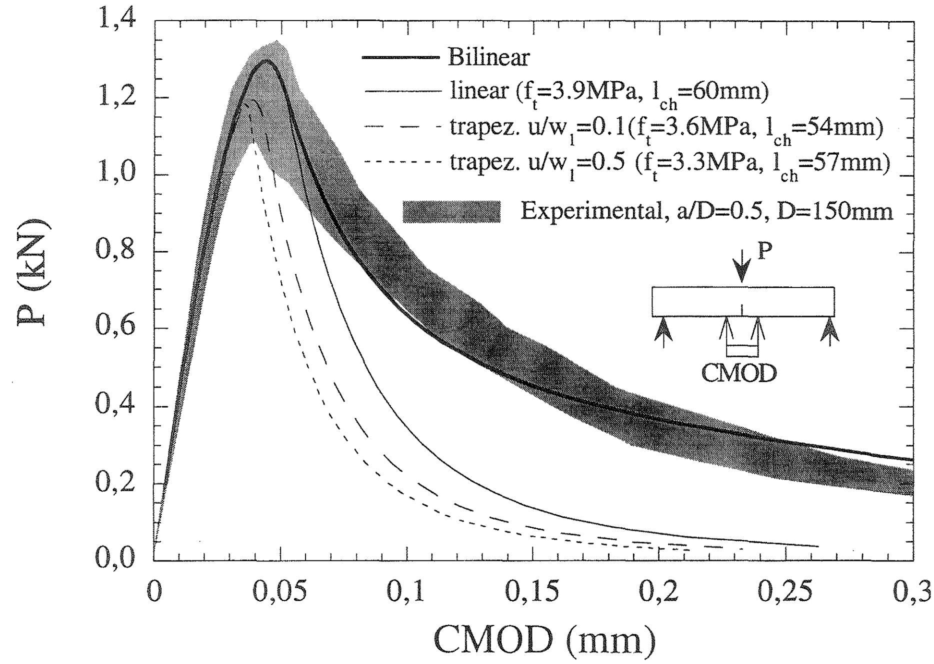

Fig.7. Load-CMOD curves for notched beams. Experimental results and numerical predictions for the softening functions shown in Figs. 5 and 8.

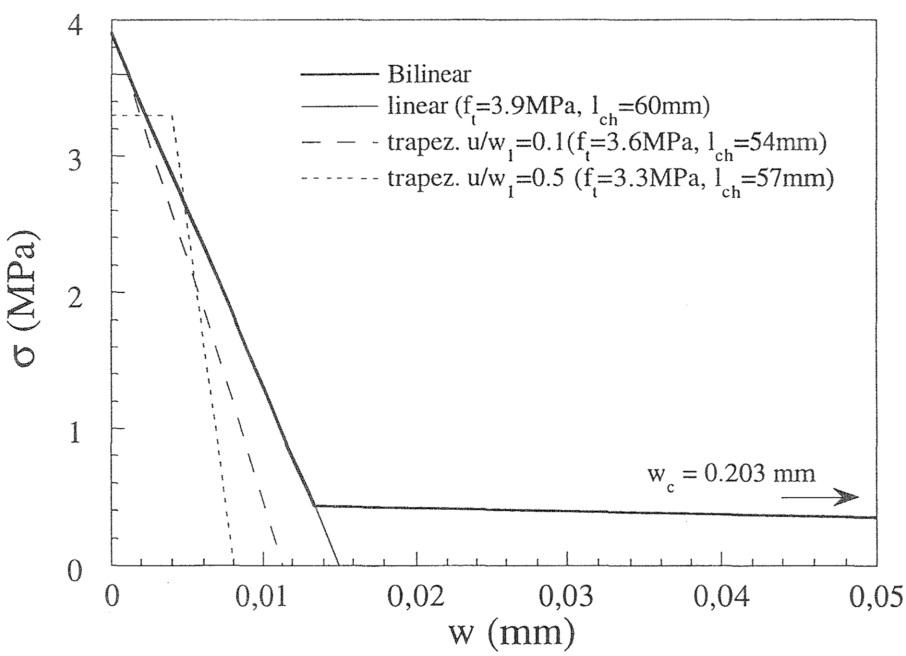

Fig.8. Softening functions used in Fig. 7.

4. Comments and Conclusions

The foregoing results show that size effect curves are inconclusive when used to determine the tensile strength or the initial part of the softening curve. For the three point bend geometry, in a wide range of sizes and for notched as well as unnotched beams, almost the same size effect curve can be obtained with linear and trapezoidal softenings, each one with different values of f_t. This proves that the linear softening function is precise enough to adequately reproduce the observed maximum load size effect for current laboratory beam sizes.

The same trend is observed in the case of the Brazilian square prism geometry, although the predicted value of the tensile strength appears less sensitive to the softening function selection.

To pick out the suitable softening curve it is essential to resort to additional post-peak experimental results, such as the complete, stable, load-displacement curve or load-CMOD curve.

5. Acknowledgments

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