

## STUDY ON INTERNAL FRICTION ANGLE AND TENSILE STRENGTH OF PLAIN CONCRETE

Y. Fujita, and R. Ishimaru  
Kaoru Architect Office., Japan  
S. Hanai  
Department of Architecture Engineering , College of Industrial Technology,  
Nihon Univ., Japan  
Y. Suenaga  
Emeritus Prof., Yokohama National Univ., Japan

### Abstract

In general, 37 degrees is applied to the internal friction angle value ( $\phi$ ) of concrete and an 8~10% value of compressive strength is for the tensile strength value of it. However, the theoretical solution of the internal friction angle is calculated as 57.5 degrees by the equivalence that linear elastic theoretical solution of the concrete split test is equivalent to the wedge theoretical solution of it. We experimented with the partial compression test of plain concrete to ensure this fact. As a result, it was proved that the test result equal to the wedge theoretical solutions at  $\phi=55$  degrees and  $F_t=0.05F_c$ .

Key words: Internal friction angle, Tensile strength, Wedge theory, Split test

### 1 Introduction

We proposed a theory named "Wedge Theory" in Suenaga and Ishimaru (1974a) regarding the bearing strength of plain concrete. Successively, Suenaga and Ishimaru (1974b), and Asakawa et al. (1994), discussed

theoretical analysis methods based on the Wedge Theory concerning shear strength of steel reinforced concrete beams and columns, using a comparison study with experimental values. In these studies, we have made calculations based on the values of  $\phi = 37^\circ$ ,  $F_{tt} = F_t / F_c = 0.1$  for the internal friction angle and tensile strength of concrete, as they are considered appropriate for the material characteristics. In Suenaga and Ishimaru (1974b), we indicated that the ratio variation between the experimental values and theoretical ones were within the range 1.0 through 0.7, with the theoretical values liable to be higher than the experimental ones. We also indicated that all these results stemmed from our original assumption of  $F_{tt} = 0.1$ . Further research, however, produced three possible reasons for the above phenomenon as follows; the fact that our Wedge Theory gives an upper bound solution,  $\phi$  values,  $F_{tt}$  values. Viewed in the light of our Wedge Theory, as mentioned later, the relation between the Wedge Theory and theoretical formula of split testing on cylinders shows that the internal friction angle is theoretically deduced to be  $\phi = \tan^{-1}(\pi/2) = 57.5^\circ$ . Looking into the difference of these two theories, we conducted experiments to research  $\phi$  values of concrete (or mortar) and the true relationship between tensile strength and compressive strength of concrete, whose results were then statistically analyzed. We concluded that, based on the Wedge Theory, experimental values and theoretical ones conform to each other under  $\phi = 55^\circ$ ,  $F_{tt} = F_t / F_c = 0.05$ , of which the details will be described in this thesis.

## 2 Split test of plain concrete

### 2.1 Outline of the experiment

#### 2.1.1 Shape and loading method of the specimen

As shown in Fig.1, the shape of the specimen was designed to be 150w×80L×200H, and the loading was conducted on the specimen installed between two loading plates, into which heat treatment steel plates had been machined in advance, on the upper and lower side of the specimen, by using a all-powerful loading machine (5 metric ton range). The inner surface of the loading plate was coated with grease in advance.

#### 2.1.2 Material quality, building and curing process of the specimen

Premixed mortar is adopted for the building specimens so that the material quality does not vary so much depending on each lot. Mortar is mixed with water into a water-cement ratio as to obtain 270 kgf/cm<sup>2</sup> ( 26.5N/mm<sup>2</sup> ) for the strength at the age of 4 weeks and cured in a metallic mold yielding 15 pieces per lot. Cured mortar is then kept in the laboratory until the day of

the testing with its surface covered with a wet cloth.

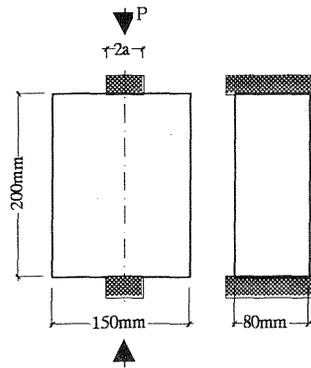


Fig.1. Shape and loading method of the specimen

### 2.1.3 Experimental parameters and the number of test specimens

Experiments are conducted with parameters fixed at  $\beta$  values of the Wedge Theory, with  $2a$  value in Fig.1 varied. The identification of each testing (called "Series"),  $\beta$ ,  $2a$ ,  $S$  value (loading area), and the number of testing specimens ("Numbers") are shown in Table 1 below. The number of specimens were fixed within the range of 38 to 40, which would give a narrow variation of the t-value. 6 pieces of specimens for compression testing were prepared per each lot, with their dimensional size  $100 \phi \times 200H$ .

Table 1. Experimental parameter

Series	$\beta=H/2a$	$2a$ mm	$S$ cm <sup>2</sup>	Numbers
A	20.00	10	8.0	40
B	14.30	14	11.2	40
C	10.00	20	16.0	40
D	8.00	25	20.0	40
E	5.00	40	32.0	40
F	4.00	50	40.0	38
G	3.45	58	46.4	38
H	2.94	68	54.4	38
I	2.50	80	64.0	38
J	2.00	100	80.0	38

### 2.1.4 Loading test

We conducted this experiment for Series A through J per each lot from April 1993 until December 1994, in which 40 specimens were tested for each testing Series A through E, and 38 for Series F through J. The standard specimen age was determined to be 28 days. Compression testing was conducted prior to the loading experiment, so that the compression strength of the specimen in each lot was measured.

## 2.2 Experiment results

### 2.2.1 Wedge failure and compressive slip failure

Prior to the analysis of the testing results, the failure types for wedge failure and compressive slip failure will be explained as different loading conditions give different failure types. The Wedge Theory is quoted again from Suenaga and Ishimaru (1974a) as below, which means that the solution of partial compressive strength "p" in a compression test is given in Eq.1 under the parameter  $t = \cos \alpha$ .

$$p = \frac{At^2 + Bt + C}{t - \mu} \quad (1)$$

In this equation,

$$A = \tau_0 - \mu Ft \quad (2)$$

$$B = (\mu \beta - 1)Ft \quad (3)$$

$$C = \tau_0 + \beta Ft \quad (4)$$

$$\tau_0 = \frac{\sqrt{1 + \mu^2} - \mu}{2} Fc$$

$$\mu = \tan \phi$$

$$\beta = \frac{h}{a}$$

$t_0$ , which gives a minimum value of Eq.1, will be, within the range of  $t > \mu$  and as

$$\frac{dp}{dt} = 0$$

Therefore,

$$t_0 = \mu + \sqrt{\mu^2 + \frac{B\mu + C}{A}} \quad (5)$$

Such a geometrical condition that gives a wedge failure is;

$$\beta \geq t_0 \quad (6)$$

In the range of Eq.6, wedge-typed failure occurs, and partial compressive strength is to be determined as below, with Eq.5 substituted for Eq.1

$$p_{\min} = B + 2A\mu + 2\sqrt{(A\mu)^2 + AB\mu + AC} \quad (7)$$

In Eq.7, the  $\beta$  value is also determined as below, with Eq.5 substituted for Eq.6,

$$\beta \geq \frac{K + \sqrt{K^2 + 4A^2}}{2A} \quad (8)$$

In this equation,

$$K = 2A\mu + \mu^2 F_t + F_t \quad (9)$$

The minimum  $\beta$  value which causes wedge failures will be calculated with  $\phi$  and  $F_{tt}$  as parameters, as shown below in Table 2.

Table 2. Limit values of  $\beta$  (h/a)

$\phi$ deg.	F <sub>tt</sub> (F <sub>t</sub> /F <sub>c</sub> )					
	0.04	0.05	0.06	0.07	0.08	0.09
37	2.24	2.31	2.39	2.47	2.56	2.66
39	2.36	2.45	2.54	2.64	2.75	2.87
41	2.50	2.60	2.71	2.83	2.96	3.12
43	2.66	2.77	2.90	3.05	3.22	3.42
45	2.83	2.97	3.14	3.32	3.54	3.80
47	3.03	3.21	3.41	3.65	3.95	4.30
49	3.27	3.49	3.75	4.07	4.48	5.01
51	3.54	3.83	4.18	4.64	5.24	6.08
53	3.88	4.26	4.75	5.44	6.42	7.91
55	4.30	4.83	5.57	6.70	8.58	12.30
57	4.85	5.63	6.86	9.04	14.00	35.00

Secondly, in the case of  $t_0 < \beta$ , the failure types would be the compressive slip failures shown in Fig.3, as was previously mentioned in Asakawa et al. (1994), and the p value is calculated in Eq.10 as shown below;

$$p = \frac{\tau_0 \cos \phi}{\cos \theta \sin(\theta - \phi)} \quad (10)$$

The denominator of Eq.10 becomes the highest value  $(1 - \sin \phi)/2$  in the case of  $2\theta - \phi = 90^\circ$ , where the lowest value of p will be as follows based on Mohl-Coulomb's failure criterion;

$$p_{\min} = \frac{2\tau_0 \cos \phi}{1 - \sin \phi} = F_c \quad (11)$$

And the slip angle under this condition will be given by;

$$\theta = 45^\circ + \frac{\phi}{2} \quad (12)$$

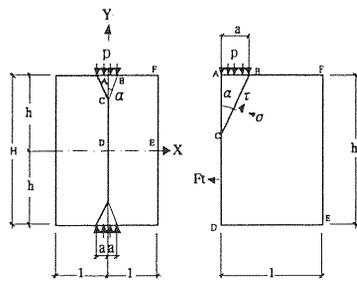


Fig.2. Mode of fracture in partial compression according to "Wedge Theory" and stress relations along fracture lines.

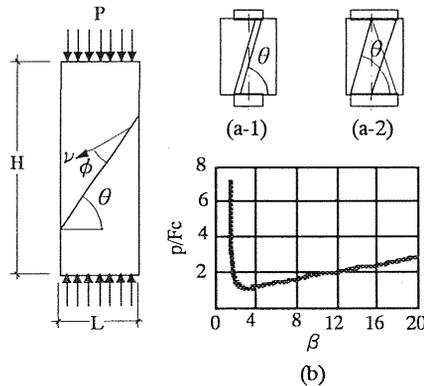


Fig.3. Failure mechanism in pure compression of a rectangular disk.  
 (a-1,2) slip failure line at  $\tan^{-1}(h/a) < \theta$   
 (b)  $p/Fc$ -  $\beta$  relation curve

The relation between  $p/Fc$  and  $\beta$  ( $\beta=1.5$  through  $20$ ) under the condition of  $\phi=55^\circ$  and  $F_{tt}=0.5$  is described in Fig.3, for which the calculation of Eq.5 is employed in the case of  $\beta \geq \beta_0$ , and Eq.10 in the case of  $\beta < \beta_0$ . In Eq.10, the  $p/Fc$  value increases drastically when  $\beta$  falls below  $2.5$ , however, any phenomenon which represents this kind of data could not be detected in the experiments thus far. Additionally, Noguchi and Tomosawa (1995) confirm that the value shows a gradual increase while  $\beta$  stays between  $2$  and  $1.5$ . If  $\tan^{-1}\beta$  becomes smaller than a slip angle  $\theta$ , which is to be calculated based on Eq.12, the slip angle becomes constant ( $\theta$ ), and  $p/Fc=1$  is obtained as shown in Eq.11 based on the assumption of introducing a slip curve as shown in Fig.3 (a-1). The relation as explained above is kept unchanged until it reaches the  $\beta$  value as shown in Fig.3 (a-2), and in case of a lower  $\beta$  value, the load amount required for the failure increases and  $p/Fc$  value is considered to increase as the slip lines on both sides do not exist in a certain portion of the longitudinal cut area. In the case of  $\phi=55^\circ$  and  $F_{tt}=0.05$ ,  $\beta=1.59$  is found in Fig.3. Therefore, in the analysis of actual values obtained in the experiment, Eq.5 should be employed in the case of  $\beta \geq \beta_0$ , Eq.10 in the case of  $\beta < \beta_0$ , and in the case of  $\tan^{-1}\beta < \theta$ ,  $p/Fc=1$  should be employed for calculating the theoretical values.

### 2.2.2 Experiment results

The 10 Series of compression tests were conducted on 390 pieces of specimens, which were taken from 26 lots. For the analysis of the compression the test results, the means of the test values of four pieces out of six in total were adopted, with the maximum and minimum values neglected. The results of the compression test are shown in Fig.4(a) through Fig.4(e), with the values of  $p/F_c$  indicated in the vertical axis and  $\beta$  in the horizontal axis respectively. The dots on each figure indicate the distribution of test results. Thicker lines represent the highest and lowest values of the confidence interval around the mean of experiment results, which was calculated by the t-test with its confidence coefficient at 95%. Thinner lines indicate the calculated values based on the theoretical equation as mentioned before. The combination of parameters, which were employed for the comparison with the test results are as follows; internal friction angle; 11 different values by the step of  $2^\circ$  between  $37^\circ$  and  $57^\circ$ . Tensile strength ratio; 9 different values by the step 0.01 between  $F_{tt}=0.01$  and 0.09 for each  $\phi$  above, which results in 99 combinations in total. Of all these, only one combination of  $\phi=55^\circ$  and  $F_{tt}=0.05$  proved to make all its  $\beta$  values above the 95% of confidence coefficient in the t-test of experimental values. In this case, the F-test conducted on the experimental values, which were compared with theoretical values, showed a significant difference narrower than 5% regardless of any  $\beta$  value.

Consequently, the following remarks were obtained;

- 1) Estimated values of population mean are most suitable for those based on  $\phi=55^\circ$  and  $F_{tt}=0.05$ .
- 2) As an internal friction angle  $\phi$  becomes closer  $\phi=57.5^\circ$ , the solution given based on the Wedge Theory, and the estimated distribution of population mean of experimental values become closer to theoretical solutions.

Fig.4(f) shows the comparison study of values based on  $\phi=55^\circ$  and  $F_{tt}=0.05$  and the means of experimental values, which indicated significant conformity.

Fig.5 (a) through Fig.5 (j) show the result of normal distribution, into which the histogram of experimental values together with the means of experimental values and their distribution were calculated, with the relative frequency employed as the horizontal axis.

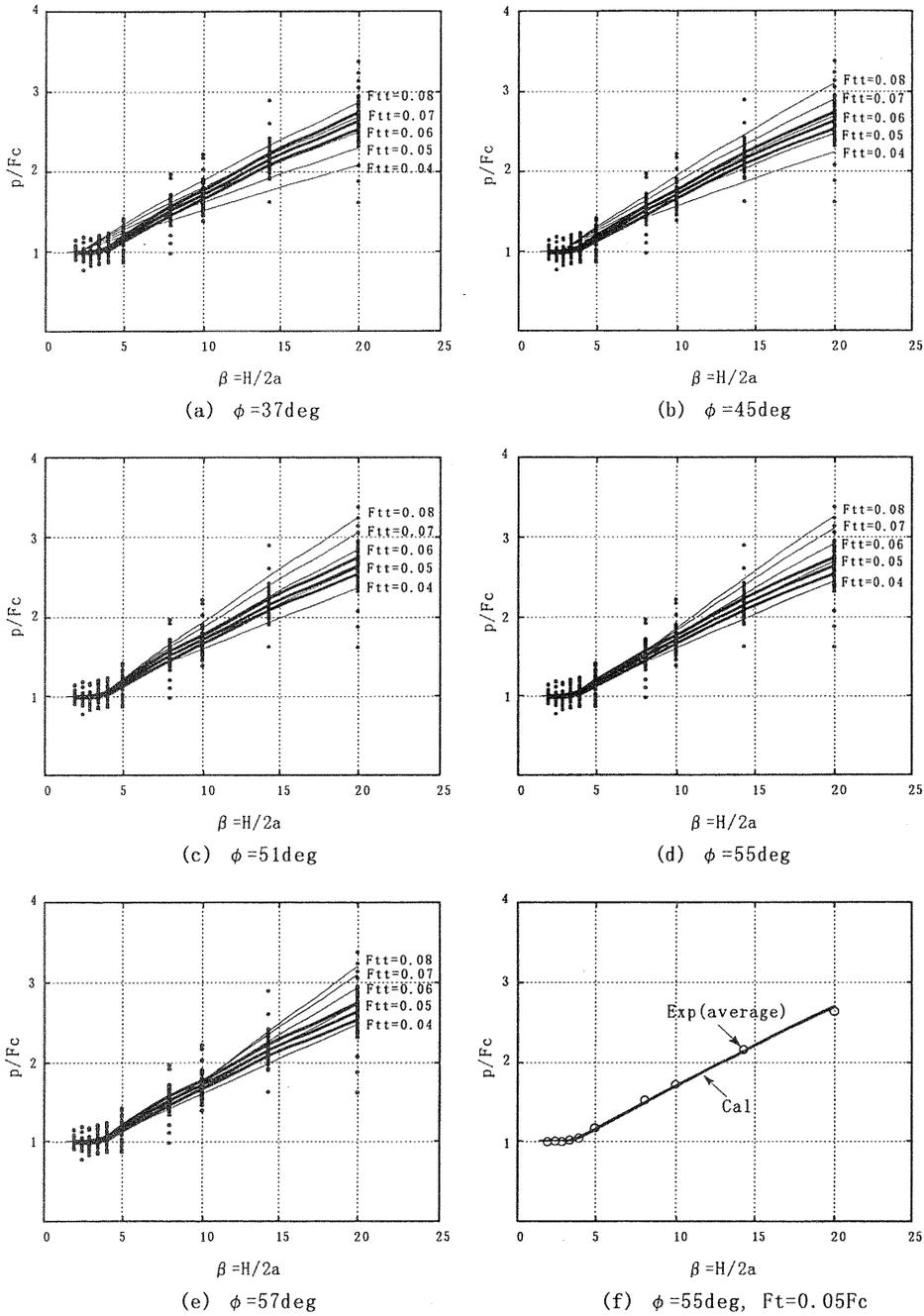


Fig.4 Experimental results

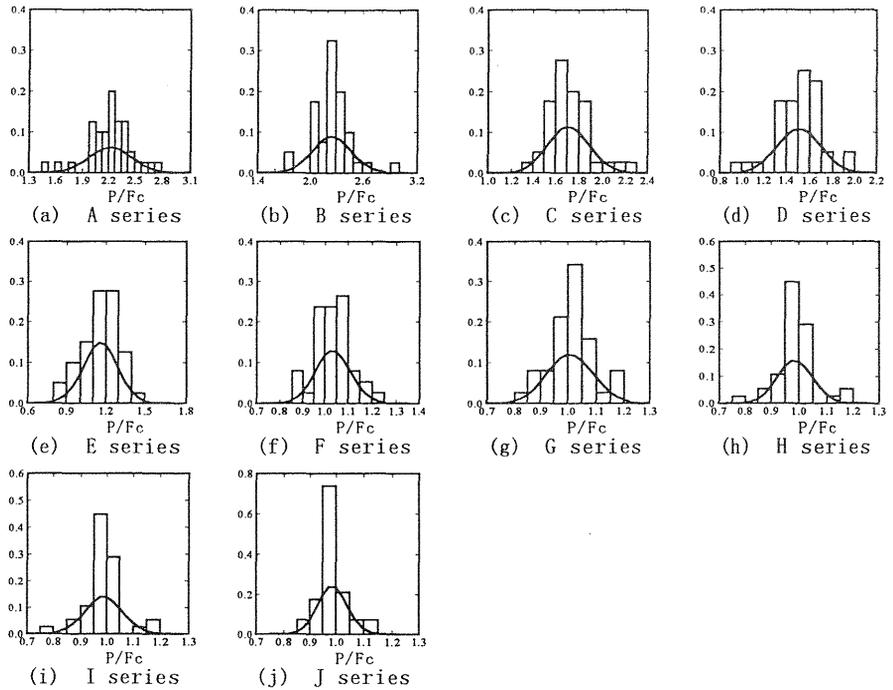


Fig.5 Results of normal distribution

### 3. Theoretical Background

For the calculation of tensile strength of concrete, split tests of concrete cylinders are normally conducted, and our intention was to introduce our Wedge Theory to this testing method. It is assumed that the bearing surface 2a becomes smaller, such results as to be similar to those of cylinder split tests should be obtained in the light of the Wedge Theory. Eq.7 is changed into;

$$\frac{p}{\beta} = \left( \mu - \frac{1}{\beta} \right) Ft + \frac{2A\mu}{\beta} + 2\sqrt{\left( \frac{A\mu}{\beta} \right)^2 + \left( \frac{A\mu^2}{\beta} - \frac{A\mu}{\beta^2} \right) Ft + \frac{A\tau_0}{\beta^2} + \frac{AFt}{\beta}} \quad (13)$$

Therefore,

$$\lim \left( \frac{p}{\beta} \right) = \mu Ft \quad (\beta \rightarrow \infty) \quad (14)$$

is obtained, which means that the relation as below approximately exists while the  $\beta$  value is rather high.

$$p/\beta = \mu Ft \quad (15)$$

Eq.15 is further changed into,

$$\beta = H/2a, \quad H=d, \quad P=2ap$$

Therefore,

$$p/\beta = 2ap/H = P/d = \mu Ft \quad (16)$$

is obtained.

On the other hand, as the tensile strength of concrete, obtained in the split test, is;

$$Ft' = 2P/\pi d \quad (17)$$

$P/d$  is deleted from Eq.16 and Eq.17, and

$$\mu = \pi Ft'/2Ft \quad (18)$$

In this equation, if it is assumed that  $Ft'$  is equal to  $Ft$ ,

$$\mu = \pi/2 \quad (19)$$

is obtained, and for the internal friction angle

$$\phi = \tan^{-1}(\pi/2) = 57.5^\circ \quad (20)$$

This proves that the internal friction angle  $\phi = 55^\circ$ , which was obtained in the experiment, is very close to the theoretical solution.

#### 4. Conclusion

As a result of the compression test conducted on plain concrete specimens, the internal friction angle  $\phi$  and the ratio between tensile strength and uniaxial compressive strength  $Ft$  of concrete (or mortar) proved to become very close to the solution obtained based on the Wedge Theory in the case of  $\phi = 55^\circ$  and  $Ft = 0.05$ .

#### 5. References

- Asakawa, T., Ishimaru, R., Yoshida, H. and Suenaga, Y. (1994) Study on shear capacity of RC columns confined with rectangular steel pipe used for the precast concrete wall structure reducing to columns at the ground story. *J. Str. Const. Eng., AIJ.* 460, 143-152
- Noguchi, T. and Tomosawa, F (1995) Effect of size and shape of specimen on mechanical properties of high strength concrete in compression. *J. Str. Const. Eng., AIJ.* 473, 19-28
- Suenaga, Y. and Ishimaru, R. (1974a) Dynamic analysis on concrete in combined stress (Part 3). *Trans. AIJ.* 220, 1-7
- Suenaga, Y. and Ishimaru, R. (1974b) Dynamic analysis on concrete in combined stress (Part 4). *Trans. AIJ.* 221, 9-16