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NEW 2-DIMENSIONAL ANALYTICAL METHOD FOR DETERMINATION OF THE SHAPE PROPERTIES OF CONCRETE CRACK SURFACES USING A LASER-BEAM

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Abstract

An measurement setup using of a laser-beam, which setup is based on the structured light projection method, was developed to obtain 3-dimensional images of concrete crack surfaces. The 3-dimensional surface-model(image) is sliced into profiles, and sampling points are marked on each profile at regular intervals. A small piece of crack surface is idealized as a section between one sampling point and the next one. In order to analyze the shape properties of the crack surface, five 2-dimensional functions are formulated. To examine the validity of the proposed 2-dimensional method, the crack surface image in one concrete specimen of a tensile experiment was measured and analyzed by this method. The analytical results show that two parameters have reasonable accuracy.
Key words : Incline density, Depth density, Mean depth, Normalized depth

1 Introduction

Visual inspection of a concrete crack surface shows that the surface has asperity with a limited range of depths of the crack, and the 3-dimensional shape of the surface is the ultimate stage in the concrete-cracking process. Thus, if the shape properties of a concrete crack surface could be quantified, the cracking mechanism could be treated as a mathematical model.

In 1988, Li and Maekawa measured the inclination angle of profiles of splitting-crack surface sections using a digitizer. In 1993, Mihashi and Umeoka measured a crack surface using a spot-laser and analyzed the fractal dimensions of the crack surface. However, these two kinds of measurement operations are very complex and the amount of data that can be obtained is limited.

A crack surface is a rough surface. Sharp changes in the shape of a rough surface make accurate measurement difficult. However, there has been no discussion of the problems concerning measurement of crack surfaces, especially the problem of measurement accuracy, which greatly affects the reliability of measured data.

In this study, a measurement setup using of a laser-beam, which is based on the structured light projection method, was developed to obtain 3-dimensional images of concrete crack surfaces. The structured light projection method is a more efficient method of measurement than conventional methods. In order to analyze the shape properties of a crack surface, five 2-dimensional functions are proposed.

To examine the validity of the proposed 2-dimensional method, the crack surface image of one concrete specimen in a tensile experiment was measured and analyzed by this method. The analytical results show that two parameters (measurement sensitivity and sampling interval) have reasonable accuracy.

2 A method for measuring the shape of a concrete crack surface based on the structured light projection method

In the structured light projection method, A/D transformation of the obtained image enables several-hundred times more data to be obtained instantaneously in one operation compared to the spot-laser method by Mihashi et al . However, this method has never been used to measure a concrete crack surface .

In this paper, we describe shape measurement procedures based on the structured light projection method and image processing for the calculation of coordinates on a crack surface.

2.1 Measurement setup

A laser beam is projected through a narrow slit onto an object surface, and by observing from a slanted direction, the shape of a cross section can be obtained. The 3-dimensional coordinates of the object can be determined by conducting simple analysis of the image coordinates. This is the measurement principle of the structured light projection method.

In order to measure of the surface of a concrete crack, the authors constructed a measurement setup consisting of a laser, optical instruments, slid-

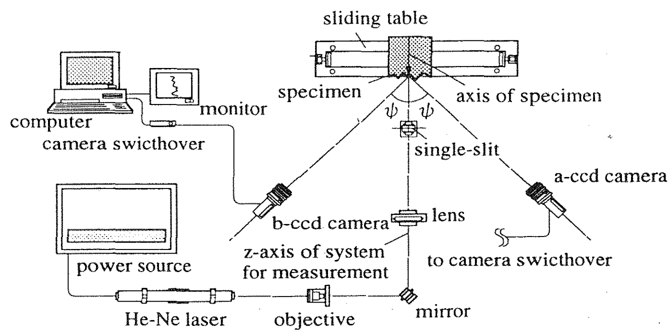

Fig. 1. System for measurement of a crack surface by the structured light projection method

ing table, and computer (Fig. 1). In the optical setup, an He-Ne laser is expanded and collimated by two lenses. The collimated light is projected through a narrow single-slit onto the concrete surface. The projected light is recorded by two CCD cameras (observation angle is $\pm\psi$ to each optical axis). The reason why two cameras are used is to compensate for the lack of image data, because a part of the slit image is lacking due to the shade of the crack itself. Thus, one image is obtained, then the object is moved an appropriate distance and the next image is recorded.

A composited image is obtained as follows. First, the laser beam in Fig. 1 is irradiated onto the object (specimen) and the first image is taken, and then the data is fed into the computer. The displacement of the specimen is prescribed by shifting the specimen, using the sliding table, in a perpendicular direction to the laser incidence axis, and a new crack image is taken. Cross-sectional images of the specimen are obtained by repeating the same process.

The crack surface image data, expressed in units of image pixels (hereafter, called “dots”) by A/D transformation, are transformed into 3-dimensional coordinates by an analytical procedure.

These images are composited by the image processing technique shown in Fig. 2.

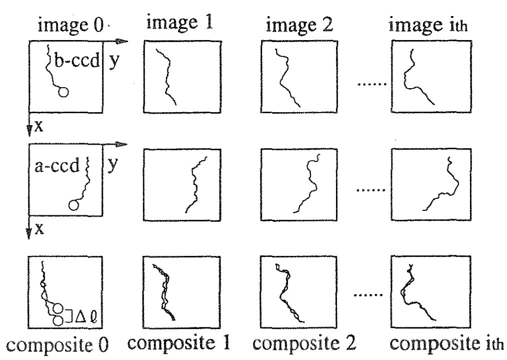

Fig. 2. Procedure for making a composited image from crack images

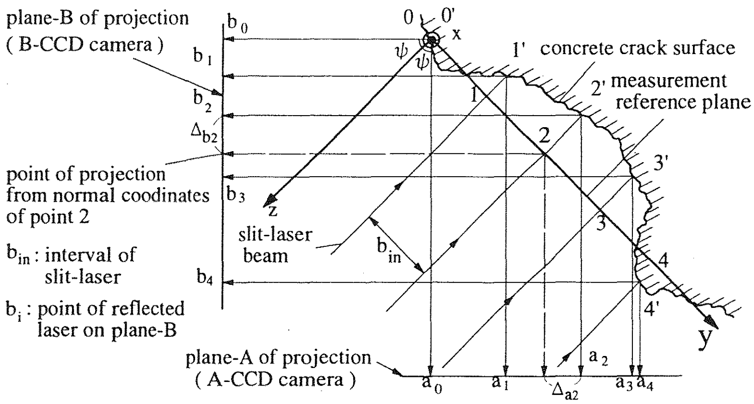

Fig. 3. Coordinate axes in the measurement system

In the coordinate system of the optical setup shown in Fig. 1, the z-x plane, parallel to the single-slit laser beam, is set as shown in Fig. 3. Planes of projection A and B are set as the image-acquisition planes of CCD cameras A and B, respectively, with the perpendicular x-y plane as the measurement reference plane. The axis of the specimen is aligned with the z-axis of our system (optical axis), making the crack reference plane, determined from a small piece of the crack surface, and the measurement reference plane almost parallel to each other.

Consequently, the 3-dimensional coordinate points (Fig. 3) of a concrete crack surface can be derived from pixel data of a composited image (Fig. 2) by a simple-method of analytic geometry.

3 Two-dimensional analytical functions for describing the shape properties of crack surfaces

For analyzing 3-dimensional coordinate values, several functions are needed to determine the shape properties of a crack surface. In other words, we need to define some analytical functions that can describe the shape properties of a crack surface. In order to recognize the properties of a crack surface, these properties must be extracted and analyzed through the application of a mathematical procedure to the measured coordinate points.

In this paper, 2-dimensional analytical functions are proposed for describing the shape properties of a crack surface. Analytical results using these functions are compared.

3.1 Proposal of new 2-dimensional analytical functions

In 2-dimensional analysis of a crack surface, the model is cut into cross sections from the 3-dimensional coordinates and divided into profiles, from which inclination and depth are calculated. The standard measurement area is an area of approximately 60㎜×60㎜ on the reference x-y plane. The direction of analysis can be set at random to the x-axis. Here, only two direc-

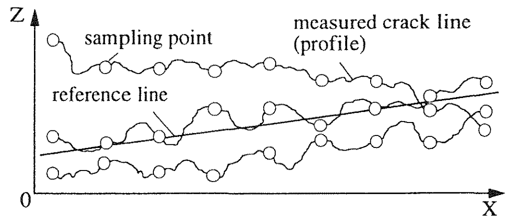

Fig. 4. Least-squares approximation of the reference line

tions are selected: the x-axis and y-axis directions. Fig. 4 shows the cross-sectional profiles cut in the x-axis direction from the 3-dimensional coordinate data of a measured crack surface. Sampling points are marked on each profile at regular intervals. In order to calculate the inclination and depth of a small piece of the crack surface, a line that serves as a reference (hereafter, called the reference line) is determined. The reference line is determined through linear approximation by applying the method of least squares to the sampling points in all cross-sectional profiles.

3.2 Modification of the incline density function Ω_S

The incline density distribution of concrete crack surfaces has already been presented by Li and Maekawa. Its function was named the contact density function. To this basic function, we introduced the concept of a reference line and carried out minor modifications. Li' s contact density function must be renamed the incline density function for shape analysis of concrete rough surfaces.

The element with two adjoining sampling points in Fig. 4 is assumed to be a small piece of the crack surface. The angle that this element makes with the x-axis of the measurement reference plane is denoted by $\bar{\theta}_{si}$ ($-\pi/2 \leqq \bar{\theta}_{si} \leqq \pi/2$), as shown in Fig. 5. This is corrected by the angle θ_0 between the reference line and the x-axis.

Thus, $\bar{\theta}_{si}$ becomes:

$$\bar{\theta}_{si} = tan^{-1}\{(z_{i+1} - z_i)/(x_{i+1} - x_i)\} \tag{1}$$

From Eq. 1, θ_{si} can be given by

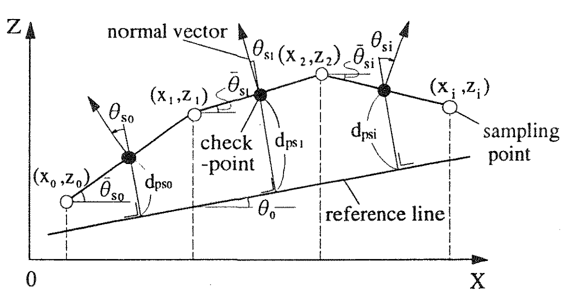

Fig. 5 Inclination angle and depth of a small piece of crack surface

$$\theta_{si} = \bar{\theta}_{si} - \theta_0 \tag{2}$$

The incline density function $\Omega_S(\theta_S)$ of a small piece of the concrete crack surface is in the integral interval of $-\pi/2$ to $\pi/2$.

Eq. 3 is obtained :

$$\int_{-\pi/2}^{\pi/2} \Omega_S(\theta_S) d\theta_S = 1 \tag{3}$$

The area of a small piece of crack surface is the width between the profiles of the measured crack surface, which are sliced at intervals equal to the sampling intervals, multiplied by the distances between adjoining sampling points. The total area of a small piece of crack surface is denoted as A_S^{suf}.

Ω_S then becomes a function that satisfies Eq. 4. It is the sum area corresponding to θ_S.

$$dA_{\theta_S} = A_S^{suf}\, \Omega_S(\theta_S) d\theta_S \tag{4}$$

3.3 Definitions of the depth density function K_S and the mean depth

The checkpoint for estimating the depth of a small piece of the concrete crack (d_{PS}) is placed in the middle point of the adjoining sampling points in Fig. 5. The distance between the checkpoint and the reference line is then calculated. If the checkpoint is above the reference line, d_{PS} is positive, and if the checkpoint is below the reference line, d_{PS} is negative. The minimum value d_S^{mxd} (hereafter, called the deepest value of the crack) and the maximum value d_S^{mxh} (hereafter, called the highest value of the crack) vary according to the crack surface being measured.

For consistency, the depth of a small piece of the crack surface, d_{PS}, is transformed to normalized depth, $\eta_S (-1 \leqq \eta_S \leqq 1)$, using Eq. 5.

$$\eta_s = 2\left(d_{ps} - d_S^{mxh}\right) / \left(d_S^{mxh} - d_S^{mxd}\right) + 1 \tag{5}$$

The depth density distribution K_S is defined by Eq. 6 using the normalized depth η_S.

$$\int_{-1}^{1} K_S(\eta_s) d\eta_s = 1 \tag{6}$$

The total surface area of the concrete crack surface, A_S^{suf}, can thus be written as

$$dA_{\eta_s} = A_S^{suf}\, K_s(\eta_s) d\eta_s \tag{7}$$

The area of a small piece of the crack surface corresponding to crack depth and normalized crack depth is described as $A_S(d_{PS})$ and $A_S(\eta_S)$. The mean crack depth d_S^{men} and mean normalized depth η_S^{men} can then be written as Eq .8 and Eq .9, respectively

$$d_s^{men} = \int_{d_s^{mxd}}^{d_s^{mxh}} |d_{ps}| A_s(d_{ps}) \mathrm{d}d_{ps} \Big/ \int_{d_s^{mxd}}^{d_s^{mxh}} A_s(d_{ps}) \mathrm{d}d_{ps} \tag{8}$$

$$\eta_s^{men} = \int_{-1}^{1} |\eta_s| A_s(\eta_s) d\eta_s \Big/ \int_{-1}^{1} A_s(\eta_s) d\eta_s \tag{9}$$

3.4 Definition of increased ratio of crack surface γ_s

When a crack is formed in concrete, the surface area of the crack increases more than the area projected onto the reference plane for recognizing the shape properties of the crack surface.

The increased ratio of crack surface (γ_s) is defined as the geometric value of the ratios of the total length of crack profiles and the total length of crack reference lines in the x-axis and y-axis directions. The ratio of these two lengths in the x-axis direction is calculated, as shown in Fig. 6. The sum of distances between sampling points on the cross-sectional profiles in the x-axis direction is divided by the length of the reference line corresponding to this. The sum of distances between sampling points is calculated by the curvilinear integral. The ratio in the y-axis direction is obtained by the same procedure.

The increased ratio of crack surface (γ_s) is given by

$$\gamma_s = \left(\int_{C_{cx}} S_x \mathrm{d}x \Big/ \int_{C_{bx}} \bar{S}_x \mathrm{d}x \right) \left(\int_{C_{cy}} S_y \mathrm{d}y \Big/ \int_{C_{by}} \bar{S}_y \mathrm{d}y \right) \tag{10}$$

4 Investigation of the effects that parameters of measurement and analysis have on the results of 2-dimensional analysis

In our measurement and analysis, two parameters may affect the analytical values. One is the coefficient of A/D transformation, i.e., the magnitude of the true length-pixel number ratio (measurement sensitivity is hereafter denoted by ㎜/dot) .The other is the length of the interval between two adjoining sampling points (hereafter, called sampling interval).

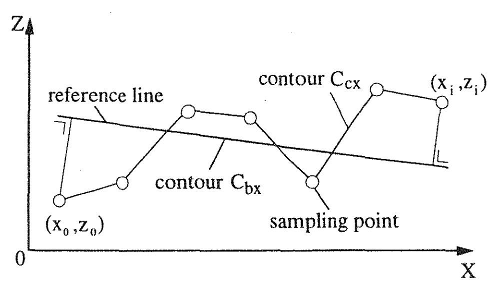

Fig. 6 Total length of crack profile and reference line

The only way to determine the optimum values of the measurement sensitivity and sampling interval is to compare the results of 2-dimensional analysis using various measurement sensitivities and sampling intervals as parameters.

Three steel specimens simulating a rough surface were made in this study. The reference measurement sensitivity value was obtained from 2-dimensional depth analysis of the steel specimens, and then the reference value was applied to the measurement and analysis of a concrete tensile crack.

4.1 The measurement sensitivity obtained from three steel specimens made to simulate rough surfaces

Depth analysis was performed on three steel specimens with three rough surfaces made by grooves, the inclination angle of which was set to $\pm \pi/4$, engraved to depths of 1.0mm, 1.5mm and 2.0mm. In the analyses, the sampling interval was kept constant at 1mm, and the measurement sensitivity was changed from 0.094mm/dot to 0.135mm/dot to 0.169mm/dot.

The results of depth analysis of the steel specimens show that if the measurement sensitivity is set to around 0.135mm/dot, we can obtain the shape of a rough surface of approximately 1.0mm in depth.

4.2 The measurement sensitivity and sampling interval for a concrete tensile crack

In this section an example of measurement and analysis of a "tensile crack" is presented. The tensile crack is acknowledged as the most basic crack surface shape due to the simplicity and uniformity of stress distribution. The effects of two parameters on the analytical results are investigated.

4.2.1 Measurement conditions

The measurement conditions are shown in Table 1. Based on the results of depth analysis of the steel specimens, the measurement sensitivity was changed from 0.114mm/dot to 0.125mm/dot to 0.159mm/dot. The measurement area in the case of measurement sensitivity of 0.114mm/dot was set at approximately 53mm×60mm. This area is slightly smaller than the prescribed rectangular area. For the other measurement sensitivities, the area was set at approximately 58mm×60mm.

In the analysis of steel specimens, the sampling interval was fixed at 1mm. However, in the analysis of the shape of a concrete tensile crack surface, the effects of changes not only in the measurement sensitivity but also in the sampling interval on the analytical values were examined, and the most appropriate parameters were determined.

The slit width was set at 0.2mm and the minimum value of sampling interval was set at 0.5mm. The sampling interval was changed from 0.5mm to 1.0mm

Table 1. Measurement conditions for the crack surface

angle of incident (rad.)	slit width (mm)	interval of laser beam (mm)	number of composited images
0.25π	0.2	0.5	121

to 3.0mm in both the x-axis and y-axis directions.

4.2.2 Outline of the concrete specimen used in the tensile experiment

Only one concrete specimen was used in the tensile experiment. The mix design of the concrete is shown in Table 2. The concrete properties are shown in Table 3. An outline of the specimen is presented in Fig. 7.

As shown in Fig. 7, the specimen is a 400mm long square prism, measuring 120mm × 120mm in cross section with a starter notch of 5mm in depth. D22 was embedded along the central axis of the prism. The D22 at both ends of the specimen was attached to an amsler-type testing machine. Tensile force was applied the specimen to introduce a crack by the testing machine. An almost perpendicular crack was formed in the specimen as shown Fig. 8.

4.2.3 Analytical results

In the analyses of shape properties of one tensile crack surface, the measurement sensitivity was changed from 0.114mm/dot to 0.125mm/dot to 0.159 mm/dot, and the sampling interval was changed from 0.5mm to 1.0mm to 3.0mm in both the x-axis and y-axis directions. All analyses were performed by the 2-dimensional procedures proposed in this paper. However, unfortunately all the analytical results of the concrete tensile crack surface can not be presented have due to space limitations. We only show the analytical results in the case of measurement sensitivity of 0.125mm/dot. The results of other cases, except γ_S, are described without figures and tables in this section.

Fig. 9 shows histograms of the incline density distribution (Ω_S) in which the measurement sensitivity was kept constant at 0.125mm/dot and the sampling interval was set to the three prescribed values of sampling interval. Fig. 10 shows the depth density distribution (K_S). Table 4 shows the ana-

Table 2. Mix design of concrete

cement	norminal strength	slump	maximum size of coarse aggregate	coarse aggregate	water cement ratio	sand-coarse aggregate ratio
normal	30N/mm^2	18cm	20mm	crushed stone	48.0%	47.7%

Table 3. Concrete properties

name	measured slump	compressive strength	splitting strength
S18-II	18cm	36.2N/mm^2	2.64N/mm^2

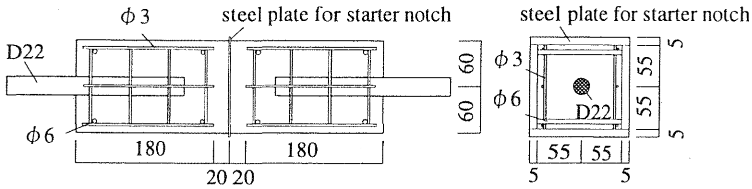

Fig. 7 Concrete specimen used in the tensile experiment

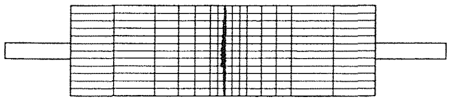

Fig. 8 Crack in concrete specimen used in the tensile experiment

lytical results of mean depth, the deepest value of the crack, the highest value of the crack, and mean normalized depth corresponding to each prescribed sampling interval. Table 5 shows the analytical results of the increased ratio (γ_s) for each setting of measurement sensitivity and sampling interval.

4.2.4 Investigation of the effect that measurement sensitivity has on the analytical results of shape properties of the crack surface

There is no great difference in the incline density distributions in the x-axis direction between measurement sensitivity 0.114mm/dot and 0.125mm/dot (Fig. 9). However, when the measurement sensitivity was reduced to 0.159 mm/dot, the inclination angle became smaller.

In the y-axis direction, the incline density values in the steep area setting of 0.114mm/dot are larger than those at other settings. This is thought to be

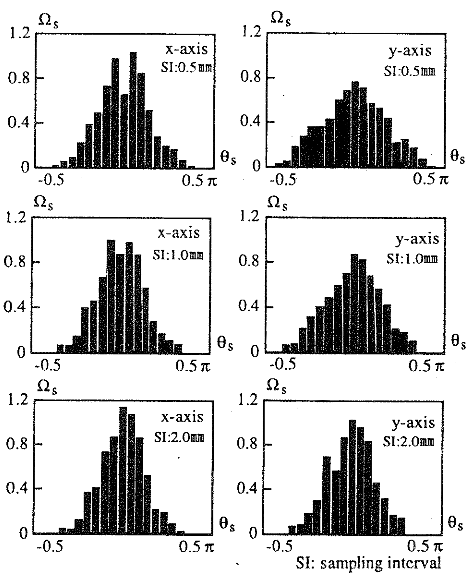

Fig. 9. Incline density (0.125mm/dot)

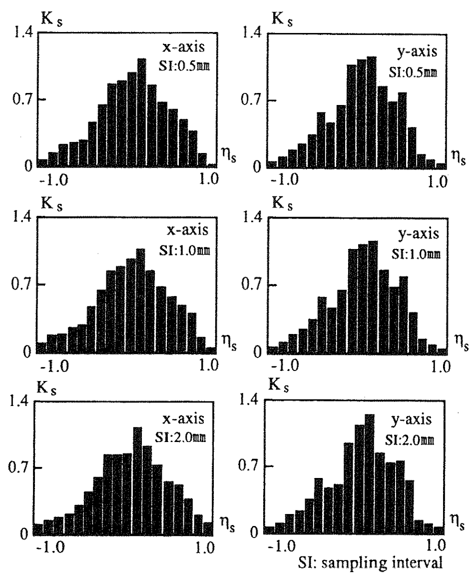

Fig. 10. Depth density (0.125mm/dot)

due to the difference in the measurement area.

A change in the measurement sensitivity did not greatly affect the depth density distribution K_S of a tensile crack surface in either direction. The tendency is shown in Fig. 10.

The analytical values of depth, mean depth, etc., in both directions are slightly higher at the measurement sensitivity of 0.114㎜/dot than at other sensitivities. There is only a small difference between the analytical values of depth at 0.125㎜/dot (Table 4) and at 0.159㎜/dot. The measurement sensitivity has almost no effect on the mean normalized depth. The results in Table 5 show that the increased ratio of crack surface (γ_s) at the highest measurement sensitivity is slightly higher than that at the lower measurement sensitivities.

The results of this study have shown that, if measurement sensitivity is set to about 0.125㎜/dot, the functions presented can be applied to the analysis of a concrete crack surface.

4.2.5 Effect of the sampling interval on the results of 2-dimensional analysis of the shape properties of a crack surface

At measurement sensitivities of 0.114㎜/dot and 0.125㎜/dot (Fig. 9), there is no large difference between the incline density distributions Ω_S at sampling intervals of 0.5㎜ and 1.0㎜ in both directions. However, at the sampling interval 2.0㎜, the steep area is reduced. When the measurement sensitivity is reduced to 0.159㎜/dot, Ω_S became flatter with gradual increases in the sampling interval.

As shown Fig. 10, the size of the sampling interval has almost no effect on the configuration of the depth density distribution K_S. Only K_S at a

Table 4. Results of depth analysis (0.125㎜/dot)

sampling interval (㎜)		0.5	1.0	2.0
x-axis	mean depth (㎜)	1.85	1.84	1.82
	deepest value of depth (㎜)	-6.26	-6.01	-5.90
	highest value of depth (㎜)	5.55	5.36	5.09
	mean normalized depth	0.32	0.33	0.34
y-axis	mean depth (㎜)	1.82	1.81	1.80
	deepest value of depth (㎜)	-6.64	-6.44	-6.41
	highest value of depth (㎜)	5.89	5.68	5.45
	mean normalized depth	0.30	0.31	0.32

Table 5. Increased ratio of crack surface in the tensile experiment

sampling interval (㎜)		0.5	1.0	2.0
measurement sensitvity	0.114 (㎜/dot)	1.391	1.309	1.242
	0.125 (㎜/dot)	1.273	1.217	1.162
	0.159 (㎜/dot)	1.292	1.217	1.171

sampling interval of 2.0mm was different to K_S at other intervals near zero normalized depth.

In Table 4(0.125mm/dot), there is almost no change in the overall behavior of depth values with changes in the sampling interval. The tendency of the analytical results at measurement sensitivity 0.125mm/dot is analogous to the tendencies at other measurement sensitivity values.

In Table 7, the difference between increased ratios of crack surface at sampling intervals of 0.5mm and 1.0mm is very small, indicating that a sufficient level of accuracy can be obtained if a sampling interval of 1.0mm is used.

5 Conclusions

We developed a new system to measure a concrete crack surface, utilizing the structured light projection method. We also proposed several new 2-dimensional analytical functions to describe the shape properties of a crack surface. This new measurement-analytical procedure was applied to steel specimens and one concrete tensile crack surface. The following is a summary of the results.

1) The 2-dimensional analytical values of the shape properties of a concrete crack surface can be obtained if the measurement sensitivity is set to about 0.125mm/dot.

2) The 2-dimensional analytical values of sufficiently high accuracy can be obtained if the sampling interval is set to about 1mm.

The 2-dimensional analytical method presented in this paper can easily be developed into a 3-dimensional analytical method, which could be used for 3-dimensional analysis of the shape properties of concrete crack surfaces.

We are currently experimenting with a 3-dimensional analytical method, and the results of measurement and analysis using this method will be reported in our next paper.

6 References

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