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Damage and Strain Softening of Concrete under Uniaxial Tension

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Abstract
Damage mechanics is used to investigate the concrete subjected to uniaxial tensile loading. Three mixtures of concrete specimens are tested to establish the relate damage equations. Moreover, a rather simple strain softening model of concrete, containing an adjustable material parameter, is deduced. It is found that the calculated results based on the proposed model are in remarkably good agreement with the sample experimental data.
Key words: damage, strain softening, uniaxial tension

1 Introduction

Since 1980s, damage theories have been applying to the field of concrete, especially for the study of concrete materials under tension. Damage mechanics provides an average measure of material degradation due to microcracking, interfacial debonding, nucleation and coalescence of voids.

This material degradation is reflected in the nonlinear load deformation behaviour of the structure, which can be described in the stress-strain relationships for concrete. Up to now, although many damage models have been proposed, and have been used with some success to describe the damage in concrete (Loland 1980, Ortiz 1985, and Mazars 1986), these models are lack of continuity for the damage variable. The reason is that the effective stresses defined by those models is first order similarity and is not superpositionable. In order to deal with this problem, we suggest an exponential model for the damage based on the uniaxial tensile test of concrete. Moreover, with the proposed damage model and under the border conditions, a simple strain softening formulation is derived and discussed with the experimental results of other researchers.

2 Experimental procedures

Three different mixtures (which denoted as 1, 2, and 3) are used to product the concrete specimens, as shown in Table 1. The materials used are portland cement, river sand and crushed aggregate (d_{max} = 15 mm).

Table 1. Mixture proportions of concrete specimens

Series	Mixture proportions			
	Cement	Water	Sand	Aggregate
DEN-1	1.00	0.75	2.85	4.11
DEN-2	1.00	0.60	2.37	3.87
DEN-3	1.00	0.50	2.07	3.25

In order to perform a direct tension test, concrete prisms were cast with deformed bars embedded in the specimen (see Fig. 1) for the purpose of loading. The specimen geometry and other specifications are given below.

Specimen prism: 100 mm × 100 mm in the section 300 mm length
Steel bars: 20 mm in the diameter 120 mm length
2 bars in each of specimen
Strain gauges: 25 mm length.

Since there was a triangular notch of 20 mm depth on both side of the specimen, the actual effective ligament section was 100 mm × 60 mm. The servohydraulic testing machine is connected to a computer through an A/D interface for data acquisition and signal generation. The axial strains of concrete are measured by the strain gauges placed on the lateral

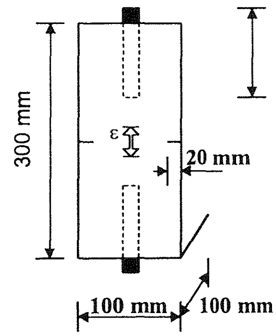


Fig. 1. Geometry of specimen.

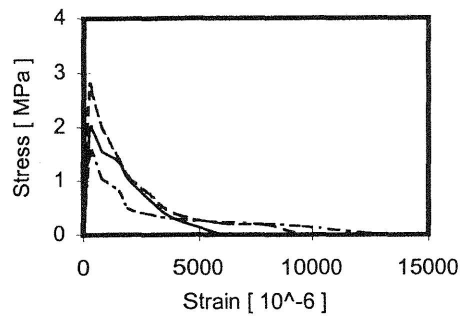


Fig. 2. Typical stress-strain curves of concrete in tension.

surfaces of the specimen. All specimens are tested under strain control mode, with the rate of 20 με /min. The main test results are shown in Table 2. Fig. 2 shows typical whole stress-strain curves obtained from the experiments.

Table 2. Main test results of concrete under tension

Series	Compressive strength (MPa)	Elastic modulus (GPa)	Tensile strength (MPa)	Strain to peak load (10^{-6})	Maximum displacement (mm)	Fracture Energy (N/m)
DEN-1	18.2	14.5	1.58	164	0.315	111.8
DEN-2	26.3	20.8	2.05	158	0.151	117.4
DEN-3	38.6	24.9	2.81	135	0.240	142.1

3 Analysis of Damage Mechanics

As Broberg (1974) defined, the damage can be expressed as follows:

$$D = \ln(A_0 / A_{eff}) \tag{1}$$

where D is damage variable, A_0 is initial area, and A_{eff} is effective area. According to the strain equivalence principle (Lemaitre 1992), we obtain

$$D = \ln(E_0 / \tilde{E}) \tag{2}$$

where E_0 is initial elastic modulus, $\tilde{E}$ is the effective elastic modulus.

From Eq. (2), we know that

$$\sigma = \tilde{\sigma} \exp(-D) \tag{3}$$

According to the relationship of $\sigma = \tilde{E}\varepsilon$ and Eq. (2), Eq. (3) can be expressed as follows

$$\sigma = E_0 \varepsilon \exp(-D) \tag{4}$$

Combining Eq. (3) and (4), we obtain

$$\tilde{\sigma} = E_0 \varepsilon \tag{5}$$

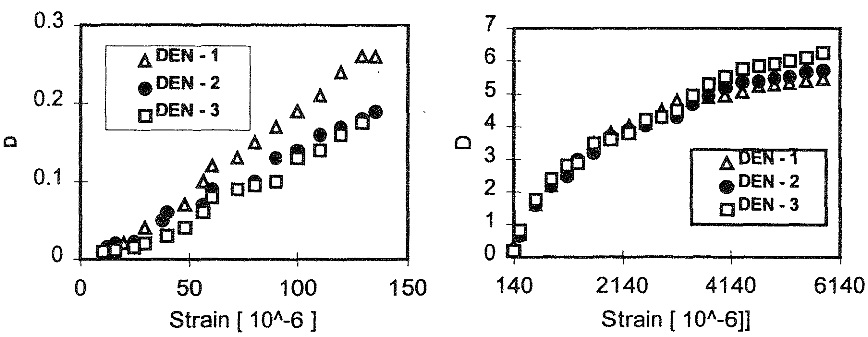
Analysis for the damage before and after peak load in the experiments is shown in Fig. 3.

According to the results of Fig. 3, the damage equations are regressed as follows

before peak load ($\varepsilon \le \varepsilon_p$): $D = C_1\, \varepsilon + D_0$ (6a)

after peak load ($\varepsilon > \varepsilon_p$): $D = D(\varepsilon_p) + C_2(\varepsilon - \varepsilon_p) + \ln(\varepsilon / \varepsilon_p)$ (6b)

where D_0 is the initial damage, in the experimental analysis we regard it as zero, $D(\varepsilon_p)$ is the damage value when $\varepsilon = \varepsilon_p$, C_1 and C_2 are coefficients.



(a) (b)

Fig. 3. Damage D vs. concrete strain ε before (a) and after (b) peak load.

Eqs. (6a) and (6b) can be generally written as follows

$$D = D_0 + C_1[\varepsilon - <\varepsilon - \varepsilon_p>] + C_2 <\varepsilon - \varepsilon_p> + \ln(\varepsilon / (\varepsilon - <\varepsilon - \varepsilon_p>)) \qquad (7)$$

where <x> = 0 , when x ≤ 0; while <x> = x, when x > 0.

By substituting Eq. (6a) into Eq. (4), we obtain the expression of ascending branch of the whole stress- strain curve:

$$\sigma = E_0 \varepsilon \exp\,(-C_1 \varepsilon - D_0) \qquad (8a)$$

Meantime, by substituting Eq. (6b) into Eq. (4), we obtain the expression of descending branch of the whole stress-strain curve:

$$\sigma = E_0 \varepsilon \exp(-D(\varepsilon_p) - C_2(\varepsilon - \varepsilon_p) - \ln(\varepsilon / \varepsilon_p)) \qquad (8b)$$

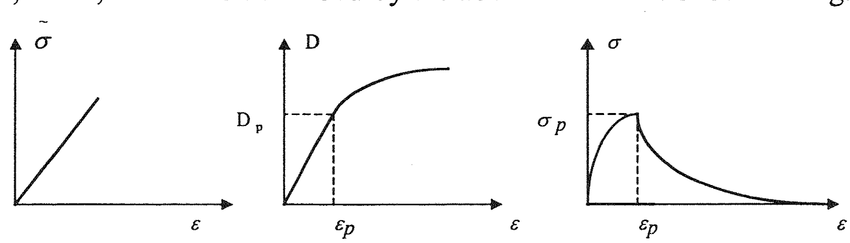
Eq. (8b) can be expressed as

$$\sigma = \sigma_p \exp(-C_2(\varepsilon - \varepsilon_p)) \qquad (9)$$

According to the boundary conditions, e.g. $\sigma\big|_{\varepsilon=\varepsilon_p} = \sigma_p$, and $l_c \int_{\varepsilon_p}^{\infty} \sigma d\varepsilon = G_F$, the coefficients C_1 and C_2 can be obtained:

$$\left.\begin{aligned} C_1 &= (\ln E_0 + \ln \varepsilon_p - \ln \sigma_p) / \varepsilon_p \\ C_2 &= \sigma_p l_c / G_F \end{aligned}\right\} \qquad (10)$$

where l_c is gauge length, G_F is fracture energy. The relationships between $\tilde{\sigma} \sim \varepsilon$, D ~ ε, and σ ~ ε described by the above model are shown in Fig. 4.



(a) (b) (c)

Fig. 4. Schematic illustrations of $\tilde{\sigma}$ vs. ε, D vs. ε, and σ vs. ε.

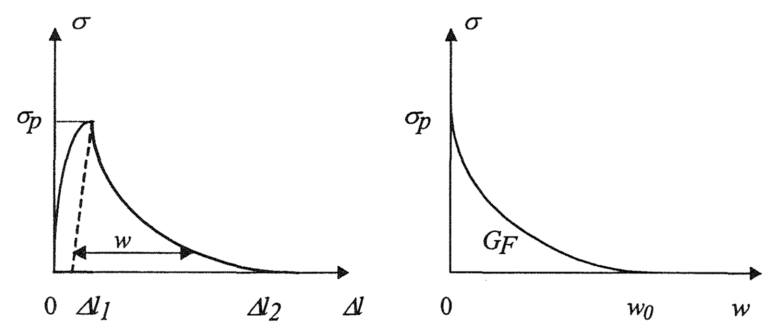
4 Strain softening model

In Fig. 5(a), the area under the complete stress-displacement curve represents the total consumed energy of concrete in the strain gauge length, which consist of the sum energy in the region of FPZ and its neighborhood. The energy consumed outside the FPZ, represented by the area on the left of the dot line in Fig. 5(a), is relatively smaller as compared to the total energy, while the area on the right of the dot line in Fig. 5(a) can be used to represented the fracture energy G_F. By calculated the supplement deformation w, we can directly obtain the strain softening curve, as shown in Fig. 5 (b).

From Fig. 5, we know that in the descending branch of the complete curve, the crack width can be simply expressed as $w = \Delta l_2 - \Delta l_1 \approx l_c(\varepsilon-\varepsilon_p)$ (where l_c is the gauge length, ε_p is the strain related to the peak load). By substituting the expressions into the Eq. (9), the expression of strain softening curve can be obtained:

$$\left.\begin{array}{l}\sigma = \sigma_p \exp(-kw) \quad \text{or} \\[2ex] \ln(\sigma / \sigma_p) = -kw\end{array}\right\} \tag{11}$$

where $k = C_2 / l_c = \sigma_p/G_F$.



(a) (b)

Fig. 5. Stress -deformation curve and strain softening curve.

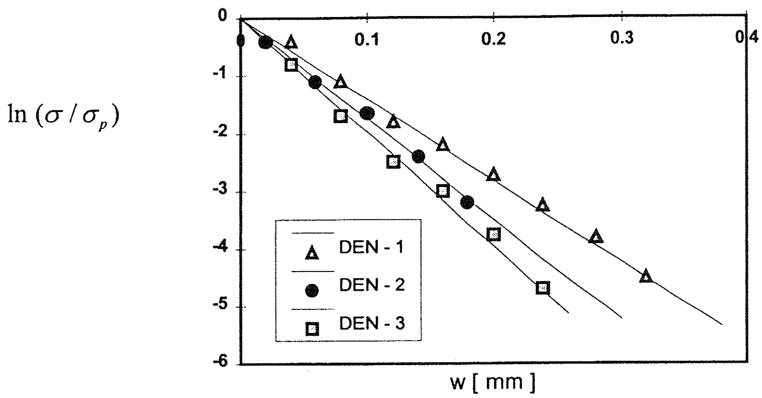


Fig. 6. Relationship of ln $(\sigma / \sigma_p) \sim w$.

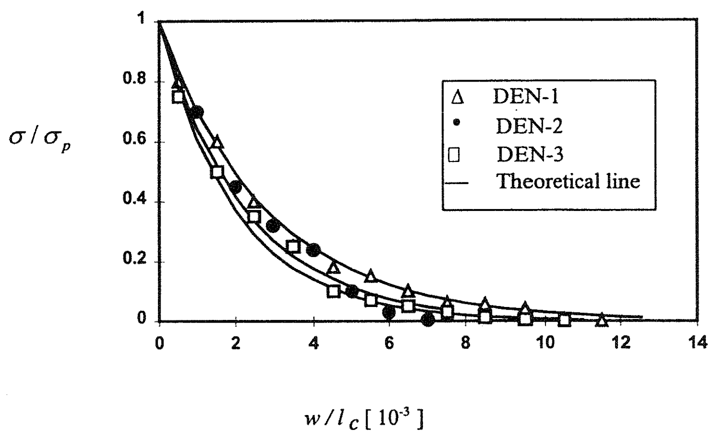


Fig. 7. Comparison between theoretical results and experimental data

The experimental results also show that there exist a good linear relationship between ln(σ / σ_p) and w, as shown in Fig. 6.

From the Eq. (11) and the results of Table 2, we can obtain the values of k for the three groups of specimens, and the relative strain softening curves. Fig. 7 compares theoretical predictions and experimental results, and all units are dimensionless for easy comparison. From the figure, we can see that the proposed strain softening curves are in good agreement with the experimental data. Moreover, in order to verify the popularity of proposed strain softening model, we apply it to the experimental results of Gopalaratnam (1985). The basic data are shown as follows:

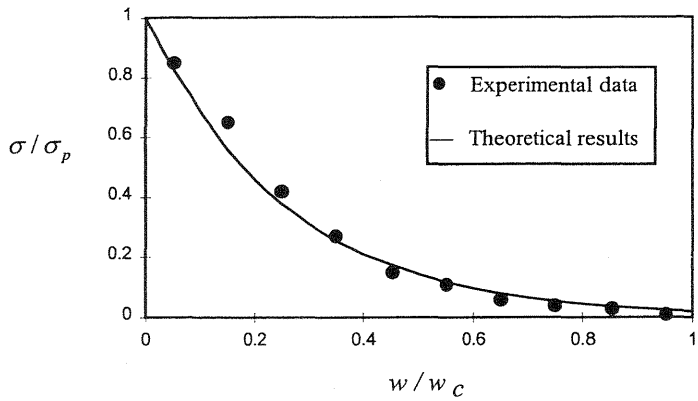


Fig. 8. Comparison between experimental data of Gopalaratnam and proposed model curves for the concrete.

Mixture :	C : S : A : W = 1 : 2.0 : 2.0 : 0.45
Size of specimen :	76.2 × mm × 19.0 mm × 305.0 mm
Gauge length :	12.7 mm
Peak stress :	3.62 MPa
Maximum crack width :	0.061 mm
Fracture energy :	56.4 N/m.

Thus, we can obtain the value of $k = 64.18$ mm^{-1}. By substituting it into Eq. (10), the strain softening relationship can be obtained

$$\sigma = \sigma_p \exp(-64.18\, w) \tag{12}$$

By comparing the calculated results of Eq. (12) to the experimental data of Gopalaratnam (1985), as shown in Fig. 8, we can find that the two results are in good agreement, indicating that the proposed strain softening model based on the damage theory, containing a variable parameter, being general and commonly applicable.

5 Conclusion

Based on the damage theory, the damage process of concrete under uniaxial tension is analyzed, relative damage equations are derived. They overcome the disadvantage of lacking of superposition for the previous damage variable. Moreover, the strain softening model is proposed, which is not only simple and accurate, but also general and commonly applicable.

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