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NUMERICAL EXPERIMENTS AND CHARACTERISTICS OF THE NEW K_R-CURVE FOR THE COMPLETE FRACTURE PROCESS OF THREE-POINT BENDING BEAMS

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Abstract
The crack extension resistance is directly associated with the cohesive force on the fictitious crack. For determining the K_R-curve, an attempt is made to carry out numerical experiments to get the complete P-CMOD curves for standard three-point bending beams. It has been verified that the results obtained in numerical experiments agree well with those measured in experiments. Through changing f_c, f_t and E of concrete, five different concrete materials with the same size of beams are numerically investigated. Furthermore, three sizes of beams with the same concrete are investigated. Then, the K_R-curves are evaluated by the proposed analytical approach. It has been found that the K_R-curves depend on the properties of concrete and are independent of the sizes of specimens investigated. The K_R-curves evaluated by the new approach can be taken as a new criterion of crack propagation for quasi-brittle materials like concrete.
Key words: Numerical experiments, K_R-curve, three-point bending, cohesive force, fictitious crack.

1 Introduction

The basic principle of the proposed new approach is that the crack extension resistance is composed of two parts. One part is the inherent toughness of a material which resists the initial propagation of an initial crack under loading and is denoted with the symbol K_{Ic}^{ini}. The other part of the crack extension resistance is contributed by the cohesive force distributed on the fictitious crack during crack propagation. Therefore, it is a function of the cohesive force distribution f (σ), the tensile strength f_t of the material and the length of the propagating crack a. Then, the crack extension resistance can be expressed as following equation:

$$K_R(\Delta a) = K_{Ic}^{ini} + K^c(f_t, f(\sigma), a) \tag{1}$$

It can be seen that for getting the crack extension resistance for a considered problem, one can carry out highly refined computation like Tvergaard and Hutchinson (1992) or can gain complete analytical solutions for the complete fracture process corresponding to the several stages during loading. In this paper, the main attempt is focused on the later. Such an aim has been progressively reached through solving three specified crack problems which are the Griffith crack with fictitious crack zones and the semi-infinite crack with a fictitious crack zone in a semi-infinite plate (see Xu, Reinhardt and Eligehausen (1997), Xu and Reinhardt (1998)) as well as the infinite strip with a finite crack (see Xu and Reinhardt (1997b)). The latter can be used to get the crack extension resistance for three-point bending beams. Using the analytical solutions the K_R-curves can be got. One only needs to measure the P-CTOD (load vs. crack tip opening displacement) curves for both problems of the Griffith crack and semi-infinite crack in a semi-infinite and the P-CMOD (crack mouth opening displacement) curves from standard three-point bending notched beams.

2 Numerical experiments on standard three-point bending beams

The main aim of numerical experiments is to gain complete P-CMOD curves for standard three-point bending notched beams of concrete. Five concrete materials were specified for a single size of the three-point bending beams by the compressive strength f_c, tensile strength f_t and Young's modulus E in the computation. Three sizes of the three-point bending beams made of the same concrete material were numerically investigated. For comparing the results achieved in numerical experiments with those measured in physical experiments, the properties of concrete materials and the basic sizes of the beams are those used by Karihaloo and Nallathambi in 1991. The detailed properties of the concretes and the beam sizes are presented in Table 1.

Table 1. Properties of materials and sizes of the beams used in the numerical experiments (from Karihaloo and Nallathambi, 1991)

Specimen or concrete mix	Specimen size SxDxB (mm)	a_0/D	Compressive strength f_c (MPa)	Elasitic Modulus E (GPa)	Tensile strength f_t (MPa)	Maximum size of aggregate d_{max} (mm)
Smaller beam	800x200x80	0.3	26.8	24.62	2.58	20
Middle beam	1200x300x120	0.3	26.8	24.62	2.58	20
Larger beam	1600x400x160	0.3	26.8	24.62	2.58	20
Concrete C1	800x200x80	0.3	26.8	24.62	2.58	20
Concrete C2	800x200x80	0.3	39.0	33.80	3.11	20
Concrete C3	800x200x80	0.3	49.4	34.65	3.50	20
Concrete C4	800x200x80	0.3	67.5	37.20	4.09	20
Concrete C4	800x200x80	0.3	78.2	40.30	4.41	20

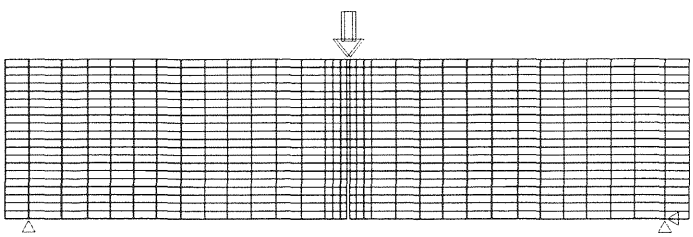

Fig. 1. Finite element mesh used in the computation

The element mesh used is shown in Fig. 1. The numerical experiments were carried out on a PC using a commercial computer program developed by Cervenka and Pukl (1992) which is based on the crack band model proposed by Bazant et al.(1983). For simulating a complete fracture process in a three-point bending notched beam, the CPU time was about 14 hours on a normal 486 PC computer in that time.

According to the main aim of investigation, the complete fracture process for each three-point bending notched beam was simulated in the numerical experiments to gain the complete P-CMOD curves. The P-CMOD curves simulated for the five specified concrete materials with the same size of the beams are plotted in Fig. 2 and those for the three different sizes of the beams with a same concrete in Fig. 3. The dead weight of the beams was considered in the computation using the data given by Karihaloo and Nallathambi(1991).

Each P-CMOD curve obtained in the numerical experiments can be considered as an average result of each group of specimens tested by Karihaloo and Nallathambi (1991). That is why the average properties of the concrete materials are used in the computation. As a comparison, the values of maximum load P_{max} obtained in the numerical experiments and those measured in the tests for the specified five concrete materials with the same beam size are plotted in Fig. 4. It can be seen that both results of the maximum load P_{max} obtained in the numerical experiments and measured in the tests agree very well.

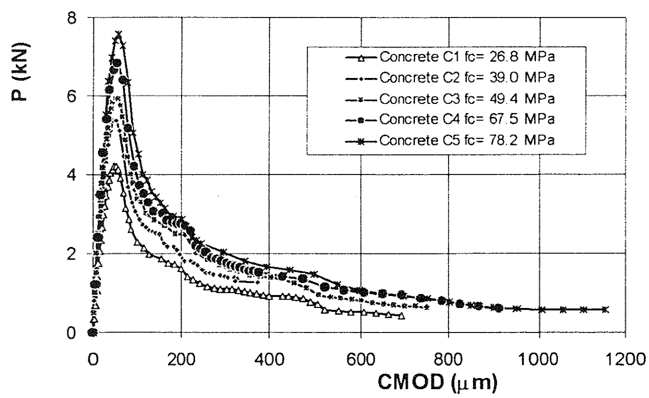

Fig. 2. The P-CMOD curves simulated in the numerical experiments for five concretes with the same size of the beams

Besides presentation of the values of maximum load P_{max} measured in the tests, a complete P-CMOD curve was presented in the literature too. The complete P-CMOD curve measured on the specimen C1-1 is compared with that simulated in the numerical experiments. Fig. 5 shows that both curves are in good agreement.

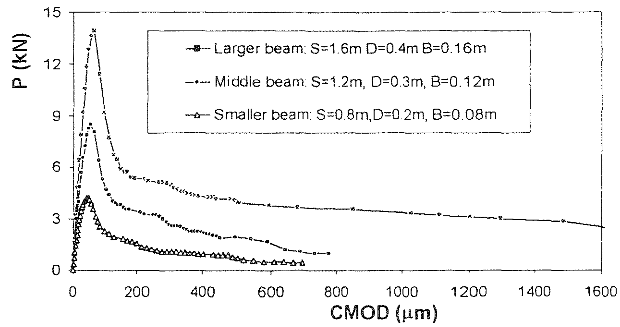

Fig. 3. P-CMOD curves from numerical experiments for three different beam sizes and same concrete

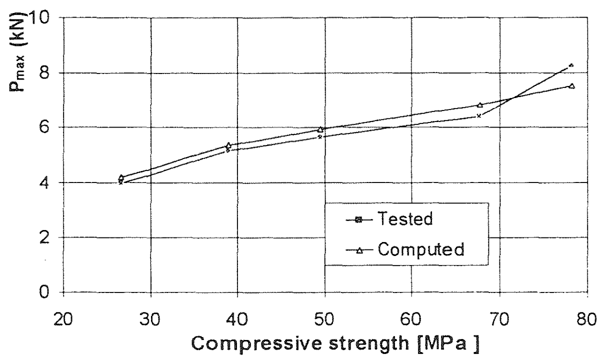

Fig. 4. The comparison between the values of P_{max} simulated in the numerical experiments and those measured in the tests by Karihaloo and Nallathambi (1991) for the five concretes

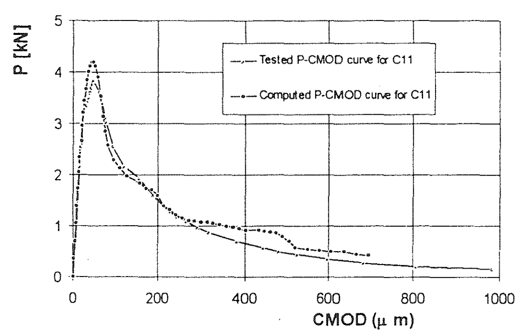

Fig. 5. Comparison between the complete P-CMOD curve from numerical experiments and tests by Karihaloo and Nallathambi (1991)

3 The proposed approach to evaluate the crack extension resistance associated with the cohesive force on the fictitious crack zone

Three typical crack lengths during the fracture process are defined so that the four different situations of crack propagation during subsequent loading stages can be distinguished. The three typical crack lengths are demonstrated in Fig. 6. a_0 is the initial length of the preformed crack. a_c is a critical crack length. At the critical situation the load arrives at its maximum value and CTOD achieves its critical value $CTOD_c$ too. a_{w_0} is a specifically characterized crack length of which the difference a_{w_0} - a_0 is the maximum length of a fictitious crack zone in a loaded body. In other words, a complete distribution of the cohesive force to meet the traction-separation law will appear in the region of a_{w_0} - a_0 , i.e. at the position of $x = a_{w_0}$ where the cohesive force is equal to the tensile strength f_t and at the position of $x = a_0$ the cohesive force is zero. Once the crack extension exceeds the length of a_{w_0} - a_0, a new stress-free crack will be formed. w_0 denotes the stress-free crack width in the softening traction-separation law.

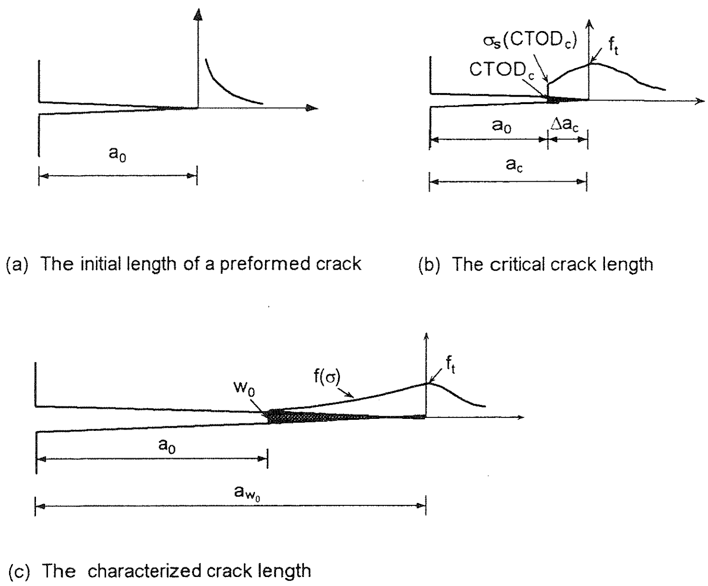

Fig. 6. Three typical lengths of the crack during fracture propagation

For the three-point bending beam, the general expression of the crack extension resistance is given as follows:

$$K_R(\Delta a) = K_{Ic}^{ini} + \int_{a_0}^{a} 2\sigma(x) F_1(\frac{x}{a}, \frac{a}{D}) / \sqrt{\pi a}\, dx \tag{2}$$

where

$$F_1(\frac{x}{a}, \frac{a}{D}) = \frac{3.52(1 - x/a)}{(1-a/D)^{3/2}} - \frac{4.35 - 5.28x/a}{(1-a/D)^{1/2}} + \left\{ \frac{1.30 - 0.30(x/a)^{3/2}}{\sqrt{1-(x/a)^2}} + 0.83 - 1.76\frac{x}{a} \right\} \left\{ 1 - (1 - \frac{x}{a})\frac{a}{D} \right\} \tag{3}$$

Now, one only needs to give the distribution functions of the cohesive force along the fictitious crack zone corresponding to the four different loading stages in the sequel. Then, insert them into the equation (2), the detailed expressions of K_R-curves for a complete fracture process in a three-point bending test can be gained (see Xu and Reinhardt (1997b). Specially, case 2 presented in the following was solved by Jenq and Shah (1985) and resolved by Xu and Reinhardt (1997a).

According to the four different stages of crack propagation, the distribution functions of the cohesive force along the fictitious crack zone are presented as follows:

(a) Case 1: $a = a_0$

$$\sigma(x) = 0 \quad \text{for } a = a_0 \tag{4}$$

(b) Case 2: $a_0 \le a \le a_c$

$$\sigma(x) = \sigma(w) + (f_t - \sigma(w))(x - a_0)/(a - a_0) \qquad \text{for } a_0 \le x \le a \tag{5}$$

c) Case 3: $a_c \le a \le a_{w_0}$

$$\sigma(x) = \begin{cases} \sigma_1(x) = \sigma(w) + \left[\sigma(CTOD_c) - \sigma(w)\right](x - a_0)/\left[a - (a_0 + \Delta a_c)\right] & \text{for } a_0 \le x \le (a - \Delta a_c) \\ \sigma_2(x) = \sigma(CTOD_c) + \left[f_t - \sigma(CTOD_c)\right](x - a + \Delta a_c)/\Delta a_c & \text{for } (a - \Delta a_c) \le x \le a \end{cases} \tag{6}$$

d) Case 4: $a > a_{w_0}$

$$\sigma(x) = \begin{cases} \sigma_1(x) = 0 & \text{for } (a_0 \le x \le (a - a_{w_0} + a_0) \\ \sigma_2(x) = \sigma(CTOD_c)(x - a - a_0 + a_{w_0})/(a_{w_0} - a_c) & \text{for } (a - a_{w_0} + a_0) \le x \le (a - \Delta a_c) \\ \sigma_3(x) = \sigma(CTOD_c) + \left[f_t - \sigma(CTOD_c)\right](x - a + \Delta a_c)/\Delta a_c & \text{for } (a - \Delta a_c) \le x \le a \end{cases} \tag{7}$$

In equation (2) K_{Ic}^{ini} denotes the inherent initiation toughness of a material. In the above-mentioned expressions, $\Delta a_c = a_c - a_0$; w is the crack opening width at the tip of the preformed crack; σ (w) is the cohesive force at the point of the tip of the preformed crack which is determined by the traction-separation law proposed by Reinhardt et al. (1986). In order to evaluate K_R-curves, detailed procedures to calculate the propagating crack length a, the point-forces σ (w) and σ ($CTOD_c$) can be seen in the work of Xu and Reinhardt (1997b).

4 The characteristics of the crack extension resistance of softening quasi-brittle materials

Using the complete P-CMOD curves simulated for the five different concretes which were shown in Fig. 2, the crack extension resistance curves during the complete fracture process in the computed three-point bending beams with the same size were calculated and are presented in Fig. 7.

It can be seen that all K_R-curves depend on the compressive strength f_c of the concrete materials and have almost the same shape and developing trend with the increase of the related crack length a/D. Especially, one can observe a point of inflection on almost the same position on the horizontal axis. This point on the K_R-curves is at the position of about a/D = 0.415. Further investigation of this characteristic point has shown that it is the critical point of unstable crack propagation for the softening quasi-brittle material considered.

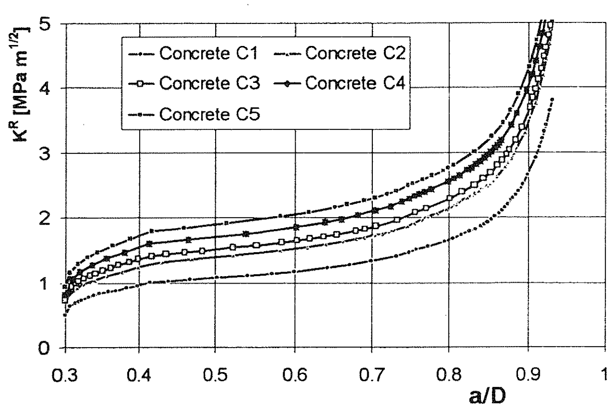

Fig. 7. The K_R-curves for different concretes

The K_R-curves gained for the three different sizes of three-point bending notched beams with the same concrete denoted by C1 were plotted in Fig. 8. It can be seen that when the material is the same and only the sizes of the specimens are different, the corresponding K_R-curves are almost independent of the sizes. Therefore, the K_R-curve as obtained according to the approach proposed in this paper can be taken as a criterion to describe the crack propagation in the complete fracture process in softening quasi-brittle materials.

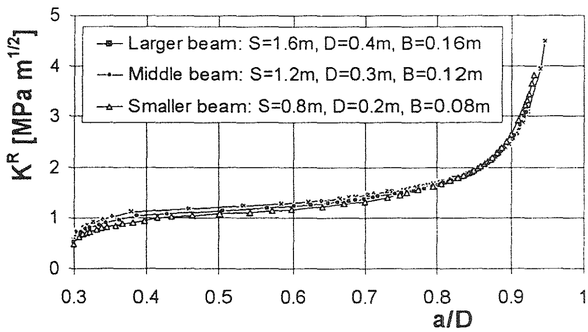

Fig. 8. K_R-curves of three beams of different size

5 Conclusions

The features of the crack extension resistance curve (K_R-curve) were investigated in this paper. According to the new approach proposed by the authors the crack extension resistance of a quasi-brittle material is composed of two parts. One part represents the inherent toughness which resists the initial propagation of an initial crack. The other part of it is contributed by the cohesive force distributed on the fictitious crack during crack propagation.

K_R-curves for different concretes and specimen sizes were obtained through combining numerical experiments and analytical solutions of the fictitious crack. The results show that the crack extension resistance curve is dependent on the properties of the materials and independent of the size of specimens. The effect of specimen geometry on the K_R-curve should be investigated in further work.

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