

Fracture Mechanics of Concrete Structures  
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## **COMPUTATIONAL MODELLING OF THE FIBRE-MATRIX BOND IN STEEL FIBRE REINFORCED CONCRETE**

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### **Abstract**

In this paper, a two dimensional lattice type model is used for simulating the load-displacement response of a notched beam made of Steel Fibre Reinforced Concrete (SFRC) and subjected to four-point bending. The material is modelled as a lattice of brittle breaking beam elements. According to a generated grain structure of the concrete, different material properties are assigned to the beam elements. Fibres are inserted in the mesh, surrounded by interfacial elements that allow for transfer of both normal and shear stresses. The stress in the elements is determined from a linear elastic analysis. Fracture is realised by removing those elements from the mesh, that exceed their tensile strength. The model is very attractive, given the small number of parameters needed. By varying the interfacial fibre-matrix strength and stiffness, a qualitative study of its influence on the mechanical behaviour of SFRC can be carried out. The results of the simulation for one set of parameters are given and compared with experiments.

Key words: fibre reinforced concrete, interface, lattice modelling

## 1 Introduction

The most general and effective way of imparting toughness to a brittle solid is to bridge any crack that propagates. Consequently, the addition of fibres in cementitious materials such as mortar and concrete, compensates their inherent weakness in resisting tensile stresses, Hannant (1976). The principal beneficial effects of the fibres occur after the development of the first major crack, when fibres prevent unstable crack propagation by bridging the crack and by restraining it from opening, Shah et al. (1995). Of prime importance is the interface between fibre and matrix, that tends to be weak and thus is a preferential crack path. To prevent immediate pullout, fibres are generally mechanically anchored so that frictional dissipation along the debonded interfaces becomes the primary source of enhanced toughness, Cotterell & Mai (1996). It is thus the fibre-matrix bond which controls the toughness and overall performance, rather than the strength of the fibres themselves.

It is now well established that the structure of the paste in the vicinity of an inclusion, whether it is a fibre or an aggregate, is significantly different from that of the bulk paste, in terms of morphology, composition and density, Bentur et al. (1985). During the last several years, many efforts have been made to improve the structure of the fibre-matrix interface and thus increase the bond strength and overall properties of the composite materials, i.g. Maage (1977), Wei et al. (1986), Njam et al. (1994), Fu & Chung (1996). Nevertheless, beyond a certain point, an increase in the bond strength leads to a decrease in toughness, as the fibres tend to break rather than to pull out, Mindess (1989). Therefore, the need of a theoretical approach of the problem through numerical simulation is undeniable for a better understanding of the influence of the interfacial bond strength on the overall behaviour of the composite.

The models describing the global response of a cement-based fibre composite can be divided into two main groups: one dealing with fibre composites at the fibre matrix (or constituent) level, which integrates the micro-mechanical properties of the components, see i.g. Siah et al. (1992), Ouyang et al. (1994), Murat et al. (1992), and the other group that considers constitutive relations and rules to model fibre composites as an isotropic or anisotropic material, see i.g. Murugappan et al. (1994), Christensen et al. (1997). The model used in the present investigation belongs to the first group. It is based on a simple numerical lattice model.

## 2 Outline of the lattice model

### 2.1 Model for plain concrete

Lattice modelling of concrete fracture has been developed and used in many configurations and shown to be highly effective in simulating crack growth with simplicity in assumptions and computations, Van Mier (1997). Since a detailed description of this model has been extensively described in various papers, Van Mier et al. (1995), only a brief review of the model is given here.

In the lattice model, the material is discretized in a triangular mesh of brittle breaking beam elements. This mesh may be regular or random, Fig. 1. As such, a directional bias is present in the regular triangular lattice and, to a lesser degree, in the random triangular lattice. This bias can be partly overcome by implementing the heterogeneity of the concrete. The lattice is therefore projected on top of a generated grain structure and different strengths and stiffnesses are assigned to the lattice beams according to their position with regard to this grain structure. When the two nodes of the beam are inside a "grain", the beam is assigned aggregate properties; beams having one node inside and one node outside a "grain" are assigned bond properties, while all other beams are assigned matrix properties, Fig. 2.

The advantage of lattice models is that, especially when the microstructure of the material is included in the model, the fracture law can become very simple, Vervuurt (1997).

In the simulations of laboratory scale specimens, only the part of the mesh where cracks are expected to grow is modelled with the lattice. The elements used for this part are linear, two-node beam elements, whereas for the remaining part of the specimen, isoparametric four-node plane stress elements are used.

The simulation is run under a unitary external load and fracturing of the material takes place by removing at each step the beam element with the highest stress relative to its strength. As the problem is linear elastic, all values are relative and the actual fracturing load can be found afterwards through the stress-over-strength ratio. After a beam has been fractured, simply a new linear elastic analysis is carried out using the reduced lattice. The analyses are performed with a standard finite element code (DIANA).

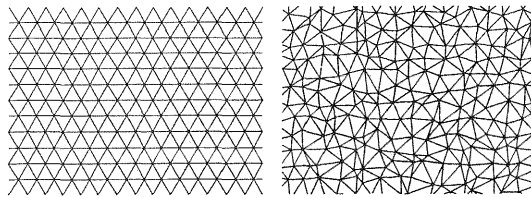


Fig.1. Regular or random lattice

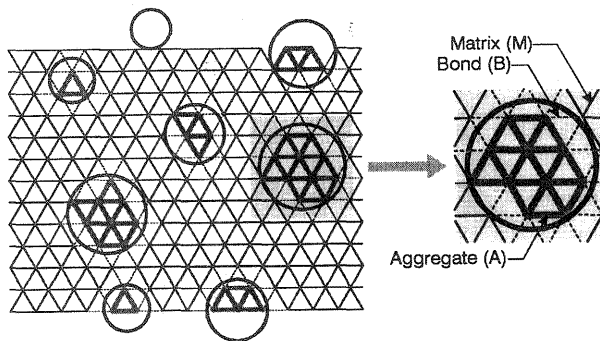


Fig.2. Implementing the heterogeneity

## 2.2 Model for Fibre Reinforced Concrete

To model the fracture behaviour of fibre reinforced concrete, some changes have to be introduced in the general lattice model, without altering the global philosophy of the model. A linear elastic analysis should therefore suffice to determine at each step which element should be removed, without introducing iterations. This is of course a rough simplification for a non-linear problem as fibre-pullout, but it has the advantage of keeping things simple.

It was thus decided to link the fibres to the original mesh through interfacial elements, of zero length, allowing transfer of both normal and shear stresses, Fig. 3, and having a simple fracture law as depicted on Fig. 4. Five new parameters appear, i.e. the normal strength  $f_{t_n}$ , the shear strength  $f_{t_s}$ , the normal stiffness  $k_n$ , the shear stiffness  $k_t$  and the residual strength  $f_r$ .

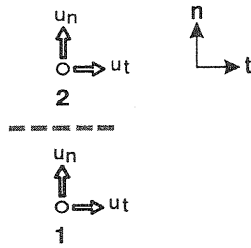


Fig. 3. Interfacial element

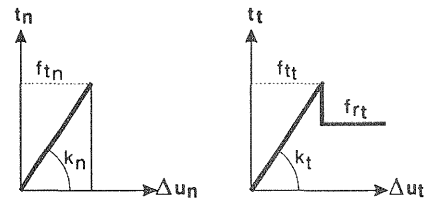


Fig. 4. Interfacial fracture law

As to model the fibres, two-node truss elements are used, which break once their constant axial stress overcomes the strength. If a random mesh is used, the nodes of the mesh at height of the fibres must be rearranged in a straight line, before they can be linked to the fibres. A mechanically anchoring of the fibres can be modelled by changing the value of the interfacial parameters over the length of one fibre.

Again, simulation is started under a unitary external load and at each step, the element with the highest stress-over-strength factor is determined from a linear elastic analysis. If this element is a beam or truss element, fracturing of the material is realised by removing the cited element. If the critical element is an interface element, two options are possible. Has the strength been exceeded in normal direction, then the corresponding stiffness is set to zero, whereas in shear direction, the stiffness is set to zero and nodal forces are introduced to model friction. The value of these forces are tuned by the residual strength, that can be from 0 to 100% of the shear strength. The model considers a constant friction, overruling as such pullout of the fibres. The value of this friction is absolute and depends on the external load, which must be adapted before running each next step.

The present model does not admit any restoration of the interface, i.e. a better bond due to friction or closing-up of a crack. Once the bond has failed (by reducing its stiffness to zero), it cannot be restored. This means that the model is not valid if traction zones develop into compression zones.

### 3 Problem and Model Parameters

#### 3.1 Physical experiment

Four-point bend tests are carried out on Steel Fibre Reinforced Concrete beams. The beams have a dimension of  $600 \times 150 \times 150 \text{ mm}^3$  and their central section is reduced by a 30 mm notch and two semi-circular

grooves of 12 mm diameter, Fig. 5. To facilitate simulation, fibres are aligned along the beam length instead of randomly distributed. The way of doing this is explained in Fig 6 : concrete is poured in layers and between each layer, fibres are placed in the plane of the notch. In the example treated below, the beam is composed of eleven layers of 49 fibres each. Hooked steel fibres (N.V. Bekaert) having a length of 30 mm and a diameter of 0.5 mm, are used. The concrete's compressive strength amounts to 42 MPa, whereas its Young's modulus is of about 36 GPa.

### 3.2 Model parameters

The advantage of the lattice model is that it is a simple and transparent model. Only a few, single valued parameters are required for the part of the specimen that is modelled with the lattice. These parameters can be divided in two groups: i.e. parameters according to the global elastic behaviour of the mesh and parameters needed in the fracture law.

To describe the elastic behaviour of the complete lattice, the Young's modulus ( $E=36$  GPa) and Poisson's ratio ( $\nu=0.2$ ) of the modelled material are available as input. The size and modulus of the beam elements must be adjusted such that the elastic stiffness and Poisson's ratio of the complete lattice resemble those values of the continuum. The way of doing this has been explained in detail in Vervuurt (1997).

The parameters which have to be determined for the fracture law of the beam elements are the bending coefficient  $\alpha$ , the scaling factor  $\beta$  and the strength of the different types of beam elements (aggregate, matrix and bond), Vervuurt (1997). All but the factor  $\beta$  have been chosen similar as in earlier work, Chiaia et al. (1997). As for  $\beta$ , it can be determined from the globally measured peak stress during the experiments, Schlangen & Van Mier (1992).

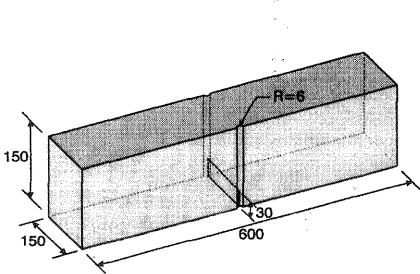


Fig. 5. Beam geometry

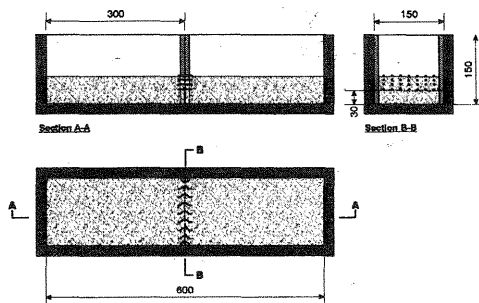


Fig. 6. Fibres placed in the beam

The addition of fibres yields eight more parameters, i.e. the cross-section and the Young's modulus of the truss elements modelling the fibres, which are chosen equal to their physical values, the strength of the fibres and the five parameters describing the fracture law of the interfacial elements. The latter may be tuned over the length of a fibre to model i.g. the hooks of the fibres.

In order to reproduce the load-transfer from concrete to fibres as closely as possible, the concrete and the steel fibres should be modelled with a high degree of accuracy. Nevertheless, a finer model leads to more elements and a longer computational effort. In order to maintain reasonable calculation times, the following example is based on a random lattice of grid size  $s = 3$  mm and randomness  $\gamma = 0.75s$ , see Vervuurt (1997).

The plane stress elements modelling the remainder part of the specimen are described by their modulus, Poison's ratio and thickness.

#### 4 Results and Discussion

Fig. 7 compares the load-deflection plots from an experiment and a numerical simulation using the parameters mentioned in Table 1. Fig. 8 shows a picture of a broken specimen, whereas in Fig. 9 the crack-path of the simulation is depicted.

Table 1. Values of the model parameters

lattice				fibre and interfaces			
thickness	t	[mm]	138 or 150	section	A	[mm <sup>2</sup> ]	0.2
height	h	[mm]	1.72	normal strength	$f_{t_n}$	[MPa]	0.3125
aggr. modulus	$E_A$	[GPa]	137	normal stiffness	$k_n$	[N/mm <sup>3</sup> ]	$10^{12}$
matrix modulus	$E_M$	[GPa]	49	shear strength	$f_{t_s}$	[MPa]	0.3125
bond modulus	$E_B$	[GPa]	49	shear stiffness	$k_s$	[N/mm <sup>3</sup> ]	$10^{12}$
aggr. strength	$f_{t_A}$	[MPa]	10	residual strength	$f_{r_i}$	[MPa]	$3.125E^{-7}$
matrix strength	$f_{t_M}$	[MPa]	5	<b>continuum</b>			
bond strength	$f_{t_B}$	[MPa]	1.25	modulus	E	[GPa]	36
bending coeff.	$\alpha$		0.005	Poison's ratio	$\nu$		0.2
global scaling	$\beta$		5	thickness	t	[mm]	150

The results of the example are based on one set of interfacial-parameters, i.e. interfacial stiffness, strength and residual strength. This set of parameters seems to model adequately the crack-path of a physical experiment, cast with 11 layers of 49 fibres each. Despite the notch and grooves, the crack grew, both in the experiment and the model, around the fibres, in the plain concrete. This results in a major brittle drop of the load-deflection plot and a low residual strength of the sample, since only the first layers of fibres are effective in bridging further crack extension, whereas the upper layers take no part in the reinforcing process.

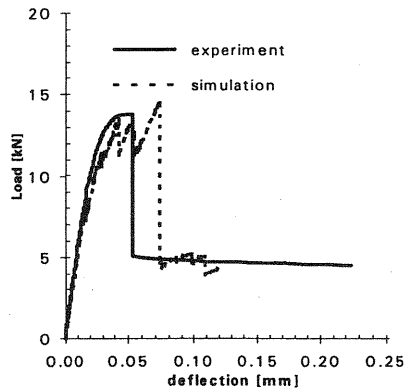


Fig. 7. Load-deflection plot from experiment and numerical simulation

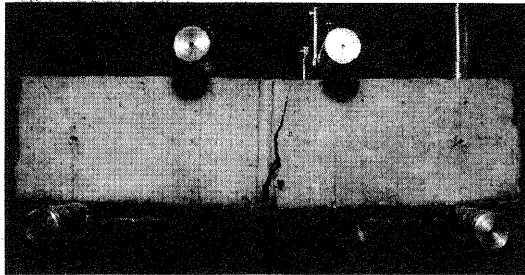


Fig. 8. Physical sample at the end of testing



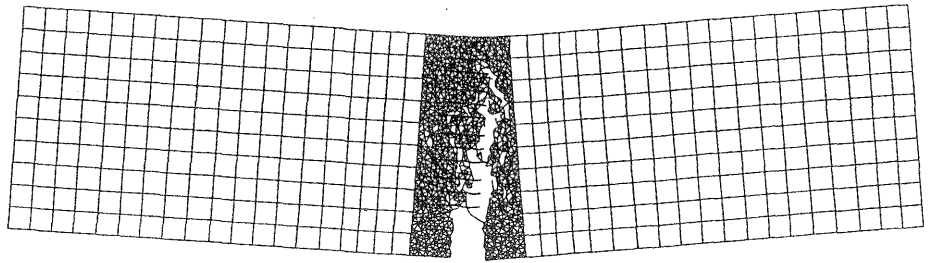


Fig. 9. Modelled sample at the end of simulation

In the experiment this was due to the fact that the central section was too strongly reinforced, so that the crack preferred to circumvent the fibres. This was rather easy given the short length of the fibres (30 mm). When less fibres are placed in each layer, the easiest crack-path goes through the fibres, resulting in a more ductile behaviour. Even less fibres lead again to a brittle behaviour, since the fibres are not effective and the crack progresses as through plain concrete. Because the model is two-dimensional, such variations in the thickness cannot be simulated. The different types of behaviour are to be modelled by varying the interfacial parameters.

Nevertheless, the purpose is not as much to compare the simulations with the experiments, but rather to understand the influence of the interfacial bond on the global behaviour. Indeed, it is known that the lattice model renders load-deflection plots which are too brittle compared to reality, Schlangen & Van Mier (1993). One possible reason for the observed brittle behaviour is the fact that the smallest particles (smaller than the grid size) are not included in the simulation. Moreover, the present model is not valid when the crack enters the compression zone at the upper part of the beam, where other failure mechanisms are pertinent. But at that time, fibre pullout is the main activity to resist complete failure. It was already mentioned that the model can not simulate pullout, since a constant friction coefficient is needed. Therefore, if the results from simulations are to be compared with the experiments, only the first part of the load-deflection diagrams may be compared. Nevertheless, the model has the main advantage of starting each new simulation under exactly the same conditions, whereas experiments are bound to be influenced by unavoidable experimental scatter. By varying one interfacial parameter at a time, it becomes very easy to determine the influence of this parameter on the overall behaviour of the sample. Each parameter can also be varied along the fibres, so that the influence of mechanical anchorage can be examined as well.

## 5 Conclusions

A new model for simulating crack-growth in fibre reinforced concrete was presented. Some preliminary results for one set of parameters showed a good adequacy to simulate the load-deflection plots and crack-paths of experiments, thus allowing to better apprehend the effect of the fibre bond on the overall response of steel fibre reinforced concrete.

## 6 Acknowledgement

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## 7 References

- Bentur, A., Diamond, S., Mindess, S. (1985) The microstructure of the steel fibre-cement interface. **J. Mat. Sc.**, 20, 3610-3620.
- Chiaia, B., Vervuurt, A., Van Mier, J.G.M. (1997) Lattice model evaluation of progressive failure in disordered particle composites. **Engng. Frac. Mech.**, 57 (2/3), 301-318.
- Christensen, A., Brincker, R. (1997) Bond properties and cracking response of fibre reinforced concrete members with main reinforcement, in **Brittle Matrix composites 5** (eds. A.M. Brandt, V.C. Li and I.H. Marshall), Bigraf and Woodhead Publ., 589-598.
- Cotterell, B., Mai, Y.W. (1996) Fracture mechanics of cementitious materials. Chapman & Hall, London.
- Fu, X., Chung, D.D.L. (1996) Effect of polymer admixtures to cement on the bond strength and electrical contact resistivity between steel fiber and cement. **Cem. Concr. Res.**, 26(2), 189-194.
- Hannant, D.J. (1976) **Fibre cements and fibre concretes**. John Wiley & Sons, Chichester
- Maage, M. (1977) Steel fiber strengths in cement-based matrixes influenced by surface treatments. **Cem. Concr. Res.**, 7, 703-710.
- Mindess, S. (1989) Interfaces in Concrete, in **Materials Science of Concrete I** (ed. J. Skalny), The American Ceramic Society, Westerville, 163-180.
- Murat, M., Anholt, M., Wagner, H.D. (1992) Fracture behavior of short-fiber reinforced materials. **J. Mat. Res.**, 7(11), 3120-3131.

- Murugappan, K., Tan, K.H., Paramasivam, P. (1994) Finite element formulation for the analysis of reinforced fibrous concrete beams. **Finite El. Analysis Design**, 18, 67-74.
- Najm, H., Naaman, A.E., Chu, T.-J., Robertson, R.E. (1994) Effects of poly(vinyl alcohol) on fiber cement interfaces. Part I : bond stress-slip response. **Adv. Cem. Bas. Mat.**, 1, 115-121.
- Ouyang, C., Shah, S.P. (1994) Fracture Energy Approach for Predicting Cracking of Reinforced Concrete Tensile Members. **ACI Struct. J.**, 91(1), 69-78.
- Schlangen, E., Van Mier, J.G.M. (1992) Micromechanical Analysis of Fracture of Concrete. **Int. J. of Damage Mechanics**, 1, 435-454.
- Schlangen, E., Van Mier, J.G.M. (1993) Lattice model for simulating fracture of concrete, in **Numerical Models in Fracture Mechanics of Concrete** (ed. F.H. Wittmann), A. A. Balkema, Rotterdam, 195-206.
- Shah, S.P., Swartz, S.E., Ouyang, C. (1995) **Fracture Mechanics of Concrete**. John Wiley & Sons, New York.
- Siah, K., Mandel, J.A., Mousa, B.R. (1992) Micromechanical Finite Element Model for Fiber Reinforced Cementitious Materials. **ACI Mat. J.**, 89(3), 277-288.
- Van Mier, J.G.M., Schlangen, E., Vervuurt, A. (1995) Lattice type fracture models for concrete, in **Continuum Models for Materials with Microstructure** (ed. H.-B. Mühlhaus) John Wiley&Sons, 341-377.
- Van Mier, J.G.M. (1997) **Fracture Processes of Concrete**. CRC Press, Boca Raton.
- Vervuurt, A. (1997) **Interface Fracture in Concrete**. PhD thesis, Delft University of Technology.
- Wei, S., Mandel, J.A., Said, S. (1986) Study of the interface strength in steel fibre-reinforced cement-based composites. **ACI J.**, 597-605.