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ON THE INTERPRETATION OF BENDING TESTS ON FRC-MATERIALS

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Abstract

In the present paper the possibility of obtaining information about the stress-crack opening relationship for FRC-materials from standard three point bending tests is reviewed. A simple bi-linear modeling of typical stress-crack opening relationships is carried out. This stress-crack opening relationship is implemented in a non-linear hinge model for a three-point bending test specimen and a parametric study is carried out. In the conclusion recommendations for testing and interpretation of test data for FRC materials are given.
Keywords: Fiber reinforced concrete, tensile strength, stress-crack opening relationship, test method.

1 Introduction

Traditionally, fiber reinforced concrete (FRC) is characterized using three or four point bending tests based on standards such as ACI 544, ASTM C 1018, or corresponding Japanese standards, JSCE-SF 4. The interpretation of these tests is typically based on the so-called toughness index and used for comparative studies of different fiber types and fiber contents. Furthermore, semi-empirical design methods are based on results of bending tests and toughness index calculation.

Recently, however, constitutive models have been proposed for FRC-materials, (Casanova and Rossi 1997, Li, Stang and Krenchel 1993). Furthermore, these models have been applied in different structural models for beams, pipes and slabs on grade. These constitutive models - based on the stress-crack opening relationship of the fictitious crack model (FCM) originally suggested by Hillerborg (Hillerborg, Modeer and Petersson 1976) - calls for much more detailed information than the information that can readily be obtained from toughness testing or even traditional fracture mechanical interpretation of bending tests with notched specimens like the RILEM test method for determination of fracture energy.

In **plain concrete** the stress-crack opening relationship $\sigma_w(w)$ – where w is crack opening and σ_w is the bridging stress – is often characterized in terms of the area under the curve, known as the fracture energy, G_F:

$$G_F = \int_0^\infty \sigma_w(w)\,dw \tag{1}$$

together with a standard approximation of the true stress-crack opening curve. Note, furthermore, that the shape of the stress-crack opening curve (i.e. the stress-crack opening curve normalized with respect to the tensile strength) is fairly independent of the concrete type in question (Hordijk 1991, Stang and Aarre 1992).

Fiber Reinforced Concrete (FRC) has been developed with the specific aim of increasing the fracture energy of concrete rather than to increase stiffness and strength which is the typical situation in other fiber reinforced composite materials. This means that the same fracture mechanical framework known from conventional concrete can be used in FRC-materials as well. On the other hand, the shape of the stress-crack opening curve is complex and greatly influenced by the type and amount of fibers used (the pull-out of fibers being the primary mechanism behind the stress transfer). Furthermore, the shape is essential for understanding the mechanical behaviour of the material. Finally, the fibers are still carrying load across the crack even for very large crack openings which are usually not relevant in structural design situations. In the light of these observations it follows that the fracture energy G_F as defined above is not a very useful tool in the characterization of material toughness and that the stress-crack opening curve itself has to be referred to and used in design situations.

The necessary information can be obtained by the direct tension test with fixed boundaries. However, this test method is tedious, expensive and requires closed loop testing facilities - thus it cannot be expected to become a standard test method in connection with design recommandations. Therefore, recently, efforts have been made to investigate the possibilities of extracting information on the stress-crack opening relation from bending tests, (Nanakorn and Horii 1996, Kitsutaka 1997). It is the aim of the present paper to investigate the possibilities of solving this inverse problem and based on the finding of this investigation to discuss possible design related testing strategies for FRC.

2 The stress-crack opening relationship

The basic test to determine the stress-crack opening relationship for FRC materials is the uniaxial tensile tests using notched specimens under deformation control.

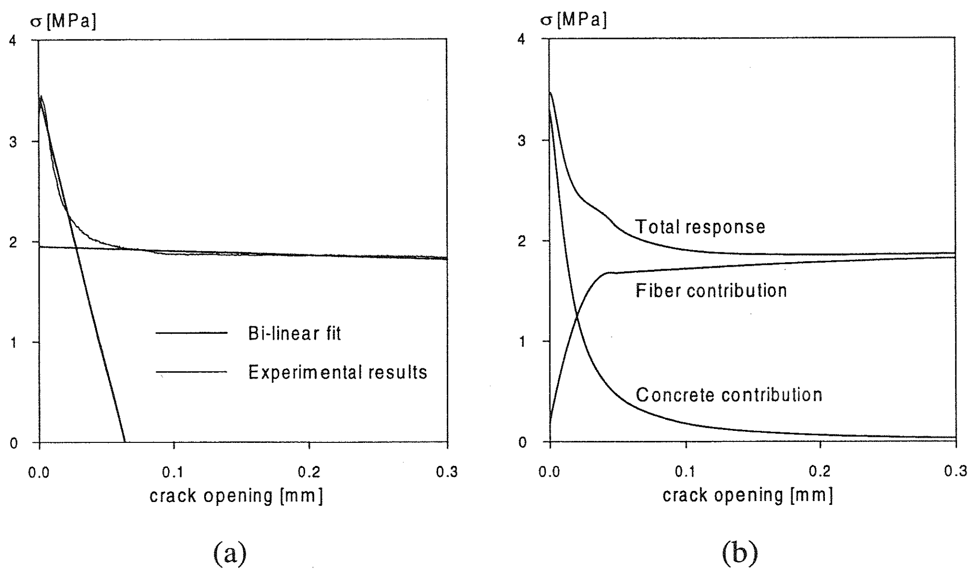


Fig. 1. Typical stress-crack opening relationship obtained from experimental measurements on steel fiber reinforced concrete containing 1 vol.% of hooked end steel fibers. In (a) is shown a bi-linear fit to the experimental results. In (b) is shown a theoretical modeling of the relationship showing the concrete and the fiber contributions.

Some discussion is currently taking place regarding whether the deformation-controlled uniaxial tensile test is the most suitable way to determine experimentally the fracture parameters of concrete and FRC, and if a uniaxial tensile test is performed how should it be designed?

Recently Stang and Bendixen (1998) made an investigation on the requirements for accurate determination of fracture mechanical parameters of FRC. It was concluded that, with sufficient rotational stiffness of the testing machine, sufficient alignment and suitable specimen size reliable measurements of the shape of the stress-crack opening relationship can be carried out. Furthermore, it was shown that these requirements can be fulfilled in a standard testing machine using a realistic specimen size. In Fig. 1 (a) a typical measured stress-crack opening relationship is shown for steel fiber reinforced concrete. Together with the experimentally obtained curves is shown a bi-linear fit. Furthermore, in Fig. 1 (b) a modeling of the stress-crack opening relationship according to Li et al. (1993) is shown. From this modeling it is clear that the first part of the bi-linear relationship reflects a combination of the concrete contribution (strength and toughness) and the initial fiber bridging action, while the second part only reflects the fiber bridging action.

3 Model for the Beam Test

In order to model a three-point bending test with a FRC material suitable for parametric investigations an analytical model is setup based on the concept of a non-linear hinge where the crack is modeled in a smeared approach.

The concept of the non-linear hinge is illustrated in Fig. 2. This concept of a non-linear hinge has previously been suggested by Ulfkjaer, Krenk and Brincker (1995) and Casanova and Rossi (1997). The hinge is modeled as incremental layers of springs which act without transferring shear between each other. The vertical boundaries of the hinge are assumed to remain straight during deformation and the angular deformation is denoted 2φ. The associated longitudinal deformation of the springs is denoted $u(y)$ where y is a vertical co-ordinate. The mean curvature of the hinge is denoted κ^* and the mean longitudinal strain is denoted $\varepsilon^*(y)$, and they are given by

$$\kappa^* = 2\frac{\varphi}{s}, \qquad \varepsilon^*(y) = \kappa^*(y - y_0) = \frac{u(y)}{s} \tag{2}$$

where y_0 is the co-ordinate of the neutral layer in the hinge. It is assumed that the hinge layers behave linear elastically as long as the tensile strength f_t is not reached. Young's modulus is denoted E. When the stress reaches f_t a fictitious crack is assumed to form with a stress-crack opening relationship σ_w which is a function of the crack opening w which in turn is a function of y. The deformation u of a layer may then be obtained from

$$u(y) = \frac{\sigma_w(y)}{E}s + w(y) \tag{3}$$

Combining Eqn. (2) and Eqn. (3) we may extract an expression for $\sigma_w(y)$:

$$\sigma_w(y) = [2(y - y_0)\varphi - w(y)]\frac{E}{s} \tag{4}$$

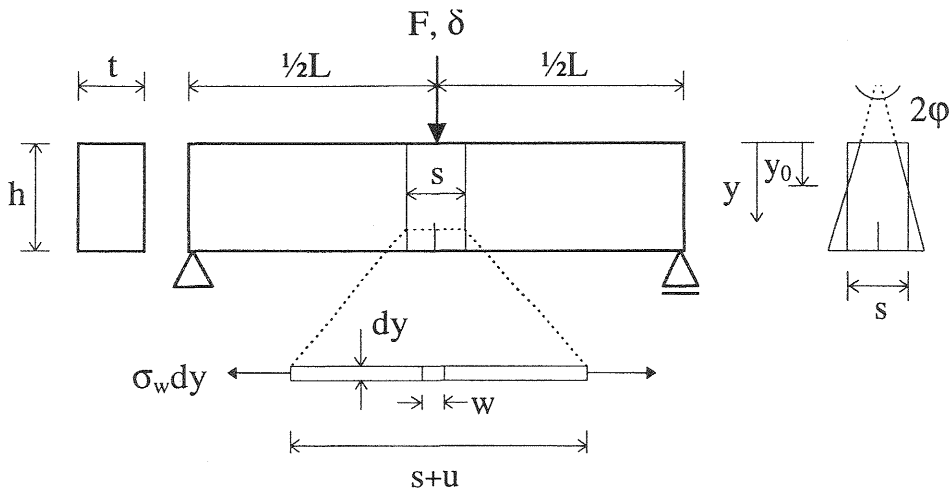


Fig. 2. Three point bending beam with non-linear hinge modeling the propagation of a crack at the mid-section. Below the beam: illustration of an incremental horizontal layer of the hinge. To the right: geometry of hinge deformation.

We now assume that σ_w may be represented by a multi-linear function:

$$\frac{\sigma_w}{f_t} = b_i - a_i w, \qquad w_{i-1} < w \leq w_i = \frac{b_{i+1} - b_i}{a_{i+1} - a_i}, \qquad w_0 = 0 \tag{5}$$

Solving Eqn. 4 and Eqn. 5 with respect to w and σ_w yields the following expressions for the crack opening and stress distributions in the cracked part of the hinge.

$$w(y) = \frac{2(y - y_0)\varphi - \zeta_i}{1 - \beta_i}, \qquad \sigma_w(y) = \frac{\zeta_i - 2(y - y_0)\varphi\beta_i}{1 - \beta_i}\frac{E}{s} \tag{6}$$

with

$$\beta_i = \frac{f_t a_i s}{E}, \qquad \zeta_i = \frac{f_t b_i s}{E} \tag{7}$$

It may be noted that within each interval of the multi-linear σ_w-function the crack opening $w(y)$ as well as the stress distribution $\sigma_w(y)$ are linear functions in y. In the uncracked part we have (cf. Fig. 3 for the definition of y^*):

$$\kappa^* = \frac{f_t}{y^* E} \Rightarrow y^* = \frac{s f_t}{2\varphi E} \tag{8}$$

Now the complete stress distribution in the hinge is established in terms of y_0 and φ. Balancing the sectional stresses with the external normal force

N and bending moment M yields a relationship between M, N and φ. Here we shall restrict ourselves to the case of zero normal force. Furthermore, we shall simplify the stress-crack opening function σ_w to be a bi-linear function where $b_1 = 1$, cf. Fig. 3.

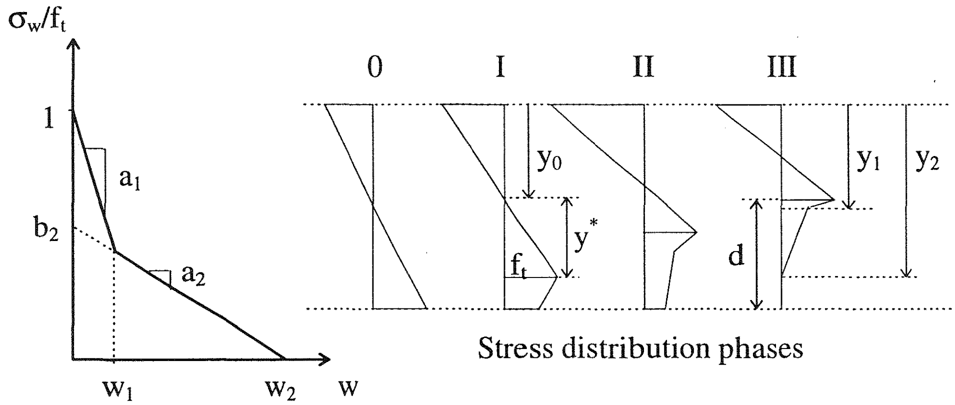


Fig. 3. Definition of a bi-linear stress-crack opening relationship and the four different phases of stress distribution during crack propagation.

The different phases of stress distribution shown in the figure are governed by the parameters y_1 and y_2 given by the general expression

$$y_i = y_0 + \frac{1}{2\varphi}[\zeta_i - w_i(\beta_i - 1)] \tag{9}$$

where i is either 1 or 2. The following normalizations are made:

$$\mu = \frac{6}{f_t h^2 t} M, \quad \theta = \frac{hE}{s f_t}\varphi, \quad \alpha = \frac{d}{h} \tag{10}$$

where d is the total depth of the crack, cf. Fig. 3. Given these normalizations the pre-crack elastic behaviour of the hinge is described by the relation $\mu = \theta$ with $0 \leq \theta \leq 1$. In the figure below the complete solution covering all the cracked phases is given. The non-linear hinge is incorporated into the midsection of the three-point bending beam. The elastic deformation of the entire beam may be given by the Bernoulli beam theory (for short beams the Timoshenko beam theory should be applied and the effect of the concentrated load taken into account). The elastic deflection at mid-span θ_e may be written as follows in the non-dimensional form:

$$\theta_e = \frac{L}{3s}\mu \tag{11}$$

This deformation includes the elastic deformation of the hinge, thus the total deformation θ_t is obtained by adding the elastic beam deformation θ_e

and the hinge deformation θ and subtracting the elastic part of the hinge deformation μ:

$$\theta_t = \theta_e + \theta - \mu = \theta_e + \left(\frac{L}{3s} - 1\right)\mu \tag{12}$$

This expression, together with the relevant expressions for the normalized moment μ from Fig. 4, establishes the load deflection curve of the beam.

Phase	α	μ
I	$1-\beta_1-\sqrt{(1-\beta_1)\left(\frac{1}{\theta}-\beta_1\right)}$	$4\left(1-3\alpha+3\alpha^2-\frac{\alpha^3}{1-\beta_1}\right)\theta-3+6\alpha$
II	$1-\beta_1-\frac{1-b_2}{2\theta}-\sqrt{(1-\beta_2)\left(\frac{(1-b_2)^2}{4\theta^2(\beta_1-\beta_2)}-\beta_2+\frac{b_2}{\theta}\right)}$	$4\left(1-3\alpha+3\alpha^2-\frac{\alpha^3}{1-\beta_2}\right)\theta-3+6\alpha-\frac{(1-b_2)\left(3\alpha^2-\left(\frac{c}{2\theta}\right)^2\right)}{1-\beta_2}$
III	$1-\frac{1}{2\theta}\left(1+\sqrt{\frac{(1-b_2)^2}{\beta_1-\beta_2}+\frac{b_2^2}{\beta_2}}\right)$	$4\left(1-3\alpha+3\alpha^2-\alpha^3\right)\theta-3+6\alpha-3\alpha^2+\frac{1}{4\theta^2}\left(1-\frac{b_2}{\beta_2}\right)\left(1-\frac{b_2}{\beta_2}+c\right)\left(1+\frac{\beta_1 c}{1-\beta_1}\right)+\left(\frac{c}{2\theta}\right)^2$

Fig. 4. Complete solution for α and μ in the three phases I, II and III. Note that the solution for phase I is equivalent to similar expressions in (Ulfkjaer et al. 1995).
Above c is given by $c = (1-b_2)(1-\beta_1)/(\beta_2-\beta_1)$

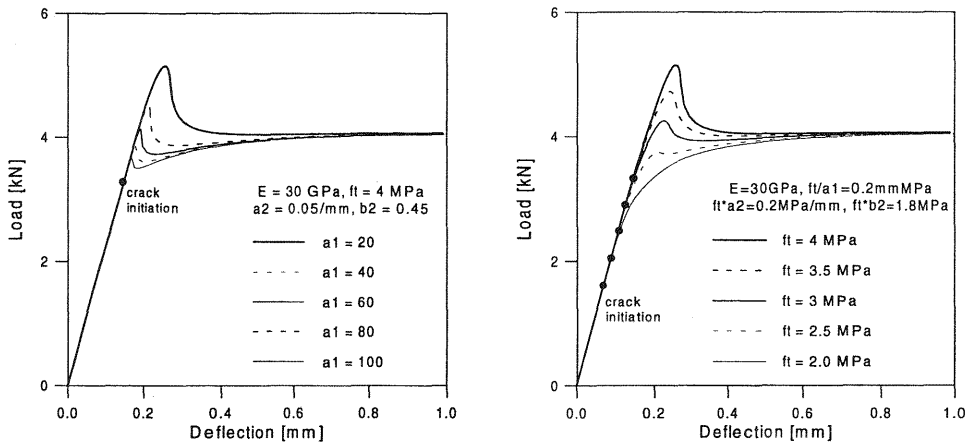
The magnitude of the hinge width s has previously been assessed (Ulfkjaer et al. 1995, Pedersen 1996) and for pure as well as fiber reinforced concrete it has been shown that $s = h/2$ is an adequate choice.

It has been verified by comparison with non-linear FEM calculations with various bi-linear stress-crack opening curves that the non-linear hinge method with $s = h/2$ gives very accurate results for the whole load deflection curve in a three-point bending test setup with notched or un-notched beams. Furthermore, it was shown by Pedersen (1996) that the non-linear hinge method – in a slightly simplified version – produces quite accurate results for the load carrying capacity of FRC pipes.

4 Parametric Study

In the following a parametric study is carried out with the aim of investigating to what extend variations in the stress-crack opening relationship cause unique changes in the load-deflection diagram from a three-point bending test.

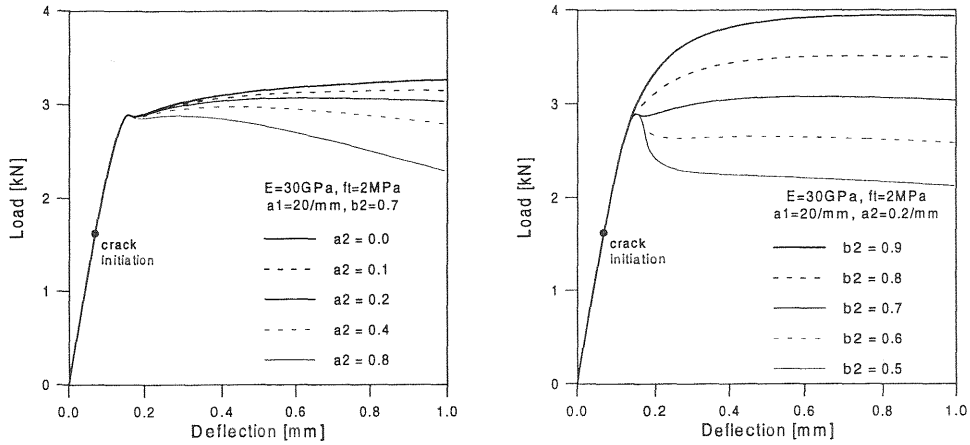
In the present investigation load-deflection diagrams are calculated with the non-linear hinge model for a 100 mm x100 mm x800 mm beam without notches. Similar curves are obtained in the case of notched beams when the presence of the notch is modeled by introducing an effective beam depth.



(a) (b)

Fig. 5. Parameter variations corresponding to changes in the concrete composition: (a) a_1 and (b) f_t.

In Fig. 5 parameter variations are shown corresponding to changes in (b) tensile strength of the concrete, f_t (keeping f_t/a_1 constant approximating a situation where the fracture energy of the concrete is kept constant) and (a) concrete toughness and/or initial fiber bridging effect, a_1. Variation in tensile strength causes the crack initiation point to change. Furthermore, the initial part of the curve is changed. The same kind of change in curve shape is observed from a change in a_1. This change is not accompanied by any change in crack initiation point. However, since the crack initiation point can bearly be distinguished changes in f_t is very easily confused with changes in a_1. On the other hand, changes in the second part of the bi-linear stress-crack opening relationship can easily be distinguished from changes in the first part, as can be seen from a comparison between Figs. 5 and 6.



(a) (b)

Fig. 6. Parameter variation corresponding to changes in fiber bridging action: (a) a_2 and (b) b_2.

5 Discussion and Conclusions

From the parameter study performed above it can be concluded that it will be very difficult to set up a general back-calculation procedure which in a unique way determines the stress-crack opening relationship for FRC materials from load-deflection test data. Especially, it is foreseen to be difficult to recover the initial part of the stress-crack opening curve.

Realizing this situation it is suggested that a design related testing procedure is based on a forward calculation and that the procedure is depending on whether information about tensile strength and initial toughness is essential.

In the normal situation where the tensile strength of the concrete is of less importance compared to the fiber effect, the designer can utilize a certain stress-crack opening relationship in his design. At the same time he can prescribe – using e.g. the model described above – a master curve for the load-deflection or load-CMOD diagram for the test specimens to be used for material property documentation. During testing it must be shown that the curves representing test results all lie above the calculated master curve.

In the more unusual situation where information about the tensile strength is also needed, bending tests are not sufficient and separate testing has to be prescribed for that property alone, e.g. in terms of uniaxial tensile testing.

6 Acknowledgement

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