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DAMAGE MODEL FOR CONCRETE INCLUDING RESIDUAL HYSTERETIC LOOPS: APPLICATION TO SEISMIC AND DYNAMIC LOADING

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Abstract
This paper is concerned with the development of a damage model for concrete materials exhibiting a residual hysteretic behaviour at a fixed level of damage. This feature is obtained by coupling damage mechanics with sliding phenomena. In its complete form, the damage variable by which the stiffness decrease is allowed, is an orthotropic second order tensor. Its evolution is driven by the tensile part of the strain tensor. The sliding between the crack lips is assumed to have a plasticity kind of behaviour with non-linear kinematic hardening. The sliding stress depends on the level of damage. Such a model assumes the evolution of two yield surfaces : a fracture and a sliding one. If unilateral effects need to be taken into account for the analysis, the damage evolution remains isotropic. In this manner, cracks closure, needed for cyclic loading is introduced.
Key words: Damage, dissipation, cyclic loading, damping

1 Introduction

One of the major drawbacks in non-linear dynamic analysis lies in the assumption of the damping matrix. If transient non-linear computations on structures need to be carried out, the expression of a damping matrix is an obligatory step. Several kinds of matrix can be imagined to achieve that purpose, such as viscous (Rayleigh, Caughey,...) or hysteretic one, Bathe (1982). A more realistic approach could consist in a better modelling of the internal dissipation.

Once cracking appears, concrete dissipates energy. Most of the constitutive models are able to reproduce realistically its behaviour in the non-linear range (La Borderie (1991), Ozbolt (1992), Feenstra (1993), ...). They often ensure predictive computations in the static case but do not handle easily a major cyclic characteristic : the residual hysteresis loops. At a fixed level of damage, concrete still exhibits dissipation due to the sliding of the crack lips. This property can be experimentally measured for a specimen during cyclic solicitations. The material constitutive laws enrichment should be a better way to model this dissipative part of the structural behaviour. The addition of viscous terms allows to avoid the illposedness of the dynamic equations when softening occurs (Sluys (1992), Dubé (1996), ...) but does not solve the problem of damping. The modelling of such features represents a more physical means to take into account the damping in the computations without using any Rayleigh type matrix.

Firstly, a short description of the model in its one dimensional formulation is exposed. It allows to understand the main assumptions and the physical meaning of different terms. A 3D extension is then proposed and its implementation in a finite element code dedicated to seismic analysis is explained. Response of the model under uniaxial loading and case study of a structure submitted to cyclic loading demonstrates the relevance of the approach.

2 Concrete modelling : constitutive relation

2.1 1D. outline

In this part are exposed the basic ideas of the material modelling in the 1 D. case. This is an obligatory step to understand the global 3 D. formulation.

Assuming a particular state potential,

$$\rho\psi = \frac{1}{2}(1-d)\varepsilon E\varepsilon + \frac{1}{2}(\varepsilon-\varepsilon_s)Ed(\varepsilon-\varepsilon_s) \tag{1}$$

the state laws are derived as follows,

$$\sigma = \frac{\partial \rho\psi}{\partial \varepsilon} \text{ and } \sigma_s = -\frac{\partial \rho\psi}{\partial \varepsilon_s} \tag{2}$$

ρ : material density
ψ : state potential
E : Young's modulus
d : damage variable
ε : total strain
ε_s : sliding strain

We can observe that the total stress is divided into two parts : a classical elasto-damage one and a sliding one :

$$\sigma = \sigma_d + \sigma_s \text{ with : } \sigma_d = E(1-d)\varepsilon \text{ and } \sigma_s = Ed(\varepsilon - \varepsilon_s) \tag{3}$$

Damage is classically driven by the elasto-damage stress and the sliding strain is only relied to the sliding part of the stress. This kind of partition, in conjunction with two failure surfaces allows the description of an hysteretic behaviour at a fixed level of damage. Details on the complementary and evolution laws will be developed for the global formulation. At this level of description, such an approach could be compared to the multi-surfaces modelling, Mroz (1967), except the fact that they are not expressed in the same space (strains-space for damage and stresses-space for sliding). The sliding strain being different from the plastic one, the thermodynamic forces associated to the total and the sliding strain are different. Such a formulation greatly differs from the classical plasticity-damage coupling. Such a choice to introduce damage in the sliding stress is guided by the idea that every inelastic phenomena in concrete is a result of cracks growth.

2.2 3 D. formulation

2.2.1 Damage model

Primarily introduced by Kachanov (1958) for creep failure problems, damage mechanics needs the introduction of a new internal variable allowing to represent the macroscopic loose of stiffness, Lemaitre & Chaboche (1990). This can be achieved through many ways, depending on the order of the damage variable : scalar, second or fourth order tensor (Murakami & Ohno (1978), Mazars (1986), Dragon (1994), ...).

A wish of physical and realistic description of oriented crack growths in concrete without neglecting the simplicity requirement lead us to a second order damage tensor formulation. In order to make easier the future numerical implementation of the model in a finite element code, the strain based Helmholtz free energy has been chosen. An effective strain tensor $(\tilde{\varepsilon})$ is defined as,

$$\tilde{\varepsilon} = (\mathbb{1} - \mathbb{d})^{1/4} . \varepsilon . (\mathbb{1} - \mathbb{d})^{1/4} \tag{4}$$

with $\mathbb{d}$ the damage tensor and ε the total strain tensor.

For symmetry conditions on the resulting stress tensor, the expression of the previous relation denotes a symetrisation of the damage operator as in Cordebois (1979). Directly introduced in the state potential, this effective strain allows the description of an elasto-damage material exhibiting orthotropic cracks :

$$\rho\psi_d = \frac{1}{2}\left\{2\mu\tilde{\varepsilon} : \tilde{\varepsilon} + \lambda Tr^2[\tilde{\varepsilon}]\right\} \tag{5}$$

with μ and λ, the Lamé coefficient defined for the virgin material.

2.2.2 Damage and sliding coupling

following the same methodology as in part 1, sliding is integrated in the behaviour through an equivalent strain coupling damage and elasticity of the sliding surface.

$$\hat{\varepsilon} = \mathbb{d}^{1/4} . (\varepsilon - \varepsilon_s) . \mathbb{d}^{1/4} \tag{6}$$

In that way the total state potential is written as follows,

$$\rho\psi = \rho\psi_d(\tilde{\varepsilon}) + \rho\psi_s(\hat{\varepsilon}) \tag{7}$$

with : $\rho\psi_s = \frac{1}{2}\left\{2\mu\hat{\varepsilon} : \hat{\varepsilon} + \lambda Tr^2[\hat{\varepsilon}]\right\} + \frac{1}{2} b\alpha : \alpha$ (8)

The stress tensors can be derived :

$$\sigma = \frac{\partial \rho\psi}{\partial \varepsilon} = 2\mu(\mathbb{1} - \mathbb{d})^{1/2} \varepsilon (\mathbb{1} - \mathbb{d})^{1/2} + \lambda(\mathbb{1} - \mathbb{d})^{1/2} Tr\left[\varepsilon(\mathbb{1} - \mathbb{d})^{1/2}\right] +$$

$$2\mu \mathbb{d}^{1/2}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_s)\mathbb{d}^{1/2} + \lambda \mathbb{d}^{1/2} Tr\left[(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_s)\mathbb{d}^{1/2}\right] \tag{9}$$

and,

$$\sigma_s = -\frac{\partial \rho \psi}{\partial \boldsymbol{\varepsilon}_s} = 2\mu \mathbb{d}^{1/2}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_s)\mathbb{d}^{1/2} + \lambda \mathbb{d}^{1/2} Tr\left[(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_s)\mathbb{d}^{1/2}\right] \tag{10}$$

The back stress is defined as : $\mathbb{X} = \dfrac{\partial \rho \psi_s}{\partial \alpha} = b\alpha$ (11)

b: material parameter

As notified for the 1 D. case, we easily recognise the partition of the stress between a damage one and a sliding one. To complete the model, we now have to define the failure and the sliding criteria as well as the dissipative potential.

2.2.3 Damage criteria and evolution laws

Based on experimental investigations, damage for brittle materials like concrete is principally governed by its tensile behaviour. To take into account this dissymetry, two damage tensors have to be introduced. The splitting between the tensile and the compressive damage tensor is achieved trough the sign of the sliding strains expressed in their own principal directions :

$$\mathbb{d} = \mathbb{d}^{+} H\left(\boldsymbol{\varepsilon}_s^{+}\right) + \mathbb{d}^{-} H\left(-\boldsymbol{\varepsilon}_s^{-}\right) \tag{12}$$

with $\boldsymbol{\varepsilon}_s = \boldsymbol{\varepsilon}_s^{+} + \boldsymbol{\varepsilon}_s^{-}$, $\boldsymbol{\varepsilon}_s^{+} = \mathbb{P}^{-1}\left\langle \boldsymbol{\varepsilon}_s^{d} \right\rangle_{+} \mathbb{P}$ and $\boldsymbol{\varepsilon}_s^{-} = \mathbb{P}^{-1}\left\langle \boldsymbol{\varepsilon}_s^{d} \right\rangle_{-} \mathbb{P}$ (13)

$\boldsymbol{\varepsilon}_s^{d}$ is the diagonal sliding strain tensor and $\mathbb{P}$ is the transformation matrix.

For each principal direction, tensile damage is evaluated with respect to the strain :

$$d_i^{+} = 1 - \frac{\varepsilon_{d0}}{\varepsilon_i^{+}} exp\left(Bt\left(\varepsilon_{d0} - \varepsilon_i^{+}\right)\right) \tag{14}$$

Damage criterion : $f_i = \varepsilon_i - \varepsilon_{d0} - \kappa_i(\varepsilon_i) \leq 0$ (15)

$\kappa_i(\varepsilon_i)$: hardening variable
ε_{d0} : initial tensile yield strain, generally equal to 1.10^{-04}
Bt : material parameter driving the slope of the softening branch.

Compressive damage is only considered as a consequence of the tensile behaviour of the material and so is taken equal to a function of the state of tensile cracking on the orthogonal directions (d_j^+ and d_k^+).

$$d_i^- = \left(\frac{d_j^+ + d_k^+}{2} \right)^{\beta} \tag{16}$$

β : material parameter relaying the damaged Young's moduli for two orthogonal directions. It has been identified in Fichant et al. (1997) on a specimen loaded under uniaxial compression. Comparison of apparent Young's modulus in longitudinal and radial directions allows the measurement of β. A classical value for concrete is, $\beta = 12$ ·

2.2.4 Sliding criteria

The sliding part of the constitutive relation is assumed to have a plasticity-like behaviour. So as to reproduce the hysteresis loops, a non-linear kinematic hardening is considered. Primarily introduced by Armstrong & al. (1966) and recently developed in Chaboche (1993), it allows to overcome the major Prager's kinematic hardening law drawback, i.e. the linearity of the state law defining the associated forces to kinematic hardening. The non-linear terms are added in the dissipative potential.

The sliding criteria takes the classical form : $f = J_2(\sigma_s - \mathbb{X}) - \sigma_y \leq 0$

$J_2(\sigma_s - \mathbb{X})$, i.e. the Von Mises equivalent stress, has been chosen in a first approximation. This is due to the existing link between sliding and shear effects.
σ_y : initial yield stress

To derive the evolution of the internal variables, classical plasticity requires the definition of a dissipative potential. A wish of non-linear kinematic hardening imposes the use of non-associated plasticity :

$$\phi = J_2(\sigma_s - \mathbb{X}) + \frac{3}{4} a \mathbb{X} : \mathbb{X} - \sigma_y \tag{17}$$

a: material parameter

Thanks to the normality rules, the evolution laws of the internal variables are expressed as follows :

$$\dot{\varepsilon}_s = \dot{\lambda} \frac{\partial \phi}{\partial \sigma_s} \text{ and } \dot{\alpha} = -\dot{\lambda} \frac{\partial \phi}{\partial \mathbb{X}} \tag{18}$$

$\dot{\lambda}$: plastic multiplier

3 Numerical implementation

3.1 Constitutive law implementation

By the definition of two different surfaces, damage tensor can be explicitly integrated as long as the tensile train value is known. That's why for uniaxial compression the algorithm becomes implicit, a kind of plane stress procedure has to be introduced. Concerning the sliding stress, a classical implicit analysis has to be carried out. Among all the different methods available to reach this goal (Euler backward or mid-point rules algorithm solved by an iterative Newton method), we chose the classical form of the so-called "return mapping" algorithm, Ortiz & Simo (1986). Indeed, it ensures convergence in the most efficient way. Details on the numerical algorithm are presented in Mazars et al. (1998) .

3.2 Finite Element code

This constitutive law has been implemented in a multilayered beam finite element code dedicated to simplified analysis : EFiCoS (La Borderie 1991). The basic assumption is that plane sections remain plane (Bernouilli's kinematic) allowing to consider a uniaxial behaviour of each layer. Each finite element is a beam which is discretized into several layers (fig. 1).

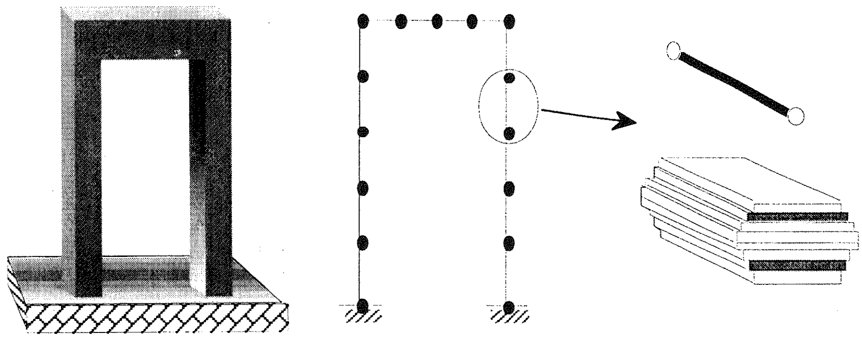

Fig. 1. Finite element code discretization

3.3 Uniaxial loading response

The two curves in figure 2 show the ability of the model to describe the hysteresis loops under traction and compression loading path. The hysteretical dissipation capacity of the model can be enlightened by plotting the consumed energy of an unloading tensile loop against the value of tensile damage in figure 3. We can easily appreciate the effect of the coupling between the state of damage and the sliding stress.

The goal of developing such a material model was to investigate the damping effects of reinforced concrete structures subject to cyclic and seismic loading. This question is discussed in the following paragraph where the case of a cyclic three points bending beam test is analysed through the hysteretical dissipation measurement.

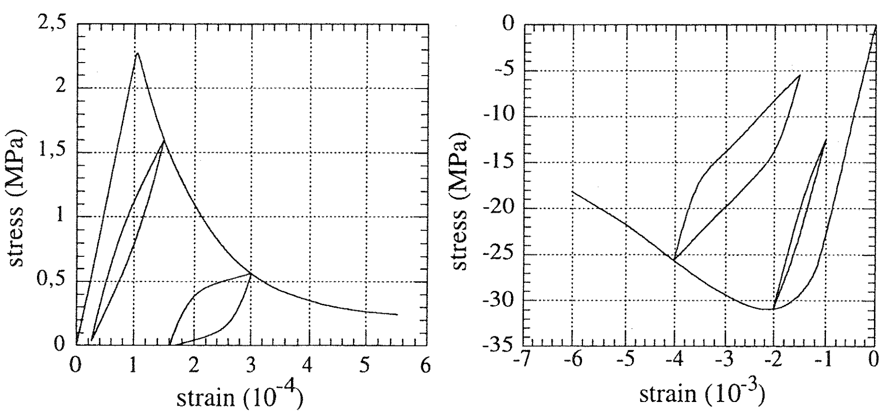

Fig. 2. Tension / compression response

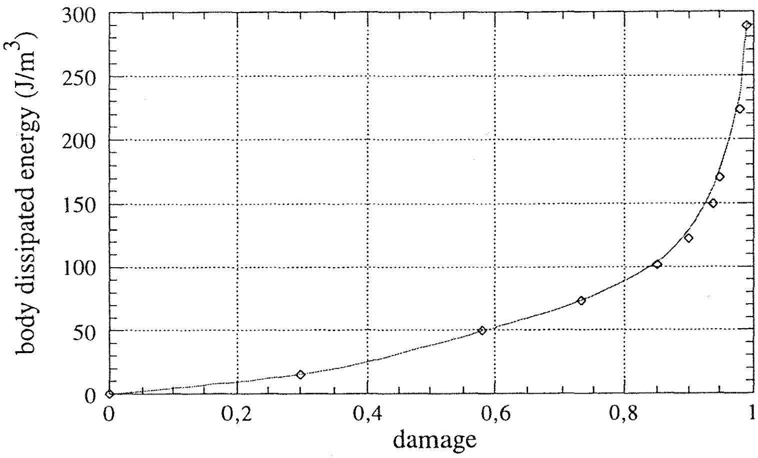

Fig. 3. Dissipated energy versus damage

4 A first application : cyclic three points bending test

4.1 Short presentation

In order to quantify the link between hysteretical dissipation and the state of cracking in a structure, cyclic 3 points bending tests have been performed on a reinforced concrete beam. The goal is to measure the relative damping during a cycle for different levels of loading. The testing machine is an M.T.S. +/- 500 kN. LVDT transducers were used to measure the vertical displacement of the middle fibber of the beam from its original position. This avoids any bias due to the supports displacement. The concrete used for casting is classical : uniaxial compression tests on normalised cylindrical specimen gave a mean value of 25 MPa. Steel reinforcement is classical too for this kind of structure element. The maximum carrying capacity of the bars has been estimated up to 400 MPa. An overview of the testing set up is presented in figure 4.

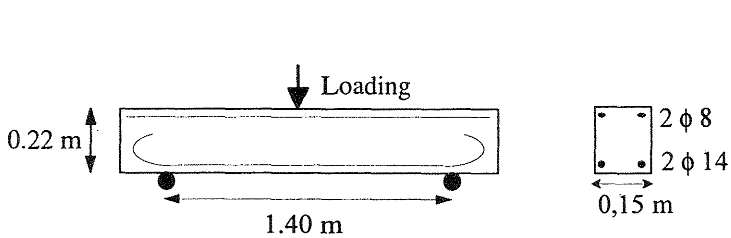

Fig. 4. Experiment set-up

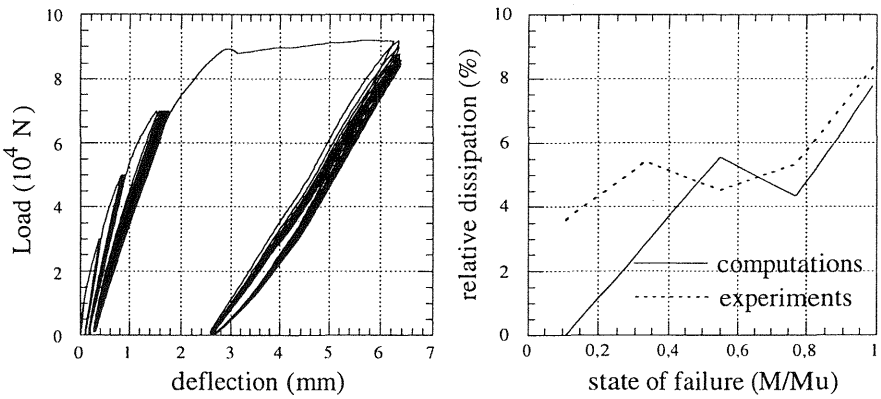

Fig. 5. Experimental result / hysteretical dissipation

A first test under a monotonic loading indicated a maximum failure load of 90 kN. A second one under repeated loading at several steps (10 kN, 30 kN, 50 kN, 70 kN and 90 kN) relieves the main results. A deflection / load diagram is presented in figure 5. At each step, only 10 cycles are performed so as to prevent any fatigue phenomena. The hysteretical dissipation is measured through the surface of the loops at each level of loading. The induced relative damping is estimated by a comparison of this value against the internal elastic energy :

$\xi_r = \frac{\omega_D}{\omega_e}$ with ω_D, the dissipated energy and ω_e the elastic energy

The comparison of this relative damping for computations and experiments in figure 5 shows that the model is able to reproduce the general trend of the behaviour : an increasing damping with the state of material cracking. The state of failure is characterised by the ratio of the actual moment against the maximum carrying capacity. We can observe that for the lowest level of loading, the model predicts no hysteretical dissipation. This is due to the fact that the material remains in its elastic domain. Dissipation being coupling to damage, we cannot in that particular case dissipate any energy.

5 Conclusion

Within the framework of seismic analysis of reinforced concrete structures, we focused our attention on the problem of damping which remains today one of the greatest unknown in structural modelling. From the realistic assumption that the total amount of energy consuming phenomena are relied to the local state of cracking, we developed a new local constitutive relation for concrete material including residual hysteresis loops at a fixed level of damage. First cyclic applications on structural elements tend to accredit this hypothesis. The use of this kind of model for dynamic and in particular seismic loading may avoid the explicit expression of an often arbitrary damping matrix, Mazars et al. (1998). More accurate investigations in this directions would allow to objectively distinguish the part taken by the material nonlinearities in the global damping of a concrete structure.

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