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PROPOSITION OF A DAMAGE INDICATOR APPLIED ON R/C STRUCTURES SUBJECTED TO CYCLIC LOADING

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Abstract

In this paper a new formulation of damage indicator is proposed for reinforced concrete members. To evaluate this factor it is considered both an energetic analysis method and the weighting of repeated cycles in their real occurring sequence. This damage indicator is a numerical value of range 0% - 100% indicating the state of damage of a structure when subjected to monotonic or cyclic loading. It allows either to decide (for repairing or destroying constructions) after earthquake or to design structural elements. Applying this method gives a regular distribution of damage up to failure and allows to give an idea of fatigue damaging. The damage indicator evaluated on the columns tested by J.G. Sieffert et al. reaches about 5% when the first tensile cracks appear, 45% when the first compression cracks occur, and 100% at failure.

Key words: Damage indicator, R/C structures, cyclic loading, fatigue

1 Introduction

The loss of life during huge earthquakes is mainly due to the destruction of houses and public buildings. Because of that reason, different technical rules and practice codes all over the world, insist on avoiding the collapse of structures, while accepting a certain amount of damage in the structural elements during medium and strong seismic vibrations.

The use of a damage indicator would allow to quantify the structural damages caused by earthquakes for each element.

The proposed damage indicator formulation for R/C structures is justified by results of tests carried out in the Civil Engineering Laboratory of the Ecole Centrale de Nantes on columns subjected to axial load and cyclic biaxial bending.

The proposed definition based on an energetic analysis method, considers the maximum transmitted energy at failure for monotonic loading and also the transmitted energy during cyclic loading. Introducing a λ_j coefficient, eventually allows to take into account the fatigue phenomena.

2 R/C members damage indicator

The evaluation of the real damage caused on reinforced concrete members relatively to failure needs a damage indicator as a quantitative ratio of damage.

An efficient indicator must be suitable for:

- different structural elements (with symmetrical or non-symmetrical behavior),
- different kinds of loading (monotonic, symmetrical or non-symmetrical cyclic loading with different number of hysteretic loops).

It must be representative of damage:

- realistic visual shape for each damaging phase,
- numerical values increasing from 0% up to 100% at failure.

Among the existing indicators, two characteristic approaches are shortly described below.

2.1 Palmgrem and Miner damage indicator

The hypotheses after Palmgrem & Miner (1924, 1945) suggest that if N_{1max}, N_{2max}, ..., or N_{imax} cycles are necessary to reach failure with subsequent cycles type 1, 2, ..., n, then the damage indicator for a series composed of N_1, ..., N_n cycles is S (S = 1 at failure).

$$S = \sum_{i=1}^{n} \frac{N_i}{N_{imax}} \qquad (1)$$

This approach is unsatisfactory for R/C members for the following reasons:

- The predicted accumulated damage does not reflect the temporal sequence of loading cycles . For example applying high stress cycles at the beginning or at the end of the loading history does not affect differently the estimated damage.

- Stresses under a certain level are assumed to cause no damage.

2.2 Meyer's damage indicator

The damage indicator presented by Meyer (1988) is based on the transmitted energy, using the maximum energy E_u under monotonic loading up to failure as a kind of normalizing factor.

The transmitted energy is divided in two parts, E_{pi} and E_{si}. Their physical meaning is described below, introducing the concepts of "primary half-cycle" (D.C.P.) and "following half-cycle" (D.C.S.). After Otes (1985) a "primary half-cycle" is considered when reaching any half-cycle with a new maximum amplitude; it is followed by a certain number of "following half-cycles" with smaller amplitudes. It means whenever a certain maximum displacement d_i, corresponding to the primary half-cycle $(D.C.P.)_i$ is exceeded, a new primary half-cycle $(D.C.P.)_{i+1}$ is established. Every D.C.P. corresponds to a certain damage degree.

The first energy part E_{pi}, deals with the transmitted energy during $(D.C.P.)_i$., the second E_{si}, with the transmitted energy during $(D.C.S.)_i$. Mathematically, Meyer's DQ is derived from DQ^+ (for positive displacements) and DQ^- (for negative displacements).

For the positive range:

$$\mathbf{DQ}^+ = \frac{\sum \mathbf{E}^+_{pi} + \sum \mathbf{E}^+_{si}}{\mathbf{E}^+_u + \sum \mathbf{E}^+_{si}} \qquad (2)$$

For negative displacements the same expression is assumed replacing superscripts "+" by "-", and collectively:

$$DQ = DQ^+ (1 - DQ^-) + DQ^- \qquad (3)$$

3 Experimental data

The different proposals given in this paper derive from test results carried out by Sieffert et al. (1990). Over 20 tests have been performed on columns under alternate cyclic and monotonic horizontal loading. The horizontal loads have been applied through different horizontal directions on the column top section. In this paper the test results of two columns (P0M and P0C for monotonic and cyclic loading) are used.

It has been observed that the experimental response of an element subjected to a 20 or 30 cycles train can be analyzed using only the first 2 cycles. Following cycles are fairly similar.

The general behavior of damage phases of the tested columns were similar for the identical maximum displacements during cycles. A summary of the damage phases is given in Table 2.

4 Studying energies

Based on the tests, the different energies have been analyzed, as follows:

- transmitted energy (applied force works),
  dissipated energy during cyclic loading (force-displacement hysteretic loops areas),
- released energy (difference between transmitted and dissipated energies).

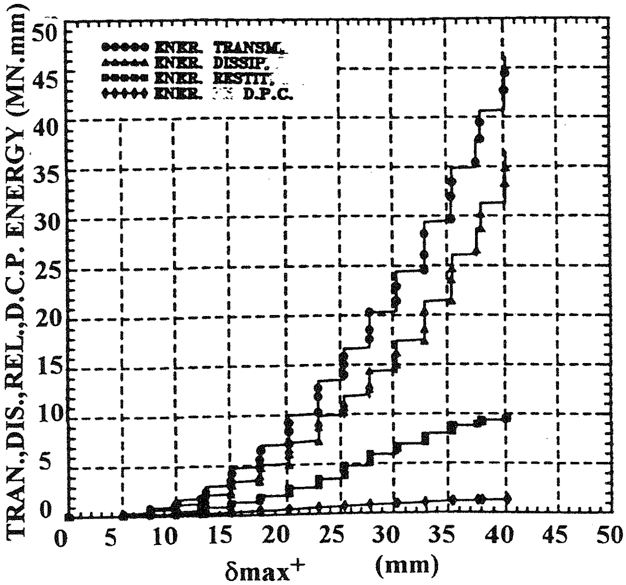

Fig. 1. Energies versus top amplitude for P0C

The transmitted energy at failure for P0C (46.1 MN.mm) is much more higher than the concerning value in the case of monotonic loading for P0M (1.6 MN.mm).

Figure 1 represents the different energies versus top positive maximum displacements for P0C (considering 2 cycles per amplitude).

A linear relation connecting the transmitted energy of D.C.P. to the positive maximum displacements has been observed.

The transmitted energy at failure for P0C and P0M are reported in Table 1.

Table 1. Energies at failure for P0C and P0M

Column id.	E_u^+ (MN.mm)	ΣE_{pi}^+ (MN.mm)	ΣE_{si}^+ (MN.mm)
P0C	-	1.3	239.4
P0M	1.6	1.6	0

The ratio $\Sigma E_{si}^+ / E_u^+$ is approximately equal to 150. It means that in the DQ^+ formula, the value of E_u^+ is negligible against the term ($\Sigma E_{pi}^+ + \Sigma E_{si}^+$), and especially against ΣE_{si}^+.

5 Damage indicator proposal

In the case of cyclic loading, the force-displacement (F-D) envelope is usually close enough to the monotonic curve, while the maximum displacement reached at failure is lower than the maximum obtained monotonically. Therefore, a difference between E_{pi}^+ and E_u^+ always stands at failure. This can be explained basically by the non-identical ways of loading.

The final DQ expression is derived from two main stages examined as follows, considering positive displacements, then negative displacements with the same principle and assuming:

$$DQ = MAX\,[DQ^+, DQ^-] \tag{4}$$

We shall only discuss here about DQ^+.

- In the first step it has been considered:

$$\mathbf{DQ1}^+ = \frac{\sum E_{pi}^+}{E_u^+} \tag{5}$$

As described in Table 1, ΣE_{si}^+ is greatly higher than E_u^+ and ΣE_{pi}^+, producing apparently a high damage indicator value using the Meyer's formula. In addition, E_u^+ can stand alone to represent a normalizing factor. However, DQ1 does not reach 100% at failure (for instance DQ1 is equal to 0.86 for P0C).

- In the second step, it has been tried different processes to correct the initial DQ1 value. It has been found that a C adaptation factor, derived from the maximum energy capacity registered at failure, could give a good connection between monotonic and cyclic DQ. C^+ is expressed as follows:

$$C^{+}=(F^{+}\text{max}.D^{+}\text{max})\text{monotonic}\ /(F^{+}\text{max}.D^{+}\text{max})\text{cyclic} \qquad (7)$$

For positive displacements, F^{+}max is the maximum force, and D^{+}max is the maximum displacement.

Consequently a final "global" formula for DQ^{+} is proposed as:

$$\mathbf{DQ}^{+}=\frac{\sum \mathbf{E}_{\mathbf{pi}}^{+}}{\mathbf{E}_{\mathbf{u}}^{+}}\mathbf{C}^{+} \qquad (8)$$

This procedure fits correctly to represent both limits (0% to 100% at failure) and also a realistic progression of the DQ factor between these limits.

6 Comparison of Meyer's and proposed damage indicators

Figure 2 allows to compare the calculated DQ, on both bases of Meyer's and proposed formulations for P0C and P0M. Furthermore Table 2 indicates the damaging phase ranges.

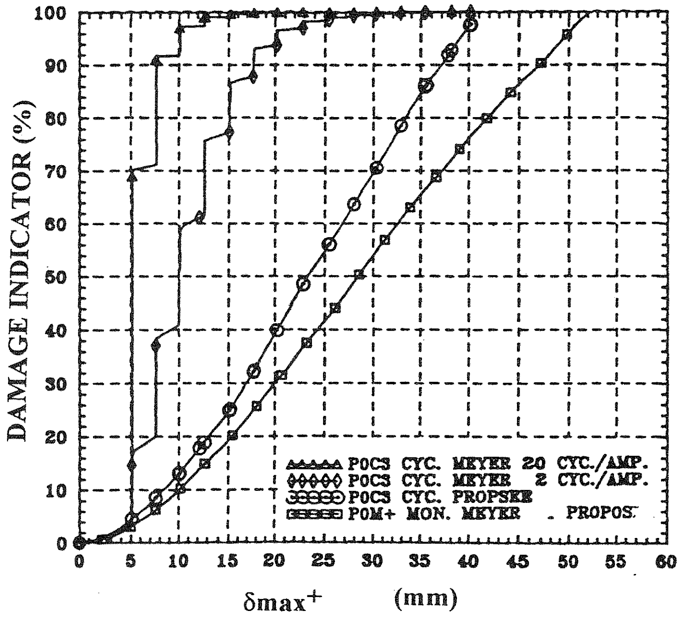

Fig. 2. Meyer's and proposed DQ (P0C, P0M)

Table 2. DQ and observed damaging phases

Damaging phases Visual observations	Dmax$^+$ (mm)	Proposed DQ (P0C) (%)	Meyer's DQ (P0C) (%)
	2.5	1.19	1.20
Phase A (First tension crack)	5.0	4.45	3.25
	7.5	8.45	71.25
Phase B (Tension crack development)	10.0	13.15	91.94
	12.5	18.85	97.44
	15.0	25.08	99.11
	17.5	32.30	99.68
	20.0	39.83	99.87
Phase C (Compression crack appearance)	22.5	48.47	99.96
Phase D (Compression crack development)	25.0	55.99	99.977
	27.5	63.64	99.988
	30.0	70.53	99.994
	32.5	78.56	99.994
Phase E (Failure of column)	35.0	86.21	99.995
	37.5	92.80	99.995
	40.0	98.00	99.996

Meyer's DQ reaching 90% in phase B and 99.9% in phase C, looks too much conservative.

As indicated in Fig. 2 and Table 2, a large difference in the evolution of DQ appears comparing Meyer's and the proposed formula. This is due to the influence of the repeated number of D.C.S. in DQ's formulas.

The reduction of the influence of the following cycle numbers in the proposal induces a better distribution of damage up to failure. Consequently with this new formulation a better approach is given to indicate the amount of damage in R/C structural elements.

7 Weighting directly D.C.S.

A third complementary step has been considered taking a similar but "local" point of view. Let us express formula (8) written in another way as (9):

$$\mathbf{DQ}^{+} = \frac{\sum \mathbf{E}_{pi}^{+} + \sum_{j=1}^{j} \lambda_j^{+} \sum_{k=1}^{k} \mathbf{E}_{sk}^{j+}}{\mathbf{E}_u^{+}} \qquad (9)$$

with:

- i : cycle number (considering all cycles)
- j : group number of constant amplitude
- k : number of cycles in group j
- Epi^{+} : transmitted energy during $(D.C.P.)i^{+}$
- $\mathbf{E}_{sk}^{j+}$: transmitted energy during $(D.C.S.)k^{+}$ at each different amplitude number j
- λj^{+} : fatigue factor for group j (positive)

It is obvious, even if Eq. (8) is globally satisfactory, that the C^{+} factors implicitly contains in types and numbers the effects of all following half cycles.

As shown previously with Meyer's formula, following half cycles can not be taken into account without weighting them. This has been noticed also during performed tests.

The λj^{+} reducing factor depends essentially on the number of cycles concerning a fixed amplitude aj. λj^{+} is calculated locally for successive $Dmax^{+}$, assuming that Eqs. (8) and (9) are equal and that C^{+} is already known.

An additional advantage of this formula is that one can expect the estimation of the identical cycle number nj^{r} that produces failure in the case of one constant amplitude a_j series. In this it is considered that the loop areas remain constant up to failure or identically that the dissipated energy is still the same during each following cycle.

According to this preliminary discussion nj^{r} is obtained as follows for this single type j cycle:

$$\mathbf{n}_j^{r} = \frac{\mathbf{E}_u^{+} - \mathbf{E}_{p1}^{+}}{\lambda_j^{+} \mathbf{E}_{s1}^{j+}} \qquad (10)$$

with:

- $Ep1^{+}$: Transmitted energy for a (D.C.P.)j

- E_{s1}^{j+} : Transmitted energy for a (D.C.S.)j.

Since λ_j is variable versus amplitude it is not possible to consider one single constant.

Figure 3 shows the estimated number of cycles producing failure versus different chosen amplitudes for P0C. The case of a monotonic loading given for n = 0 can be obtained with a maximum displacement of approximately 59 mm while it was observed 53 mm in the real monotonic experiment, corresponding to a 10% relative difference.

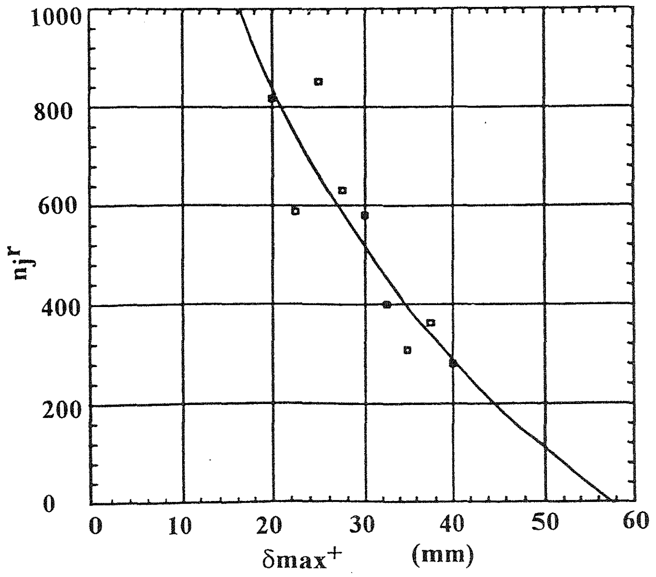

Fig. 3. n_j^r versus a_j at failure for P0C

8 Practical use of the proposed DQ

In order to calculate DQ, it is sufficient to evaluate the values of force and displacement. Simulation of structures under cyclic and monotonic loading can be carried out either numerically or by laboratory testing methods. For deciding either repairing or destroying constructions, the calculated DQ is compared with an admissible damage indicator (DQ) which should be fixed by technical rules and practice codes for different types of structures.

The proposed damage indicator reaches approximately:

- 5% for the first tension cracks,
- 45% for the first compression cracks,
- 100% at failure.

(Values are obtained for tests on columns under alternate cyclic loading).

9 Conclusions and future proposals

The following main advantages of using the proposed damage indicator can be summarized:

(1) It considers the real temporal sequence of loading cycles.

(2) It gives a regular distribution of damage up to failure, considering the weight of following half-cycles.

(3) It makes possible to predict the number of identical cycles producing failure similarly to fatigue when considering the "local" approach as mentioned previously.

Next studies should deal with codifying admissible damage indicators for different types of structures (such as buildings, industrial factories, marine structures, water tanks, nuclear power plants, etc.).

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