Fracture Mechanics of Concrete Structures
Proceedings FRAMCOS-3
AEDIFICATIO Publishers, D-79104 Freiburg, Germany

ANALYSIS OF SPECIFIC EXPERIMENTAL PROBLEMS IN DYNAMIC TESTING OF CONCRETE

H.ZHAO and G.GARY
Laboratoire de Mécanique des Solides
Ecole Polytechnique
France.

Abstract
Experimental problems encountered in the testing of concrete with techniques using bars are studied in this paper. The commonly used SHPB test (Split Hopkinson Pressure Bar), the DIHP test (Direct Impact Hopkinson Bar), and the test under confined pressure are examined. At first, it is shown that recent improvements in the data processing such as wave dispersion correction and exact time shifting are indispensable to obtain accurate measurements in those tests. The accuracy of conventional analyses which are based on the assumption of homogeneous stress and strain fields is afterwards investigated with the aid of transient numerical simulations. It is shown that only one of those conventional analyses gives an acceptable precision. The technique for dynamic compression under confining pressure is finally discussed.
Keywords : Concrete, Dynamic testing, Kolsky's bar, Numerical simulation

1. Introduction

The behaviour of concrete under impact loading is involved in the study of safety requirements of structures in extreme situations such as earthquakes, accidental impacts or explosions. The early experimental data of those materials under dynamic loading are obtained with pendulum tests, drop weight tests, etc.

A review of those works is given by Green (1958). The "state of the art" report of RILEM committee (1975) has given a comparison of results obtained from different loading devices, showing important gaps among different techniques. It is not only due to the heterogeneous nature of concrete or to different specimen dimensions. It is also because of the unavoidable imprecision of those loading and measuring devices.

The technique using bars, which offers accurate results under impact loading, has been then applied to the testing of concrete. The use of a compressive Split Hopkinson Bar (SHB) to determine the rate sensitivity of concrete has been reported in recent works (Malvern et al. 1991; Tang et al. 1992). The less frequent application of the tensile SHB arrangement to concrete specimens has also been reported (Reinhardt et al., 1986). The direct impact Hopkinson bar (DIHB) is used to perform tests at medium strain rates (Gary & Klepaczko, 1992). The technique has also been proposed to impose loading-unloading cycles at high strain rates (Zhao & Gary, 1997). As those techniques using bars have been initially developed for the testing of metals, their application to concrete raises some specific problems.

This paper presents a study of those specific problems. On the one hand, in order to ensure the accuracy of experimental devices, measuring imprecision due to the wave dispersion in split bars and due to the error in the time shift are discussed. An optimal data processing is then proposed. On the other hand, the analysis of basic measurement (forces and velocities at the faces of the specimen) is reconsidered. Transient numerical simulations of tests on concrete specimens are performed with a rate-sensitive crack model. It permits for a careful investigation of the accuracy of the stress-strain relation obtained with the conventional analysis. Finally, the techniques permitting for the application of lateral pressure are examined.

2. Concrete testing using bars, optimal data processing

2.1 SHPB and DIHP tests

The Split Hopkinson Pressure Bar, or Kolsky's apparatus (Kolsky, 1949) is somehow a universal experimental technique in the study of the constitutive law of materials at high strain rates. A typical SHPB setup is shown in Fig. 1. It is composed of the long input and output bars with a short specimen placed between them. The impact of the projectile at the free end of the input bar develops a compressive longitudinal incident wave $\varepsilon_i(t)$. Once it arrives at the bar specimen interface, a reflected wave $\varepsilon_r(t)$ is developed in the input bar, whereas a transmitted wave $\varepsilon_t(t)$ is developed in the output bar.

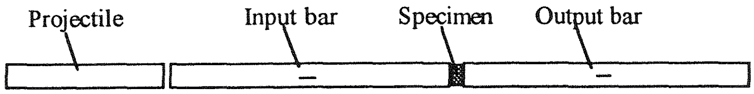


Fig. 1. Split Hopkinson Pressure Bar (SHPB) test

When the waves are known at bar-specimen interfaces, the forces and the velocities at both faces of the specimen are given by the following equations (1).

$$
\begin{aligned}
F_{input}(t) &= S_B\ E\ (\varepsilon_i(t)+\varepsilon_r(t)) \qquad & V_{input}(t) &= C_0\ (\varepsilon_i(t)-\varepsilon_r(t)) \\
F_{output}(t) &= S_B\ E\ \varepsilon_t(t) \qquad & V_{output}(t) &= C_0\ \varepsilon_t(t)
\end{aligned}
\tag{1}
$$

where S_B, E and C_0 are respectively the bars' cross-sectional area, Young's modulus, and elastic wave speed.

The use of an input bar leads to a limitation of the maximum force for a given impact velocity because of the linear relation between the stress and the particle velocity associated to the same wave. Indeed, with SHPB, the impact force and the average strain rate applied to the specimen are linked with the incident wave in the following way.

$$
\begin{aligned}
|F_{input}| &= ES_b|\varepsilon_i+\varepsilon_r| \le ES_b|\varepsilon_i| \\
|\dot{\varepsilon}_s| &= C_0|\varepsilon_i-\varepsilon_r-\varepsilon_t| \ge C_0|\varepsilon_i|
\end{aligned}
\quad \text{where } |\varepsilon_i| = \frac{V_{impact}}{2C_0}
\tag{2}
$$

The SHPB fails then to smash concrete specimen at medium strain rates. Low strain rates need low input velocities that will produce low input forces. A current solution to overcome this problem is to strike directly the specimen with a heavy striker. A scheme of the arrangement of such a so-called direct impact Hopkinson bar (DIHB) test is shown in Fig.2.

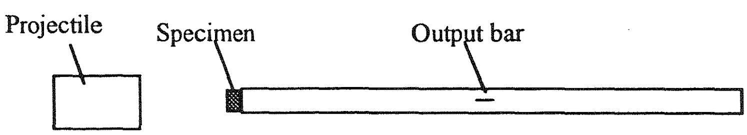


Fig. 2 Direct Impact Hopkinson Bar (DIHB) test

As measurements are only made at the output side, supplementary assumptions must be made for the input side. The usual method assumes an equilibrium state of the specimen so that the input force is equal to the output one. To calculate the input velocity, it is assumed that there is an instantaneous jump from zero to the measured initial velocity V_0 of projectile and afterwards a deceleration due to the action of input force (Dharan & Hauser, 1970; Pope & Field, 1984). When the striker is sufficiently long so that the reflected wave will not come back before the end of the test, we have:

$$V_{input}(t) = V_0 - \frac{F_{input}(t)}{S_p \rho_p C_p} \tag{3}$$

where S_p, ρ_p, C_p are respectively the cross-sectional area, density and wave velocity of the projectile.

When the striker is relatively short, the wave's reflections at its free end must be taken into account. The input velocity then depends on the characteristic time $T = 2l_p / C_p$, needed for the wave to perform a round trip in the projectile. The input velocity can be expressed in the following way :

$$V_{input}(t) = V_0 - \frac{1}{S_p \rho_p C_p} \left(F_{input}(t) + \sum_{n=1}^{N} 2F_{input}(t - nT) \right) \quad \text{if } NT \le t < (N+1)T \tag{4}$$

where $N = \text{int}(t / T)$.

2.2 Optimal data processing

Experimental results reported in the literature show often relatively important dispersions, partly due to the measuring imprecision of experimental devices. As to the technique using bars, the main difficulty of the measurement lies in the fact that the three basic waves have to be shifted from the strain gages to the specimen faces, in time and distance. This shifting leads to two different perturbations. First, waves change in their shapes (wave dispersion) on propagating along the bar. Second, one has to find the exact time shift for the three waves so that the shifted waves at the bar-specimen interfaces correspond to the same time origin.

To avoid shifting errors, the correction of the so-called wave dispersion effect should be made at first. It is realised on the basis of the analytical solution of the harmonic wave propagation in an infinite cylindrical bar (Davies, 1948; Follansbee & Franz, 1983; Zhao & Gary, 1995), which gives a theoretical dispersion relation (wave velocities versus frequencies). An accurate determination of time shifts is obtained by a method based on the simulation of the specimen response (Zhao & Gary 1996). One can simulate numerically, for an elastic specimen, the fictitious reflected and transmitted waves where the exact origins are known. If the specimen is supposed to behave elastically at the early stage of the test, the comparison between simulated and real waves allows for a precise determination of their origins. The application of those improvements in the data processing gives more accurate results. (Zhao, 1998a) They are particularly important for the accuracy in the range of small strains.

3. Analysis of measured forces and velocities in concrete testing.

Technique using split bars provides then accurate dynamic measurements at faces of the specimen (forces and velocities). From the basic experimental data given by the equation (1) (forces and velocities at input and output sides), the conventional analysis assumes the axial uniformity of stress and strain fields in the specimen. An average strain and an average stress are calculated as follows.

$$\dot{\varepsilon}_s(t) = \frac{V_{output}(t) - V_{input}(t)}{l_s}, \quad \sigma_s(t) = \frac{F_{input}(t)}{S_s} = \frac{F_{output}(t)}{S_s} \tag{5}$$

where l_s, S_s denote respectively the length and the cross sectional area of the specimen.

However, the stress-strain curves calculated by Eqn. (5) are correct only when stress and strain fields in the specimen are homogeneous. This is never true because of transient effects in the specimen. For example, when the input side of the specimen is loaded, the output side remains at rest until the wave goes through the specimen. In order to investigate this point, a natural method to do this is to simulate a test on a concrete specimen. For this purpose, a one-dimensional constitutive law for the specimen is proposed.

3.1 Chosen constitutive law of concrete, Numeric simulation using the method of Characteristics

A rate-dependant crack model (Sluys & de Borst, 1992) proposed for concrete is adopted to describe the rate-dependence and softening behaviour of the concrete-like material. In this model, it is supposed that the behaviour is purely elastic when the total strain ε is smaller than a crack threshold ε_s.

$$\varepsilon = \sigma / E \qquad if\ \varepsilon \le \varepsilon_s \tag{6}$$

When the total strain is larger than the threshold, it is assumed to be the sum of the elastic strain ε_e and a so-called crack strain ε_{cr} due to the opening of the micro-cracks cumulating in the concrete. Using a simple linear dependence of the stress on the crack strain and the crack strain rate, a four-parameters model is formulated.

$$\varepsilon = \varepsilon_e + \varepsilon_{cr} \qquad if\ \varepsilon > \varepsilon_s$$

with the two parts defined in the following:

$$\dot{\varepsilon}_e = \dot{\sigma} / E$$

$$\dot{\varepsilon}_{cr} = \frac{\sigma - E\varepsilon_s + k\varepsilon_{cr}}{\eta} \tag{7}$$

where $\sigma, \varepsilon, \varepsilon_{cr}, \varepsilon_s$ are respectively the stress, the total strain, the crack strain, and the crack threshold. E, k, η are coefficients associated with elasticity, linear softening and viscosity.

The one-dimensional governing equations and the constitutive law are written as follows,

$$\frac{\partial\sigma(x,t)}{\partial x} = \rho\frac{\partial v(x,t)}{\partial t}$$
$$\frac{\partial\varepsilon(x,t)}{\partial t} = \frac{\partial v(x,t)}{\partial x} \qquad (8)$$

where σ, ε, v are the stress, the strain, and the particle velocity in the specimen. ρ is the mass density.

The governing equation (Eqn.8) is hyperbolic and it can be then solved by the method of characteristics. Because of the form of constitutive law (Eqn.7), the characteristic network in this case is composed of families of straight lines (Zhao, 1998b). With a regular discretisation grid, the governing equations (Eqn.8) with the boundary conditions are numerically integrated.

3.2 SHPB test simulation

Using the presented numerical technique, fictitious tests can be simulated. However, it is more convincing to simulate a fictitious test close to a real one. On applying the velocities measured in a real SHPB test, corresponding simulated forces can be calculated with any arbitrary set of parameters of the model. The comparison between simulated forces and measured ones allows for an adjustment of the parameters to obtain an optimised matching between them.

For instance, a real concrete test is performed with aluminium bars (length 2m, diameter 40mm). The specimen is made of a microconcrete (36mm diameter, 36 mm length). An excellent matching is found (Fig.3) for this test with an optimised set of parameters ($E=2x10^{10}$ Pa, $k=1.5x10^{10}$ Pa, $\eta=8.5x10^{5}$ Pa.s).

The average stress-strain curve is then calculated from the waves with the classical formula (Eqn.5). Its comparison with the stress-strain curve given by the imposed model at the same strain rate history shows that this classical formula does not give a good estimate (Fig.4).

However, another possible average formula (Eqn.9) has been used (Lindholm, 1964) in the literature, which consists of calculating the stress by the average of input and output forces.

$$\dot{\varepsilon}_s(t) = \frac{V_{output}(t) - V_{input}(t)}{l_s}, \quad \sigma_s(t) = \frac{F_{input}(t) + F_{output}(t)}{2S_s} \qquad (9)$$

Fortunately, this so-called "three-waves" formula offers an acceptable result for the stress-strain relation (Fig.4), and it should be preferred to the so-called "two-waves" formula (Eqn.5) in the analysis of tests on concrete.

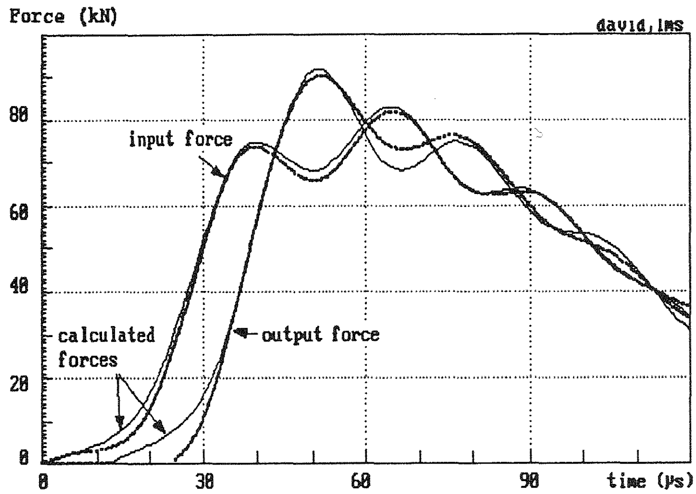


Fig. 3 Comparison between simulated and measured forces (SHPB)

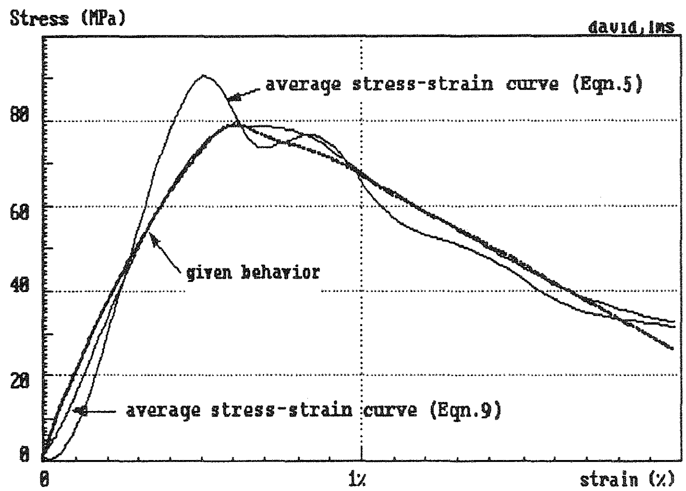
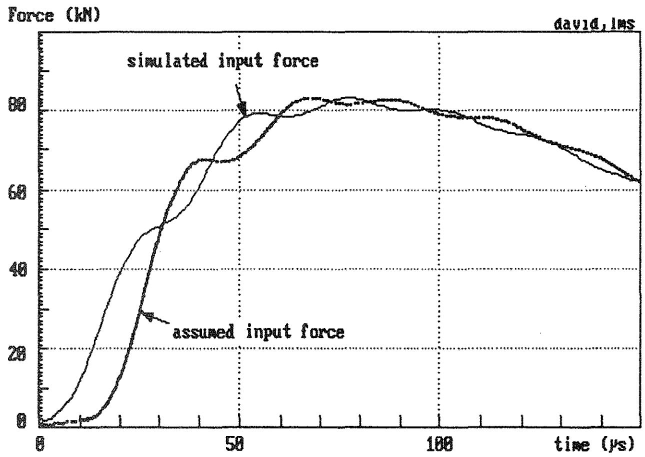


Fig. 4 Average stress-strain curves compared with the given behaviour (SHPB)

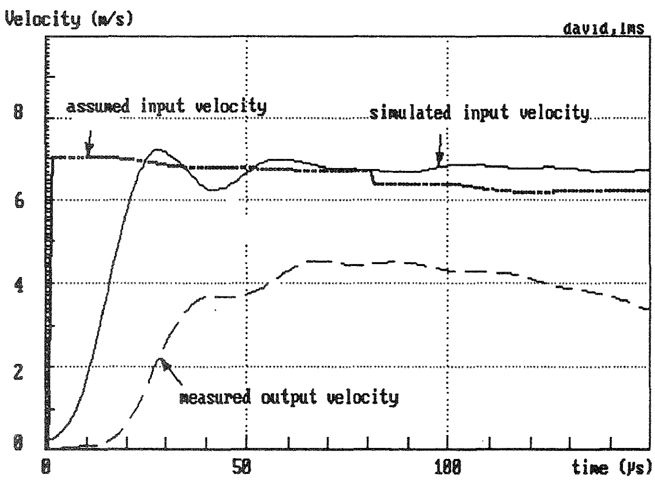
3.3 DIHB test simulation and discussions, inverse approach

In DIHB tests, the velocity and the force at the input side are obtained with supplementary assumptions because of the elimination of the input bar. Therefore, the efficiency of formula (9) strongly depends on those assumptions which are not completely analysed in the literature, probably because one has no evident means to obtain an experimental measure to check them.

It is tried here to investigate those assumptions by numerical simulations. Indeed, a fictitious test can be simulated with known boundary conditions as the force and the velocity measured at the output side in a real DIHB test. If the simulation is performed with a realistic concrete-like model, the simulated test is supposed to be representative for a real DIHB test on a concrete specimen. Therefore, the force and the velocity at the input side are known from this calculation. Comparisons between the simulated data at the input side and those assumed by the conventional DIHB analysis provide an estimate of the accuracy of DIHB conventional assumptions.



(a)



(b)

Fig.5 Comparison between simulated and assumed data (DIHB), (a) input force, (b)input velocity

The measured force and velocity at the output side for the same kind of microconcrete specimen as previously used are obtained in a DIHP system (40mm diameter aluminium output bar, 100mm diameter steel projectile) with an initial projectile velocity of 7m/s. The same model and parameter as those used in the previous simulation are applied to simulate input data.

It is shown in Fig.5 that both the simulated force and the simulated velocity are quite different from the assumed ones. This difference could not only be due to an eventually improper constitutive model. Consequently, the usual DIHB assumptions are not reliable for concrete testing, especially the assumption of the initial input velocity jump, because this infinitely small rise time induces an important error on the strain and the strain rate.

One has at least to know the rise time for the input velocity to obtain an acceptable accuracy. However, there is no evident measuring method and an assumption of an arbitrary rise time is not precise because it depends on the uniformity of contact between the striker and the specimen (Dharan & Hauser,1970).

4. The specific device for confined pressure tests.

In the modelling of concrete behaviour, it is important to perform tests under confining pressure (Gary & Bailly, 1998). Different methods can be used to provide such multiaxial loading conditions. A very simple one consists in making oedometric tests. Such a method has given good results for sand testing (Semblat, 1994) but cannot be easily applied to concrete because of unknown (and probably strong) longitudinal friction effects induced by the differential strains between the specimen and the thick confining cylinder.

To avoid longitudinal friction, another method consists of using a thin metallic ring compressed with the specimen. This method, used by Malvern et al. (1991), needs a special specimen preparation and the use of a strain-gage on the ring. It allows for a simultaneous measurement of the radial pressure and strain of the specimen, but without the ability of an easy control of the lateral pressure.

For sake of simplicity, we put the specimen into a cylindrical quasi-static pressure cell. The bars are acting as pistons and are introduced in the cell through seal rings. A scheme of the complete set-up is shown in Fig. (6). The lateral pressure can be applied with oil (up to 50 MPa), or with air (up to 10 MPa).

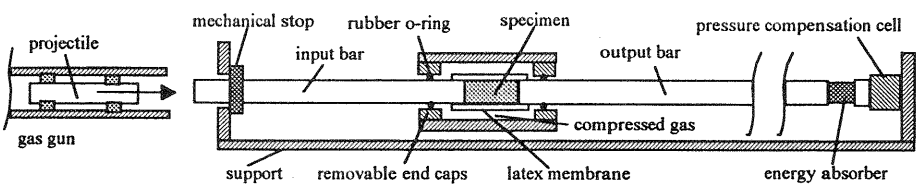


Fig. 6 Set-up of the confined pressure device

It seems reasonable to assume that a static confined pressure applied with air will not be significantly affected by the increase in the diameter of the specimen induced by the deformation (in this device, the chamber is 120 mm long and has a diameter of 75 mm so that with a 40 mm long and 40 mm diameter specimen, the volume of the fluid in the chamber is almost 10 times the volume of the specimen).

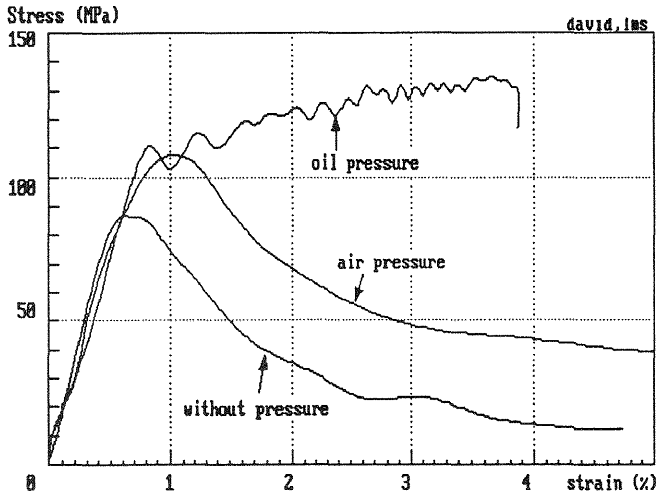


Fig. 7 Influence of the lateral pressure

The situation is not so clear with oil and it is not sure, because of transient effects in the fluid, that a measurement of the oil pressure during the test at a point in the chamber would give an exact measure of the pressure applied to the specimen. To evaluate this point, tests with oil and with air have been performed, using the same initial confinement pressure and other initial conditions. Results under oil pressure look very much like the ones presented in (Malvern et al. 1991) using a very similar device where water lateral pressure was used. When lateral pressure is applied with air, the stress strain relation shows a much lower apparent strain hardening, as presented in Fig. (7).

The same kind of results has been obtained in tests on sand specimens (Semblat, 1994) showing that oil pressure is increasing during the test. In that case, for similar tests (same specimen, same speed of loading, same initial lateral pressure), the stress associated with a given strain was minimum for air pressure tests, maximum for oedometric (zero lateral strain) tests, and in-between for oil pressure tests.

5. Conclusion

In this paper, the accuracy of tests at high strain rate on concrete is investigated. It is shown that the shifting of the waves, which are the only directly measured data, will not introduce significant error provided that the wave dispersion correction and that the exact time shift are carefully taken into account. For the application of lateral pressure using a confining pressure device, the use of air as fluid seems to be the most accurate technique.

A transient numerical simulation is developed using a method of characteristics and a realistic crack model. For the SHPB test, this simulation permits for the validation of the stress-strain relation given by so-called "three-waves" formula. For the DIHB test, this simulation shows that the usual assumption is not accurate.

6. References

Davies R.M. (1948), A critical study of Hopkinson pressure bar, **Phil. Trans. Roy. Soc.**, A240, 375-457.

Dharan C.K.H. and Hauser F.E. (1970), Determination of stress-strain characteristics at very high strain rates. **Exp. Mech.**, 10, 370-376.

Follansbee P.S. and Franz C. (1983), Wave propagation in the split Hopkinson pressure bar, **J. Engng. Mater. Tech.**, 105, 61-66.

Gary G., Klepaczko J.R. (1992), Résumé des résultats expérimentaux sur mini-béton, essai de compression, **Rapport scientifique GRECO**, 105-119.

Gary G., Bailly P. (1998), Behaviour of quasi-brittle material at high strain rate. Experiment and modelling, **European J. Mech.**, Accepted for publication.

Green H. (1958), The impact testing of concrete, in **Mechanical properties of non-metallic brittle materials**, (Ed. Walton W.H.), 300-317.

Kolsky H. (1949), An investigation of mechanical properties of materials at very high rates of loading, **Proc. Phys. Soc. London**, B62 , 676-700.

Lindholm, U.S. (1964) Some experiments with the split Hopkinson pressure bar. **J. Mech. Phys. Solids**, 12, 317-335.

Malvern L.E., Jenkinds D.A., Tang T. & McLure S., (1991), Dynamic testing of laterally confined concrete, in **Micromechanics of failure of quasi brittle materails**, Elsevier Applied Science, 343-352.

Pope P.H., and Field J.E. (1984), Determination of strain in a dynamic compression test, **J. Phys. E: Sci. Intrum.**, 17, 817-820.

Reinhardt H.W., Körmeling, H.A., and Zielinski A.J. (1986), The split Hopkinson bar, a versatile tool for the impact testing of concrete, **Matériaux et Constructions**, 19, 55-63.

RILEM committee (1975), The effect of impact loading on building, State-of-art report, Matériaux et Constructions, 8, 77-125.

Semblat J.F., (1994), **Sols sous sollicitations dynamiques et transitoires: réponse dynamique aux barres de Hopkinson, propagation d'ondes en milieu centrifugé**, Ph D. Thesis. Ecole Polytechnique.

Sluys L.J. and de Borst R. (1992), Wave propagation and localisation in rate-dependant cracked medium-Model formulation and one-dimensional example. **Int. J. Solids & Struct**, 29, 2945-2958.

Tang T., Malvern L.E., and Jenkins D.A. (1992), Rate effects in uniaxial dynamic compression of concrete, **J. Engng. Mech.**, 118, 108-124.

Zhao H. and Gary G. (1995), A three-dimensional analytical solution of longitudinal wave propagation in an infinite linear viscoelastic cylindrical bar. Application to experimental techniques. **J. Mech. Phys. Solids**. 43, 1335-1348.

Zhao H. and Gary G. (1996), On the use of SHPB techniques to determine the dynamic behavior of materials in the range of small strains, **Int. J. Solids & Structures,** 33, 3363-3375.

Zhao H. and Gary G. (1997), A new method for the separation of waves. Application to the SHPB technique for an unlimited measuring duration. **J. Mech. Phys. Solids**, 45, 1185-1202.

Zhao H., (1998a) A study on testing techniques for concrete-like materials under compressive impact loading, **Cement and Concrete Composite**, Accepted for the publication.

Zhao H., (1998b) A study of specimen thickness effects in the impact tests on polymers by numeric simulations, **Polymer**, 39, 1103-1106.