Fracture Mechanics of Concrete Structures
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# A STUDY ON CRACK PROPAGATION IN CONCRETE UNDER CYCLIC LOADING

T. Ueda, Y. Sato and Y. Kakuta
Division of Structural and Geotechnical Engineering, Hokkaido University, Japan
T. Tadokoro
Nippon Steel Co. Ltd.,
Japan

**Abstract**
Plain concrete prisms were tested under three-point-bending by a displacement control actuator. Along the predicted crack path wire strain gages were attached. From the measured strains it was found that at unloading compressive stresses were acting along the crack surface near the crack tip and that these compressive stresses caused tensile stresses in the concrete ahead of the crack tip. Both the compressive and tensile stresses increased with loading cycles. When the tensile stresses became large enough to cause cracking, the crack propagated. This phenomenon was also confirmed in the finite element analysis where a model for stress transfer along a crack obtained experimentally under uni-axial tension-compression cyclic loading was implemented.
Key words: Crack propagation, cyclic loading

## 1. Introduction

It is generally known that cracks in concrete propagate under cyclic loading. The crack propagation mechanism, however, has not been clarified much. In a previous study by Shin (1990) it was shown that the residual crack opening can not be the source of the crack propagation under cyclic loading due to the facts that a crack does not propagate during unloading in the experiment and the analysis. In another study by Horii et al. (1990) it was considered that the stress degradation in reloading causes the crack propagation as in both experimental and analytical results a crack propagates during reloading. In both studies only the crack opening was measured but not the concrete strains in the vicinity of a crack tip which more directly indicate the source of the crack propagation. In the present study the concrete strains along a crack as well as in front of a crack tip were measured. The crack propagation mechanism was examined both experimentally and analytically.

## 2. Outline of experiment

The specimen is a plain mortar prism with a notch at the center as shown in Fig. 1. The specimen was subjected to a three-point-bending. Wire strain gages with 10 mm length were attached at a spacing of 10 mm over the entire prism height in the region where a crack propagates (see Fig. 1).

The load was applied by a displacement control actuator. The compressive strength of the mortar was 78.8 MPa.

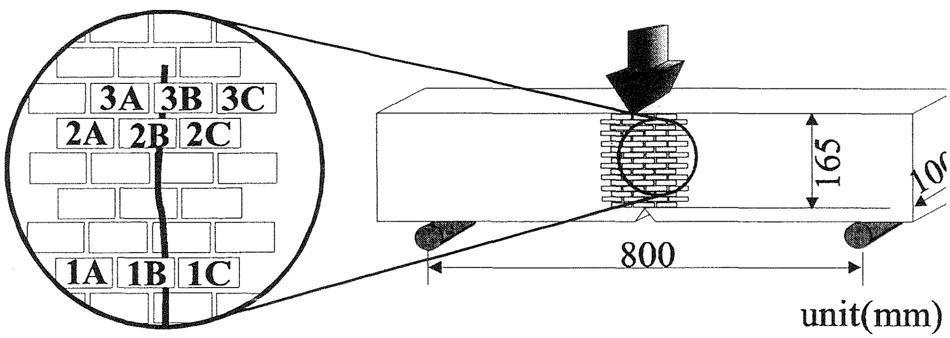



Fig.1. Specimen with gage arrangement and crack

## 3. Experimental results

The cyclic loading with 10 cycles was applied immediately after the peak load. The load-displacement curve was as shown in Fig. 2. Despite attempt to maintain the peak displacement, the actual peak displacement was increased bit by bit.

Relationships between measured strains at 1A-1C, 2A-2C and 3A-3C and load during the cyclic loading are shown in Figs. 3-7. The strain

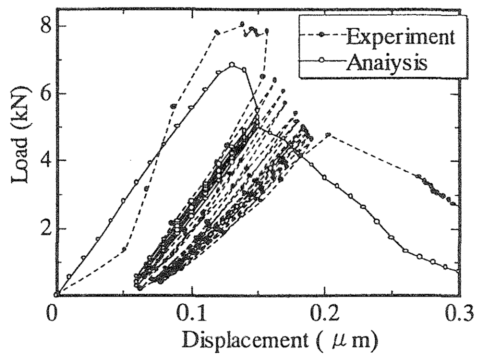
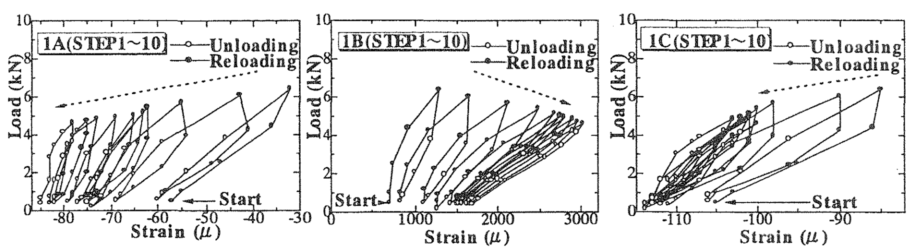



Fig. 2. Load-displacement curve

gages of 1A-1C, 2A-2C and 3A-3C were at 65 mm, 105 mm and 115 mm from the notch respectively. "Start" in those figures indicates the beginning of the cyclic loading.

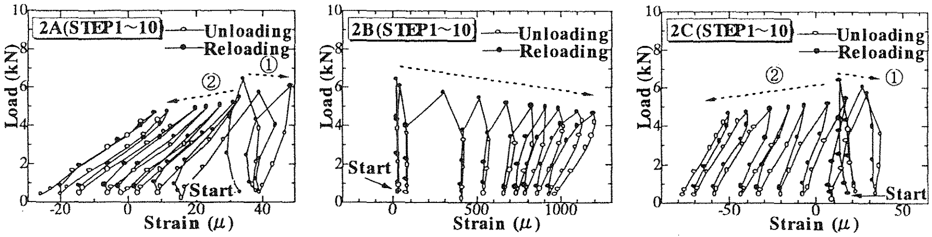
The crack propagated through the strain gages 1B, 2B and 3B as shown in Fig. 1. The tensile strain at 1B is already over 500 $\mu$ which means that the concrete has cracked at the beginning of the cyclic loading (see Fig. 1(b)). The tensile strains at 1B increased gradually with loading cycles, while compressive strains are seen at 1A and 1C and increase during unloading and with loading cycles (see Figs. 3(a) and (c)). Those compressive strains indicate that compressive stresses are acting along the crack surface and increase during unloading and with loading cycles. It can be considered that mismatch at the crack surface due to plastic deformation induced at cracking is the source of the compressive stresses. In a previous study on plain concrete under tension-compression cyclic loading by Reinhardt, et al. (1986) it was also observed that compressive stresses were acting through crack contact when the crack was closing (see Fig. 8).

At 2B the strains suddenly reach nearly 300 $\mu$ at the peak of the 3$^{rd}$ cycle and continue to increase in the following loading cycles as shown in




(a) Gage1A (b) Gage1B (c) Gage1C

Fig.3. Load-concrete strain curves




(a) Gage2A (b) Gage 2B (c) Gage 2C

Fig. 4. Load-concrete strain curves

Fig. 4(b). Thus it is considered that the crack reached there at the 3rd cycle. In the 1st and 2nd cycle tensile strains increase during unloading at 2A, 2B and 2C except in the 2nd cycle at 2A (see Figs. 4(a)-(c)). This fact indicates that tensile stresses increase during unloading. After the 3rd cycle tensile strains at 3B increase during reloading and with a number of loading cycles. At 3A and 3C tensile strains decrease with a number of loading cycles and eventually change to compressive strains. Those compressive strains at 3A and 3C increase during unloading and with a number of loading cycles. It is considered that the observed decrease in tensile strain is caused by tension softening of concrete and that the observed increase in compressive strain is due to compressive stress through crack contact as shown in the previous study by Reinhardt et al. (1986) (see Fig. 8). It can be said that relationship between load and strain in the vicinity of a crack tip is completely different between before and after cracking.

As seen in Fig. 6(b) in the 8th cycle the tensile strain at 3B suddenly starts to increase much quicker than in the 1st to 7th cycle. In the 7th cycle the tensile strain increases to over 200$\mu$ which is considered to be cracking. Before the cracking in the 8th cycle at 3A, 3B and 3C tensile strains increase during unloading except 5th to 7th cycle at 3C (see Figs. 5(a), 6(a) and 7(a)). This phenomenon is the same as that in the 1st and 2nd cycle at 2A, 2B and 2C (see Figs. 4(a)-(c)). After the cracking in the 8th cycle tensile strains increase during reloading and with a number of loading cycles at 3B (see Fig. 6(b)), while tensile strains decrease with a number of loading cycles at 3A and 3C (see Figs. 5(a) and 7(b)). This decrease in tensile strains is again considered to be caused by tension softening of concrete.

It is considered that the increase in tensile stress during unloading due to increased tensile strain occurs to balance the compressive stresses acting at the crack surface near the crack tip (see Figs. 3(a), 3(c), 4(a) and 4(c)). Since the compressive strains increase during unloading and with a number of loading cycles, the tensile strains also increase during

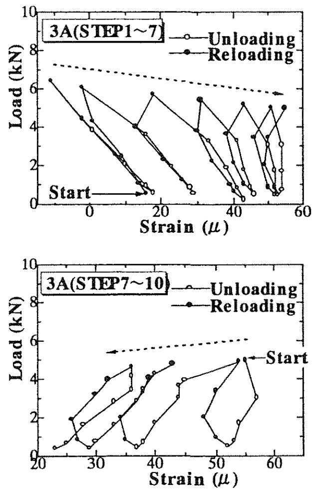



Fig. 5. Load-concrete strain curves
(a) Step 1-7
(b) Step 7-10

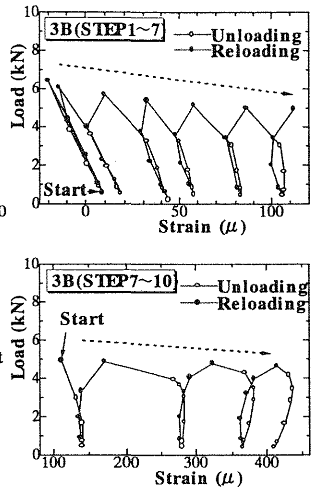



Fig.6. Load-concrete strain curves
(a) Step 1-7
(b) Step 7-10

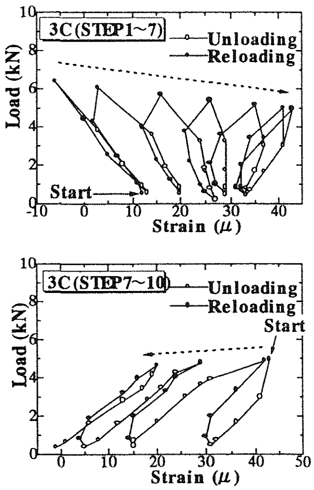



Fig.7. Load-concrete strain curves
(a) Step 1-7
(b) Step 7-10

unloading and with a number of loading cycles. When the tensile strain reaches its fracturing strain which is considered to be 150-200$\mu$, the crack propagates. Increased compressive stresses due to accumulated plastic deformation at a crack surface during loading cycle is considered to be a major source of the crack propagation. In the 3rd cycle the crack tip was at the level of 2B (105 mm from the notch) and at the level of 3B (115 mm from the notch) in the 8th cycle. It means that 4 cycles of loading extend the crack by 10 mm.

In the previous study by Shin (1990) crack propagation was only found during reloading. The same fact was observed in this study. It seems, however, in this study that the crack may not propagate without the accumulated tensile strain induced by the accumulated compressive stress along the crack surface during unloading.

## 4. Outline of analysis

A finite element program for concrete structures developed by Okamura and Maekawa (1991) was applied. The element mesh is shown in Fig. 9. The crack was modeled by link elements in which stress-strain

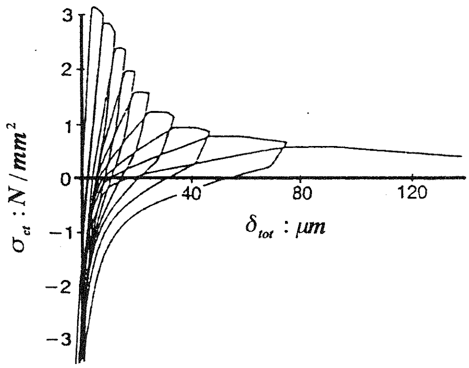



Fig. 8. Uni-axial tension test by Reinhardt et al. (1986)

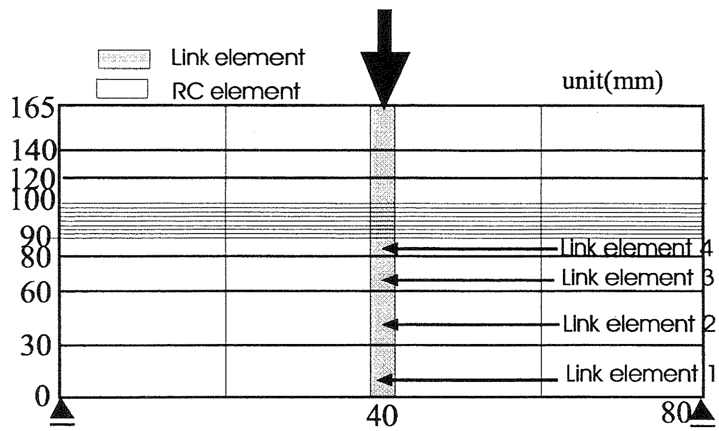



Fig. 9. Finite element mesh

relationships after the peak tensile stress under tension-compression cyclic loading were implemented. The following equation proposed by Reinhardt et al. (1986) was adopted for the envelope curve:

$$\frac{\sigma}{f_t} = \left\{1 + \left(c_1 \frac{\delta}{\delta_0}\right)^3\right\} \exp\left(-c_2 \frac{\delta}{\delta_0}\right) - \frac{\delta}{\delta_0}\left(1 + c_1^{\,3}\right)\exp(-c_2)$$

where $\sigma$ is a concrete stress at crack, $f_t$ is the tensile strength of concrete, $c_1 = 3$, $c_2 = 6.93$, $\delta$ is a crack displacement and $\delta_0$ is the critical displacement at which the stress becomes zero ($= 160\mu m$).

The inner curves for unloading and reloading were mathematically modeled to simulate the experimental results in the study by Reinhardt et al. (1986) shown in Fig. 8. In the concrete elements cracking was prevented, so that cracking only took place in the link elements.

## 5. Analytical results

Figure 10 indicates relationships between stress and displacement at the Gauss points in the link elements. In link elements 1 and 2 compressive stresses can be seen during unloading. Stress distributions along the link elements during loading, unloading and reloading are shown in Fig. 11. During loading it can be seen that tensile stresses decrease due to tension softening after the peak stress was reached. Tensile stresses are developed in front of the crack tip to compensate the stress reduction in the tension softening region. This is the mechanism of crack propagation during loading.

During unloading the tensile stresses continuously decrease and finally become compressive stresses. In order to balance the compressive stresses some tensile stresses start to act along or in front of the crack. Figure 12 indicates stress distributions along the link elements near the crack tip. During unloading tensile stresses decreases in the region below the crack tip, while tensile stresses increase in the region above the crack tip. It implies that the tensile stress to compensate the compressive stresses acting along the crack near the crack mouth can exist in the region above the crack tip where the concrete is still in elastic stage but not in the region below in which the tension softening takes place. The tensile stress at a Gauss point (point A in Fig. 12) increases to the peak stress in tension (point B), which indicates that the crack propagates to point B during unloading. It can be said, therefore, that the compressive stress induced by the plastic deformation at the crack surface during cracking and the following loading cycles is a source of the crack propagation

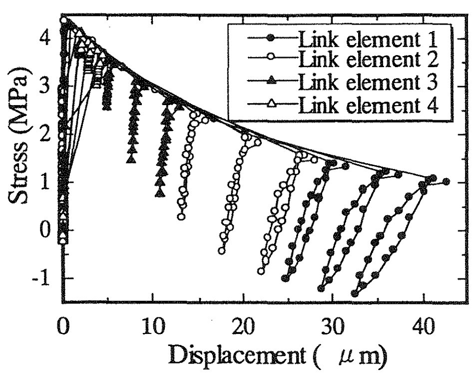



Fig. 10. Relationship between transferred stress and opening displacement in link element

during unloading.

The analysis by Horii et al. (1990) indicated that the stress degradation during loading cycles is the source of crack propagation. This study also considers the stress degradation which is less than that in the study by Horii et al. (1990). However, it was indicated in this study that plastic deformation can be also a cause of the crack propagation.

Crack propagation predicted by the analysis was 0.9 mm in one cycle which is less than that observed in the experiment. The loading cycles with a constant peak displacement which was given for the analysis is considered to be a cause of the underestimated crack propagation because the actual peak displacements increased. The assumed stress degradation which may be less than the actual one can be another cause of the underestimation.

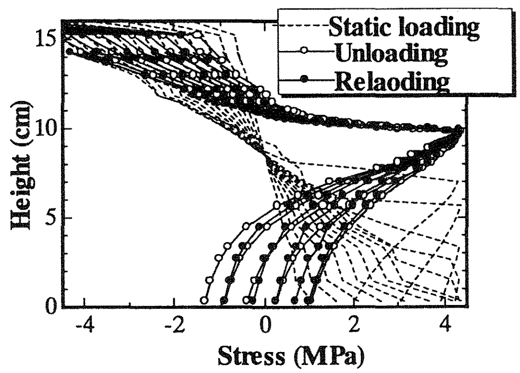



Fig.11. Concrete stress distribution along link elements

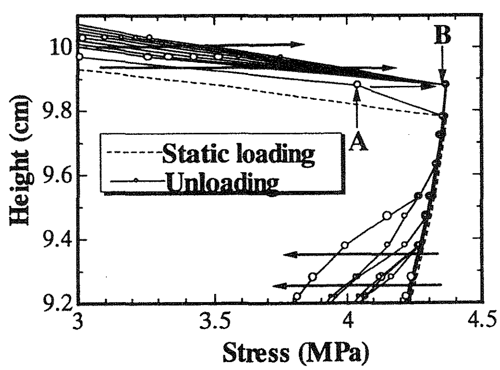



Fig. 12. Concrete stress distribution along link elements (Near crack tip)

## 6. Conclusions

By conducting cyclic loading tests of a plain concrete prism as well as a finite element analysis of the prism under cyclic loading the following conclusions are drawn:

- It was observed in the experiment that compressive stresses along a crack increase during unloading and that tensile strains ahead of the crack tip increase during unloading. The increased tensile strain is considered to be a major source of the crack propagation during loading cycles.
- The finite element analysis with relationship between crack stress and crack displacement under tension-compression cycle also indicates the compressive stress along a crack and the tensile stress ahead of the crack tip which causes the crack propagation during unloading.

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