Fracture Mechanics of Concrete Structures
Proceedings FRAMCOS-3
AEDIFICATIO Publishers, D-79104 Freiburg, Germany

FRACTURE OF CONCRETE UNDER MIXED LOADING. EXPERIMENTAL RESULTS AND NUMERICAL PREDICTION

J.C. Gálvez, D.A. Cendón, J. Planas, G.V. Guinea and M. Elices
Departamento de Ciencia de Materiales, E.T.S.I. Caminos,
Universidad Politécnica de Madrid,
Spain

Abstract
A novel testing procedure for mixed crack propagation in concrete is presented: *Four Point Bending* of notched beams under the action of two independent force actuators. In contrast to classical procedures, this method allows non-proportional loading and crack trajectory modifications by changing the action of one actuator. Different experimental crack trajectories, under mixed mode and non-proportional loading, are presented together with the corresponding curves of load-CMOD and load-displacement. The tests were developed for three sizes of geometrically similar beams, and the results are useful for checking the accuracy of mixed mode propagation numerical programs.The programs should predict the crack trajectory and a complete group of experimental records for three different sizes of beam. The Cohesive Crack Model is incorporated into a finite element code with satisfactory results.
Key words: Concrete, fracture, mixed mode, finite element method.

1 Introduction

Mixed mode crack propagation in concrete is a complex problem, even in two dimensions. During the last two decades much work has been done to improve the knowledge of this problem in order to develop analytical and numerical tools to describe the initiation and propagation of the cracks in mixed mode I/II in concrete structures.

A relatively large number of experimental results of the crack initiation and the propagation in mixed mode on notched beams are based on the Iosipescu geometry (Iosipescu 1967): Arrea & Ingraffea (1982), Bazant & Pfeiffer (1986), Bocca et al. (1991) and Schlangen (1993), among others. Other important sets of tests have been developed by the Delft group on notched specimens partially cracked in tension (Nooru-Mohamed & Van Mier 1990; Nooru-Mohamed 1992). More detailed information on other geometries and experimental data are found in Gálvez et al. (1998). The advance has been important, but some difficulties remain. The Iosipescu geometry and the non-symmetric three point bend tests give trajectories of the crack that are easy to predict within the wide scatter of the results. The tests of the Delft group led to an overlapping of several cracks and showed dependence on the loading history. In the opinion of the authors, there is no adequate *benchmark* to verify analytical and numerical models for mixed mode fracture of concrete.

The numerical aspects of the cohesive crack model has been included in finite element programs (Valente 1995, Reich et al.1993 and Çervenka 1994) as well as in boundary element programs (Saleh and Aliabadi 1995). In this work, it is assumed that the crack grows in the direction normal to the maximum principal stress (the Maximum Tangential Stress criterion). This hypothesis has been verified for materials of almost linear elastic behaviour (Gálvez et al. 1996), but it has to be verified for mortar and concrete with more involved trajectories of the cracks.

The purpose of this contribution is to provide additional experimental information on mixed mode fracture of concrete: a novel testing procedure, crack trajectories, load-displacements and load-CMOD curves that may help research in this field. A numerical prediction of the experimental tests is also presented.

2 Experimental program

2.1 Mixed mode I/II fracture test

Fig. 1a shows the geometry of test specimens and the forces diagram of the testing procedure. Forces P1 and P2 are supplied by independent force actuators, which permits non-proportional loading of the specimen. Different combinations of P1 and P2 provide different trajectories of the crack. Stable tests were achieved by applying P1 through a servocontrolled testing machine and P2 through a spring boundary condition (Fig. 1b).

2.2 Material and specimens

Five identical concrete mixtures of concrete were used for the specimens, composed of Portland cement, siliceous sand as fine aggregates, and siliceous crushed coarse aggregates of 5 mm maximum size. The cement was supplied in bulk to guarantee the homogeneity of the tests. The water/cement ratio was 0.45 and the mean settlement measured with the Abram's cone for the five mixes was 100 mm, approximately.

On each mixture, three homotetic sizes of specimens were cast. The dimensions and number of specimens are detailed in Table 1. The fracture energy, compressive strength, Young's modulus and tensile strength were determined in accordance with RILEM 50-FMC, ASTM C39, ASTM C469 and ASTM C496, respectively. Mechanical properties of the concrete are detailed in Table 2.

The specimens were cast horizontally in one layer in rectified steel moulds. The specimens were left in the moulds 72 hours, covered with saturated sacking at room temperature. They were then left in a curing room at 20°C and 99% relative humidity for 28 days; from then until testing they were immersed in lime saturated water at room temperature. The notches were machined with a low speed diamond cutting disc. The support surfaces were ground to avoid spurious displacements due to crushing of the irregularities of the surface.

The specimen nomenclature is the following: Mixture; Size; Number of specimen.

2.3 Experimental procedure

To obtain different crack trajectories, the extreme values of the spring stiffness (K) were adopted: (a) In type 1 tests, K was 0 (Fig. 2a), (b) in type 2 tests, K was ∞ (Fig. 2b). In type 2 tests, the boundary condition at point B was imposed by an actuator that prevented vertical displacement throughout the test.

During the tests the following parameters were recorded: the Crack Mouth Opening Displacement (CMOD), the load P, the load-point displacement of force P, the displacement of point B (Figs. 2a and 2b) and the reaction force at point B (test type 2).

The tests were performed in CMOD control. The CMOD rate was

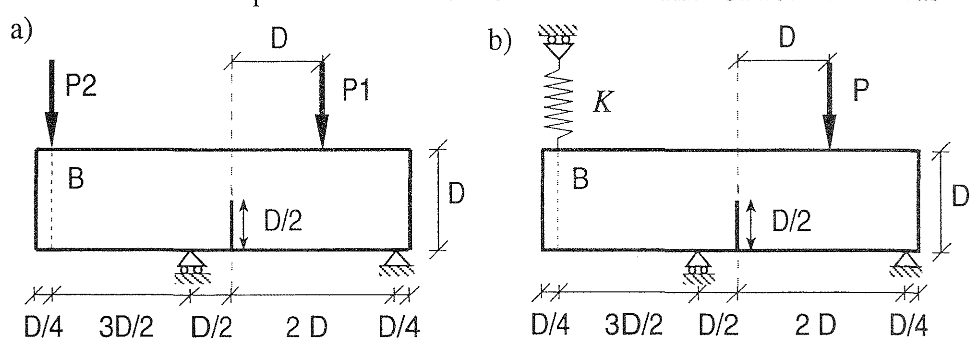

Fig. 1. Geometry and forces diagram of testing procedure under non proportional mixed mode. (a) Idealized. (b) Experimental device.

0.004 mm/min until 40 % of the peak load in the descending branch and 0.08 mm/min until the end of the test. Type 1 tests were performed without displacement control at point B.

2.4 Testing equipment

Load P was supplied by a servocontrolled INSTRON 1275 testing machine, the reaction at point B through a servocontrolled force actuator INSTRON 1287. The load P and reaction at point B were measured with 5, 25 and 100 kN INSTRON load cells with ± 0.5 % error at full scale. Details of the extensometers are given in Gálvez et al. (1998).

Table 1. Dimensions and number of specimens per batch.

	D (mm)	*L* (mm)	*B* (mm)	Number of specimens	Objective
D1	75	340	50	8	MP & MM
D2	150	675	50	4	MM
D3	300	1350	50	2	MM

D: Depth; *L*: Length; *B*: Thickness
MP: Mechanical properties; MM: Mixed mode tests

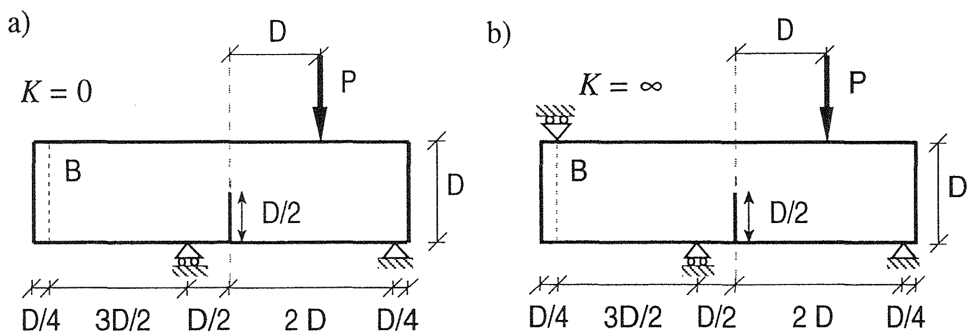

Fig. 2. Geometry, forces and boundary conditions in mixed mode tests. (a) Type 1 (K= 0). (b) Type 2 ($K = \infty$).

Table 2. Mechanical properties of the mixtures.

Batch	f_{ck} (MPa)	f_{ct} (MPa)	G_c (N/m)	E (GPa)
1	54	3.0	69	38
2	56	3.2	70	38
3	56	2.8	61	38
4	61	3.0	75	39
5	57	3.0	69	39

3 The cohesive crack model on mixed mode fracture

3.1 Numerical simulation technique

A numerical simulation of the mixed mode tests was made by the incorporation of the cohesive crack model into a finite element code. The main stages of the process are the calculation of the crack path for each size of specimen and type of test, and the incorporation of the cohesive crack model to the crack path.

The crack trajectory is calculated by means of the maximun principal stress criterion. The numerical prediction is made with the FRANC2D code.

The cohesive crack model incorporation uses non-linear springs. 100 non-linear equidistant springs are used for the numerical simulation, each following the softening law of the concrete and oriented in a direction normal to the faces of the crack. For the numerical simulation, the ABAQUS® code is used and the arc-length technique adopted.

3.2 Softening law

The softening law is an integral part of the cohesive crack model. This function, a material property, relates the stresses acting across the crack faces to the corresponding crack openings. In this work, a bilinear approximation was used. The parameters of the curve were determined using the general bilinear fit (GBF), developed by Guinea et al. (1994), on notched three point bend beams of D = 75 mm .

4 Experimental results and numerical prediction

4.1 Trajectories of the cracks

Fig. 3 shows the experimental envelope of the trajectories of the cracks and the numerical prediction. The accuracy of the prediction lends further support to the hypothesis that the elastic fracture crack path is a good approximation for concrete structures, even though fracture is clearly non-linear. So far, this result has been shown to be valid for stable tests. Further testing is needed before it can be used for unstable crack growth.

4.2 Load-displacement and load-CMOD curves

Fig. 4 and Fig . 5 show the records of load P vs CMOD and displacement of the application point of force P. The agreement is good, particularly for the smallest (D = 75 mm) and medium (D = 150 mm) sizes.

The peak load is accurately predicted in all cases by the numerical model, except for the biggest size (D = 300 mm) with type 2 tests.

The numerical prediction of the descending branch in the curves of load P vs CMOD (Figs. 4a, 4c, 4e, 5a, 5c and 5e) is good, as is that of load P vs displacement of the aplication point of the load P (Figs. 4b, 4d, 4f, 5b, 5d and 5f), with the exception of the biggest size (D = 300 mm); this suggests that the experimental fracture energy in mixed mode is greater in

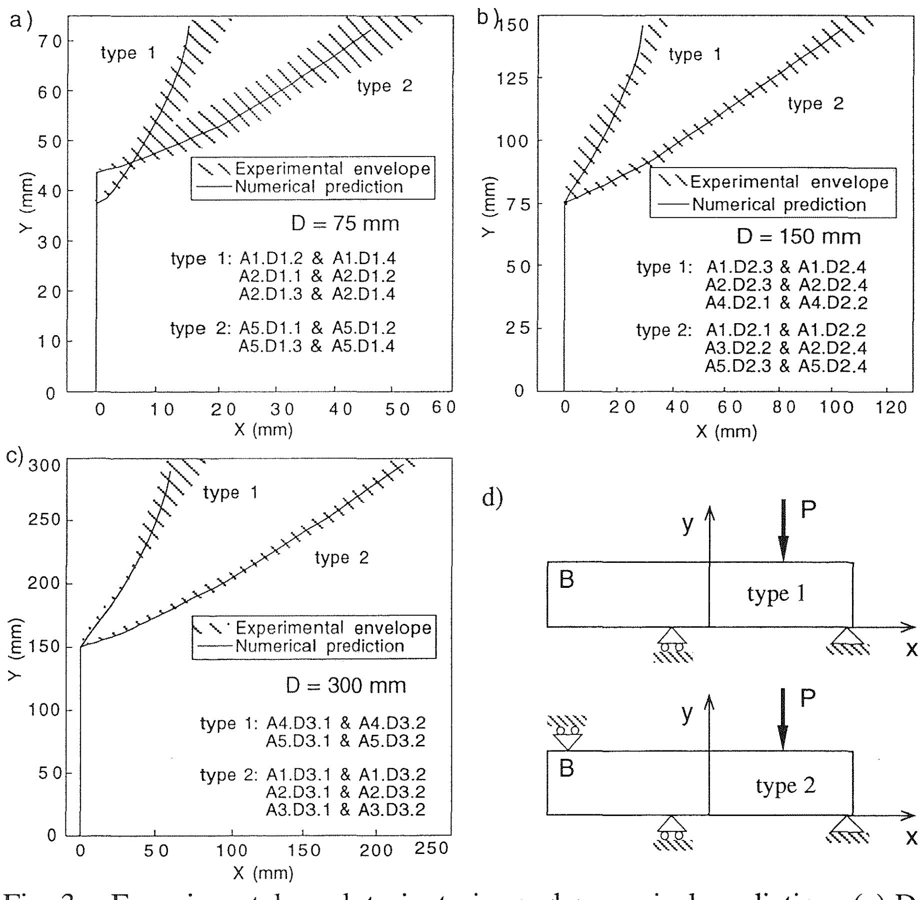

Fig. 3: Experimental crack trajectories and numerical prediction. (a) D = 75 mm. (b) D = 150 mm. (c) D = 300 mm. (d) Axes of reference.

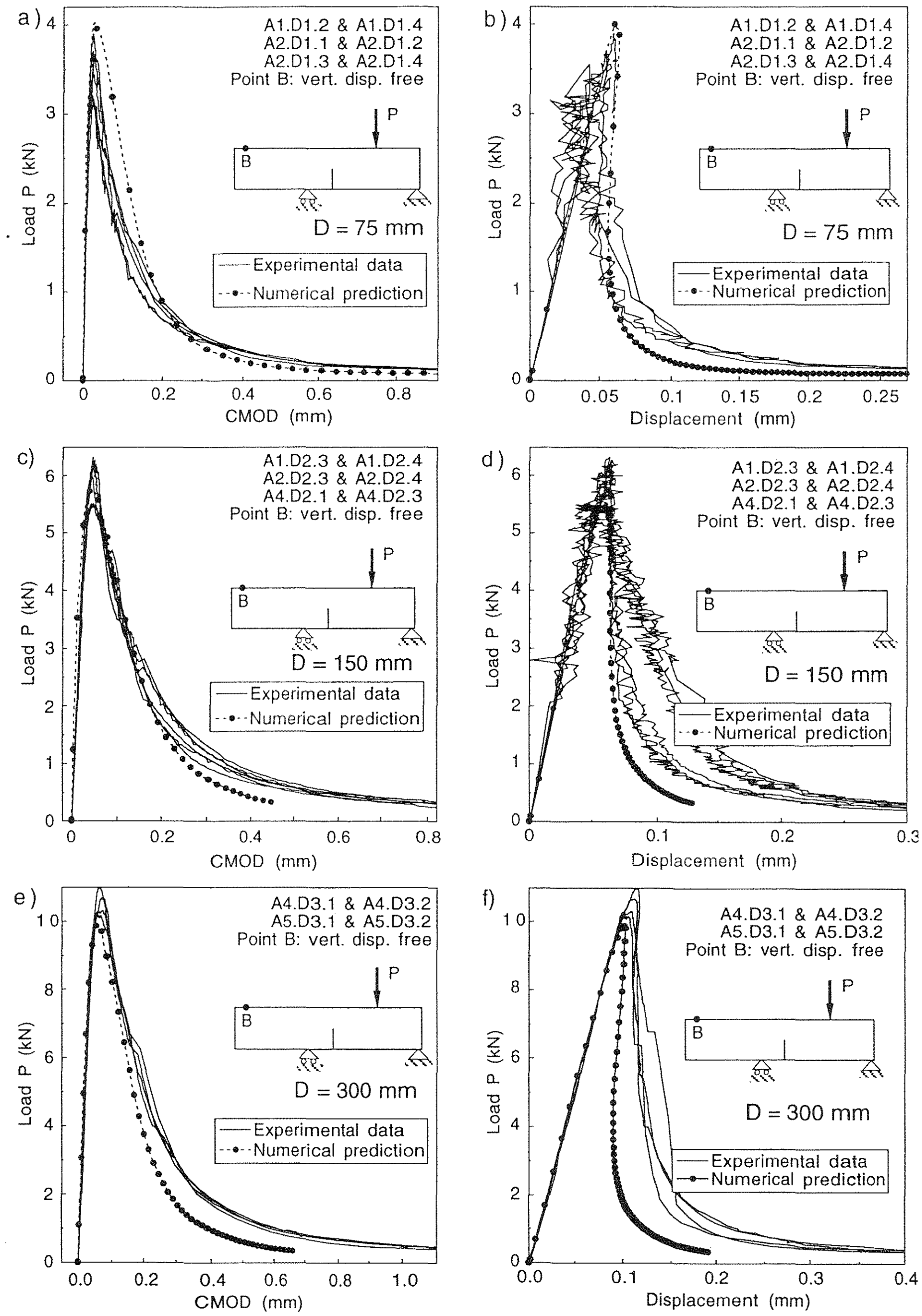

Fig. 4: Curves load-CMOD and load-displacement for the *type 1* tests and sizes D1 (D = 75 mm), D2 (D = 150 mm) and D3 (D = 300 mm).

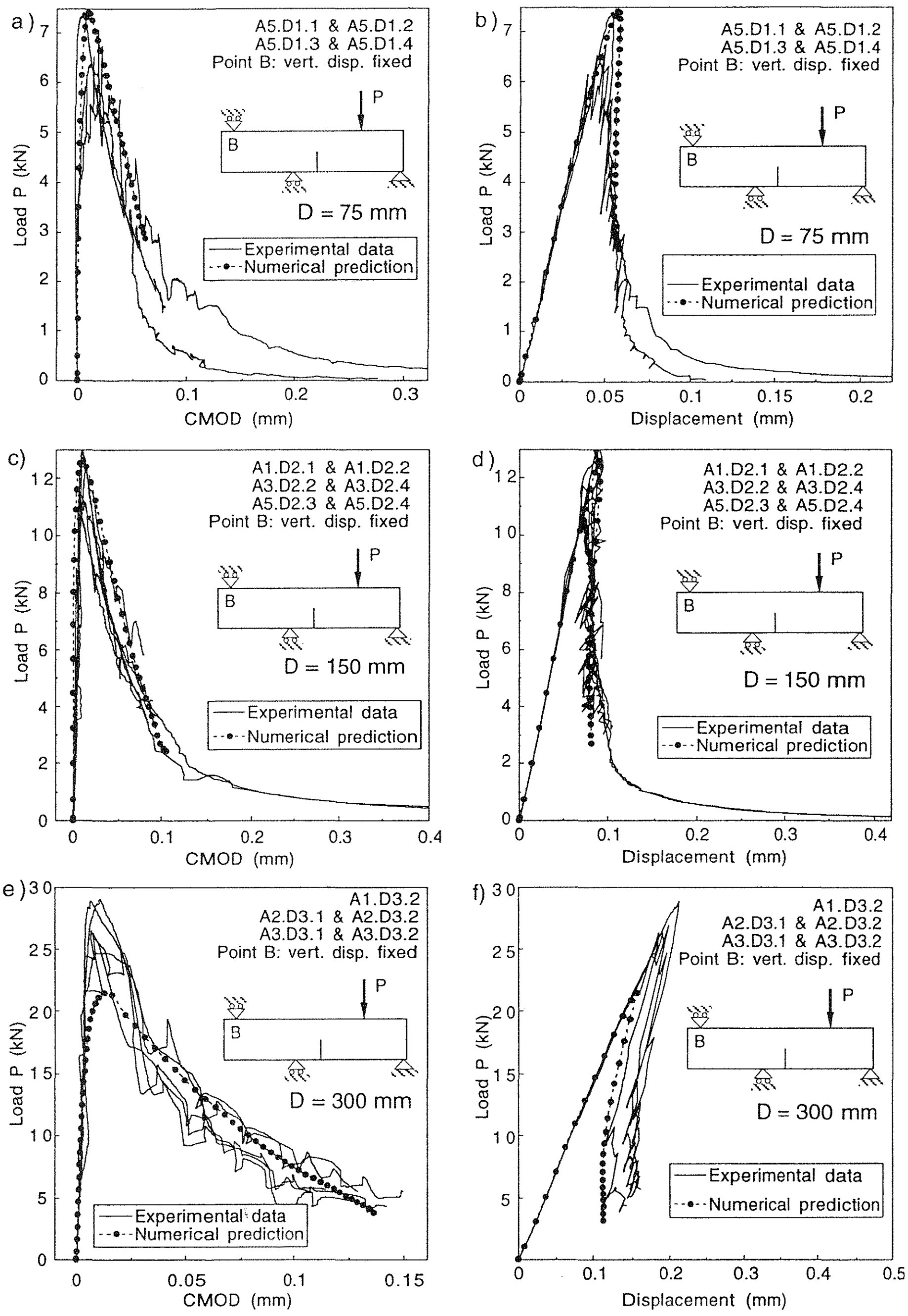

Fig. 5: Curves load-CMOD and load-displacement for the *type 2* tests and sizes D1 (D = 75 mm), D2 (D = 150 mm) and D3 (D = 300 mm).

the largest specimens than in mode I with small specimens.

5 Conclusions

- The trajectory of the cracks in concrete structures can be approximated by means of a linear elastic model, even thought that the fracture behaviour of concrete is clearly non-linear.
- The incorporation of the cohesive crack model into a finite element program gives a good prediction of the mixed mode fracture of concrete. More specifically, the experimental load P vs CMOD and load P vs displacement curves are well predicted by the numerical model, specially for the smallest and the medium sizes of the tested beams.
- For the largest sized beams, the numerical prediction, based on the fracture properties of concrete measured with the smallest beams, leads to worse prediction of the descending branch of the experimental load P vs displacement curves. It suggests an apparent size effect on the descending branch.
- Further research is required to explain the apparent size effect on the descending branch of the curves load P vs displacement.

Acknowledgements: the authors wish to thank Portland Valderribas Inc. for supplying the cement, Prof. A. Ingraffea for providing the computer program FRANC. The authors also gratefully acknowledge finantial support for this research given by the Comisión Interministerial de Ciencia y Tecnología, Spain, under grants MAT 97-1022 and MAT 97-1007-C02-2.

6 References

Arrea, M. and Ingraffea, A. (1982) **Mixed Mode Crack Propagation in Mortar and Concrete**. Report 81-13, Dpt. of Structural Engineering, Cornell University.

Bazant, Z.P. and Pfeiffer, P.A. (1986) Shear fracture tests of concrete. **Materials and Structures**, 19, 111-121.

Bocca, P., Carpinteri, A. and Valente, S.(1986) Mixed node fracture of concrete. **Int. J. Solids Structures**, 27, 1139-1153.

Cervenka, J. (1994) **Discrete Crack Modelling in Concrete Structures**. Ph. D. Thesis, University of Colorado.

FRANC2D: A Two-Dimensional Crack-Propagation Simulator.

Version 2.7, P. Wawryzynek and A. Ingraffea.

Gálvez, J.C., Elices, M., Guinea, G.V. and Planas, J. (1996) Crack trajectories under mixed mode and non-proportional loading. **International Journal of Fracture**, 81, 171-193.

Gálvez, J.C., Planas, J., Elices, M. and Guinea, G.V. (1998) Mixed mode fracture of concrete under proportional and non proportional loading. *To be published..*

Guinea, G.V., Planas, J. and Elices, M. (1994) A general bilinear fit for the softening curve of concrete. **Materials and Structures**, 27, 99-105.

Hilleborg, A., Modeer, M. and Petersson, P. (1976) Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. **Cement and Concrete Research**, 6, 773-782.

Iosipescu, N. (1967) New accurate procedure for single shear testing of metals. **Journal of Materials**, 2, 537-566.

Nooru-Mohamed, M.B. (1992) **Mixed Mode Fracture of Concrete: an Experimental Approach**, Ph. D. Thesis, Delft University.

Nooru-Mohamed, M.B. and Van Mier, J.G. (1990) Geometrical and structural aspects of concrete fracture. **Engineering Fracture Mechanics**, 35, 617-628.

Reich, R., Plizarri, G., Cervenka, J. and Saouma, V. (1993) **Numerical Models in Fracture Mechanics of Concrete**, (ed. Wittman), Balkema, Rotterdam, 265-286.

Saleh, A. and Aliabadi, M. (1995) Crack growth analysis in concrete using boundary elements method. **Engineering Fracture Mechanics**, 51, 533-545.

Schlangen, E. and Van Mier, J.G. (1993) Mixed-mode fracture propagation: a combined numerical and experimental study. **Fracture and Damage of Concrete and Rock**, 166-175.

Valente, S. (1995) On the cohesive crack model in mixed-mode conditions, in **Fracture of Brittle Discordered Materials: Concrete, Rock and Ceramics**, (eds. G. Baker and B. Karihaloo), E & FN Spon, London, 66-80.