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STRAIN SOFTENING AND STRUCTURAL ANALYSIS OF BEAMS FAILING IN COMPRESSION

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Abstract
In the last few years it has been accepted that localization occurs for concrete loaded in compression. Localization means that the descending branch of the stress-strain curve is dependent on specimen size, and the complete stress-strain curve can therefore not be regarded as a pure material property. The observed size effect on the softening branch and its influence on beams failing in flexure is discussed. Further the effect of localization on structural analysis is addressed from a fracture mechanics point of view. It is shown that the size effect on the stress-strain curve is of some importance for the moment capacity of over-reinforced beams and of great importance for the ductility of under-reinforced beams.
Key words: concrete, strain softening, size effects, strain gradients

1 Introduction

Material models for compressive failure of concrete are normally based on a uniaxial compressive stress-strain curve obtained from tests, where uniform deformation of the concrete specimen is assumed. While this assumption is reasonable for the ascending branch of the stress-strain curve, this is not the case of the softening branch. In fact, experimental studies have shown that the deformation after peak stress is more or less localized to certain zones, e.g. van Mier (1984), Rokugo and Koyanagi

(1992), Markeset (1994), Vonk (1993), van Mier (1997). Due to the localization phenomenon the complete stress-strain curve becomes specimen size dependent

Most load-bearing elements, such as beams, columns and slabs, experience strain gradients. As localization of deformations occurs when the failure stage is approached, it is clear that one single stress-strain curve obtained from centric loading test will not realistically represent the behaviour of concrete subjected to strain gradients. Improperly characterizing the post peak behaviour of concrete can lead to poor and unsafe prediction of the ductility and ultimate axial deformation of columns and the rotational capacity, balanced steel ratio, and ultimate moment capacity of beams.

The observed size effect on the softening branch and its influence on the analysis of concrete members subjected to compressive strain gradients is discussed in this paper.

2 Strain softening and size effect

From a Round Robin test, (van Mier et al. (1997)) it was found that the pre-peak stress strain curve was independent of the slenderness when low-friction loading platens were used. In the softening regime, however, an increase of ductility in terms of stress and strain with decreasing specimen slenderness was found in all experiments. It was observed that a strong localization of deformation occurs in the softening regime, irrespective of loading system used. The compressive failure is found to take place in a zone of limited length. The length of this damage zone is reported to be of order 2 to 3 times the width of the specimen, see e.g. Rokugo and Koyanagi (1992), Markeset (1993, 1994), Jansen and Shah (1997). In the zone where damage is occurring, the displacement continues to take place under decreasing load. Outside this zone the concrete expands axially as unloading occurs.

Within the damage zone extensive lateral deformations occur. The lateral deformations have to do with longitudinal distributed cracks, which form and expand during the failure process. In this damage zone two different fracture processes may develop. A sliding mode of failure occurs when the inclined micro-cracks coalesce to form inclined localized shear bands and a tensile mode of failure (axial splitting) occurs when a critical lateral deformation is exceeded. The influence of each failure mode may, however, depend on factors such as: water-cement ratio, type of aggregate and maximum size, amount of steel fibres and stirrups.

The total amount of shear bands (within the damage zone) is strongly dependent on the specimen height as illustrated in Fig. 1. Whereas few shear bands are observed for specimen with slenderness 2.0 (height 200 mm), the number increases with decreasing specimen height.

The influence of specimen length on the softening branch of the stress-strain curve is illustrated in Fig. 2 for a high strength concrete. Formation of shear band and damage zone is illustrated on specimens with slenderness 3.0 and 4.0, respectively.

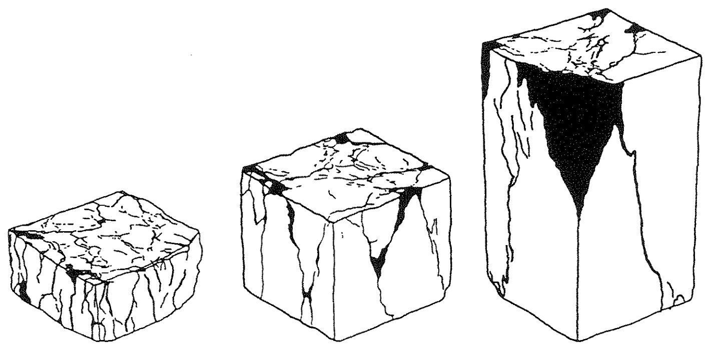
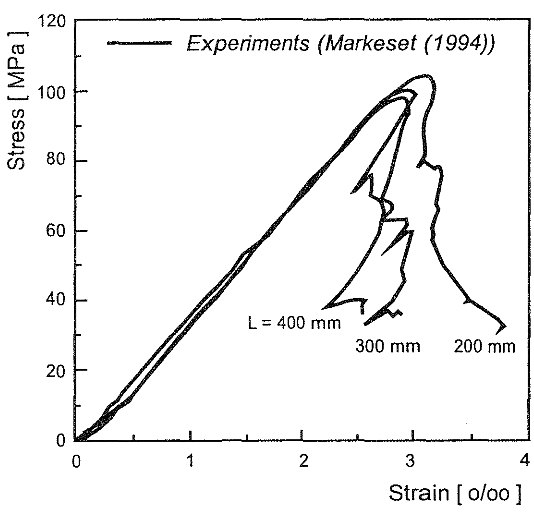


Fig. 1. Crack patterns of normal strength concrete prisms, van Vliet and Mier (1996)



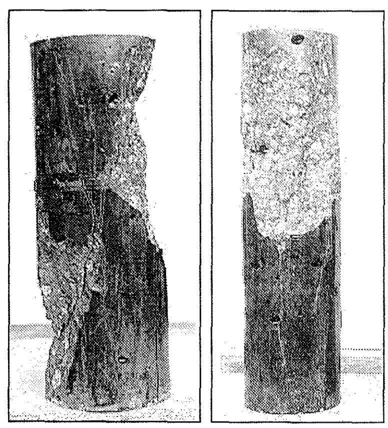


Fig. 2. Influence of specimen length on the complete stress-strain curve, and formation of shear band and damage zone, Markeset (1994). Cylinder diameter is 100 mm

The size effect on strength and ductility of reinforced concrete beams failing in flexure was studied by Adachi et al. (1995). The beam notations and dimensions, steel reinforcement ratio, $A_s/(BD)$, and yield strength/tensile strength of the tensile reinforcement are listed in Table 1. Moment displacement curves for the beam tested are given in Fig. 3. No stirrups nor compression reinforcement were applied in the constant moment area.

Table 1. Structural variables for the beams tests shown in Fig. 3
B=width, D=height, L=length of beam

Name of specimen	**Size of specimen BxDxL (mm)**	**Reinforcement ratio (o/o)**	**Yield strength/ tensile strength**
B07 1	300x600x5400	0.62	385/566
B21 1		1.64	361/570
B07 2	150x300x2700	0.52	364/503
B21 2		1.77	378/551
B07 4	75x150x1350	0.50	439/561
B21 4		1.51	439/561
B07 8	37.5x75x675	0.75	368/497
B21 8		2.23	348/485

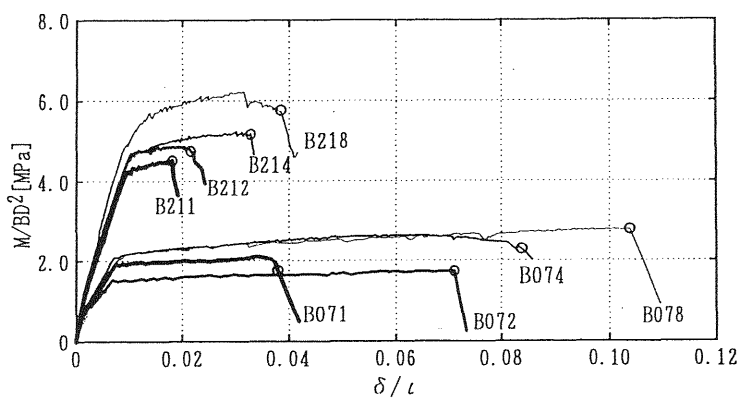


Fig. 3. Normalized bending moment-displacement curves, Adachi et al. (1995)

From Fig. 3 it can be seen that there is a size effect on the strength for the beams with the highest reinforcement ratio. For the specimens with the lowest reinforcement ratio a pronounced influence of size on the ductility is observed. It is, however, not clear whether the observed size effect in strength for the series B21 only is a result of the strain localization in compressive softening, since the tensile reinforcement is in the yielding region when maximum moment is achieved. Most likely the observed size effect in strength is caused by a combination of strain localization in the compressive zone and strain hardening in the reinforcing steel.

The size effect or the strain gradient effect on the deformation capacity of the concrete compression zone was studied by Meyer (1997). Three different sizes of eccentrically loaded prisms were tested. The eccentricity was kept constant for all dimensions. Three different concrete strength (C25, C40 and C60) were tested without transversal reinforcement. In addition the confinement effect from stirrups was studied on specimens with concrete strength C40. He reported a decrease of ultimate strain, i.e. the strain in the most compressed fibre at maximum load capacity, with increasing depth of the compressive zone for both unconfined and confined specimens. However, it was observed that the activation of transversal reinforcement decreases the size dependence of the deformation capacity. The relationship between ultimate strain and the depth of the compression zone on unconfined specimens is shown in Fig. 4.

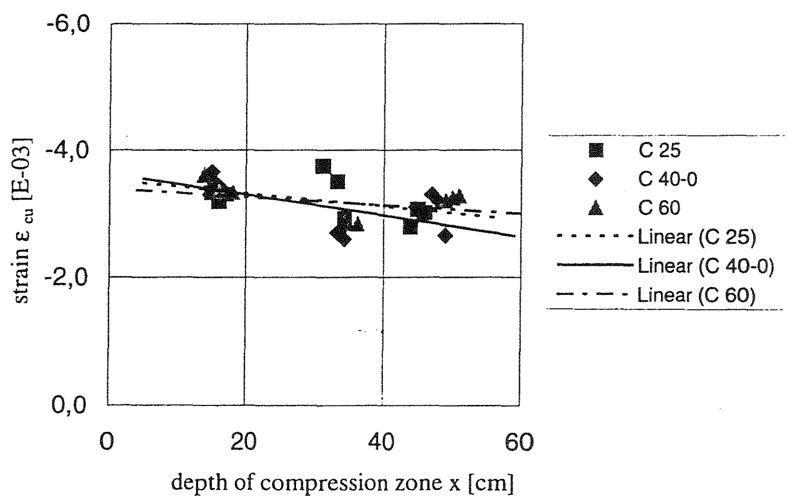


Fig. 4. Ultimate concrete strain, ε_{cu}, versus depth of the compression zone, Meyer (1997)

3 Compressive softening models based on fracture mechanics

Analogue to the Fictitious Crack Model in tension, Hillerborg et al. (1976), Hillerborg (1988) proposed to model the fracture of concrete in compression by a stress-strain curve for the ascending branch and a stress-deformation curve for the descending branch. Hence, the softening behaviour is assumed to concentrated to a localized zone, whereas the material outside the crack is assumed to be mechanically intact, Fig. 5. The average strain on a certain length L containing one localization zone can then be written as:

$$\varepsilon_m = \varepsilon + \frac{w}{L} \tag{1}$$

where w is the deformation in the direction of the compressive stress within the localized zone.

A similar model was proposed by Bazant (1989). However, here the failure was not localized to a line crack as proposed by Hillerborg, but assumed to occur within a band of finite length.

A model proposed by Markeset (1993), named the Compression Damage Zone (CDZ) model, is based on the hypothesis that compressive failure of concrete occurs within a damage zone, L^d, of limited length. This length seems to be of order 2.0 to 3.0 times the width of the specimen. Within this zone the failure mode is a combination of distributed axial splitting and sliding in a localized band. The model is illustrated in Fig. 6.

The first curve in Fig. 6 is the stress-strain curve for the material loaded up to the compressive strength and then unloaded. This curve is valid for the concrete material in the whole specimen. The second curve shows the relationship between the stress and the average additional strain ε_d within

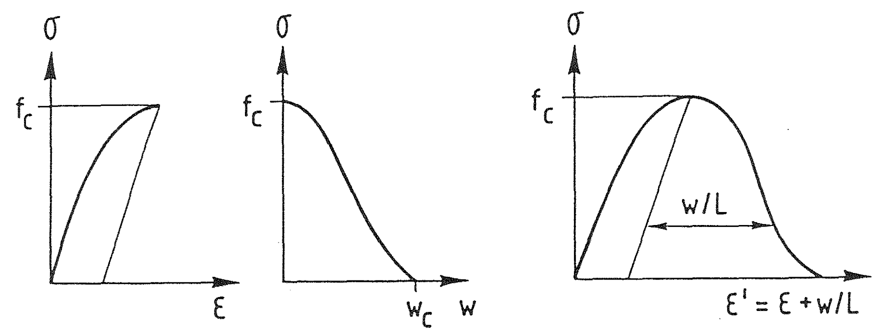


Fig. 5. Stress-deformation curve for the localized zone as a part of the complete compressive stress-strain curve for a gauge length L, Hillerborg (1988)

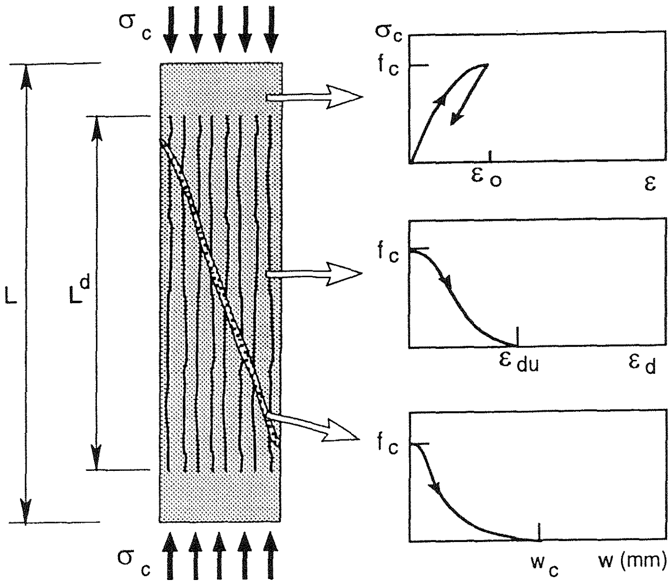


Fig. 6. Illustration of the CDZ model on a specimen loaded in uniaxial compression, Markeset (1993)

the damage zone, related to the formation of longitudinal cracks and corresponding additional lateral strain within the damage zone. Here the energy absorbed during the longitudinal tensile fracture process is assumed to be proportional to the fracture energy, G_F, in pure tension, i.e. ε_{du} becomes a function of G_F/f_c. The third curve is a stress-deformation curve, related to localized deformations in shear band. The input parameters to the CDZ model is further discussed in Markeset (1993) and Markeset and Hillerborg (1995).

The composition of the complete stress-strain curve is illustrated in Fig. 7. The average compressive strain ε_m for a specimen with length $L>L^d$ may then formally be given as:

$$\varepsilon_m = \varepsilon + \varepsilon_d \frac{L^d}{.L} + \frac{w}{L} \tag{2}$$

For a specimen of length L equal or less than the damage zone length L^d the average strain becomes:

$$\varepsilon_m = \varepsilon + \varepsilon_d + \frac{w}{L} \tag{3}$$

In modelling the case where a strain gradient is present, the CDZ model makes a further assumption regarding the development and interaction of several damage zones. The longitudinal deformation of the most stressed

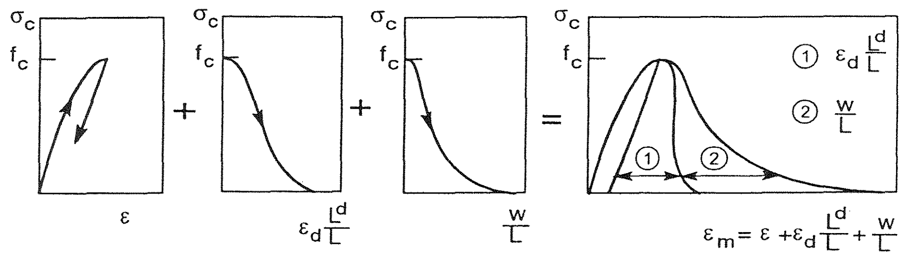


Fig. 7. Composition of the complete stress-strain curve, Markeset (1993)

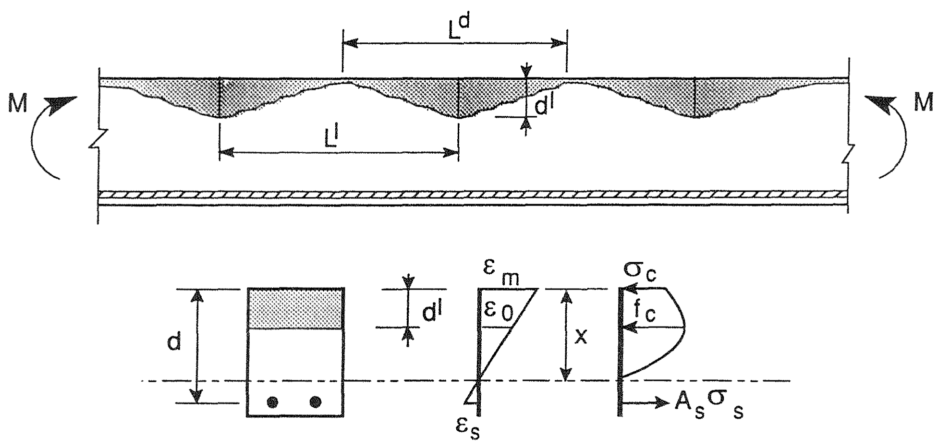


Fig. 8. Damage localizations in a bent beam, Markeset (1993)

fibre can not develop freely, but is governed by the rather uniform deformation of the underlying less stressed fibre. As soon as a localization starts to develop at one point the stress decreases at that point and the forces are partly taken over by the underlying fibre. The decrease in stress in the most stressed fibre spreads along a distance L^d in the stress direction. If the material is homogeneous and the gradient constant along the specimen, another localization appears at a constant distance L^l, see Fig. 8. Assuming that the damage zones are close together, the distance L^l becomes then approximately equal to length of each damage zone L^d. Inserting $L=L^d=L^l$ into Eq (2) the average strain in the most stressed fibre can be formally written as:

$$\varepsilon_m = \varepsilon + \varepsilon_d + \frac{w}{L^l} \tag{4}$$

It seems reasonable that the length of the damage zone L^d is proportional to the depth d^l, of the damage zone. This has also been suggested by Hillerborg (1988). The length L^d in bending is believed to be twice the length in pure compression.

Stirrups will cause a confinement of the compression zone, leading to a triaxial compressive situation, which may increase the compressive strength. They may also counteract the development of longitudinal cracks and in that way increase the compressive fracture energy, and thus also the value of ε_{du} in the σ-ε_d curve in Fig. 6. Accordingly, stirrups will make the compression zone stronger and tougher. The CDZ model has been further developed to take this confinement effect into account, Meyer (1997).

4 Discussion

Vonk (1993) studied the influence of specimen size on the compression post-peak energy for normal strength concrete prisms, Fig. 9. The figure shows that the post-peak energy per unit area increases with increasing specimen length. This is not in accordance with experiments reported by e.g. Markeset (1994) and Jansen and Shah (1997). From compression test on concrete cylinders with length-to-diameter ratios above 2.0, they found that the post-peak energy per unit area was independent of the length. For a normal strength concrete Jansen and Shah found the constant post-peak energy to be in the order of 20 to 25 N/mm. This coincides with the values found by Vonk for a slenderness ratio above 2.0. The CDZ model incorporates both these aspects by using Eq (3) for slenderness ratio less than 2.5 and Eq (2) for slenderness ratio above, assuming the length of the damage zone being 2.5 time the width of the specimen. From Fig. 9 it can be seen that the model is able both to simulate the increase in post-peak energy for small slenderness ratios, as well as the constant post-peak energy observed for specimen with slenderness ratios above 2.0-3.0 as indicated by the two horizontal lines in the figure.

The experimental observed size effect on the rotational capacity of reinforced beams (Cederwall and Sobko(1990)) are studied by means of the model proposed by Hillerborg, Eq (1), and the CDZ model, Eq (4), in Fig. 10. As it can be seen, the size dependence of the rotational capacity is realistically described by the CDZ model, whereas the model suggested by Hillerborg seems to over-estimate the size effect. This strong size effect is not surprising, since this model describes the concrete compressive failure only by localization, i.e. without including the tensile fracture process.

The size effect on the structural response on reinforced beams has been systematically studied using the CDZ model, Markeset (1993). Calculated relations between the relative bending moment and the relative curvature for mechanical ratios 0.2, 0.35 and 0.8 are shown in Fig. 11. No strain hardening is assumed for the tensile reinforcement. The figures clearly show the influence of beam depth on the ductility for low reinforcement ratios and on the ultimate moment for high reinforcement ratios. The stronger size effect on the high strength concrete (ND70) beams compared

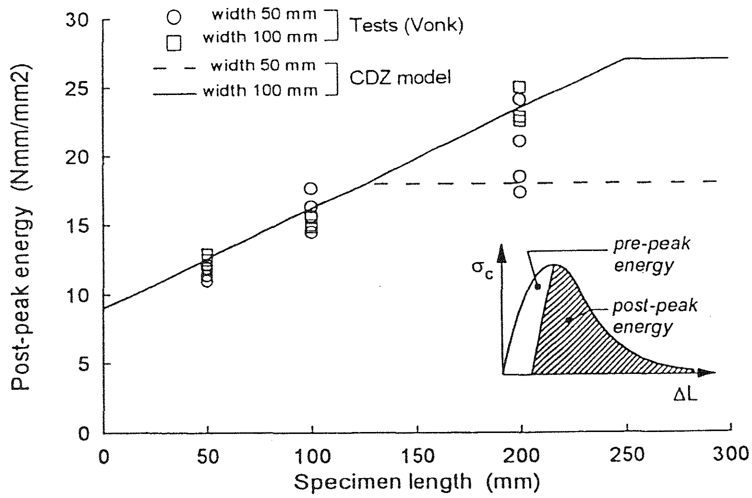


Fig. 9. Post-peak energy for different specimen geometries, Markeset and Hillerborg (1995)

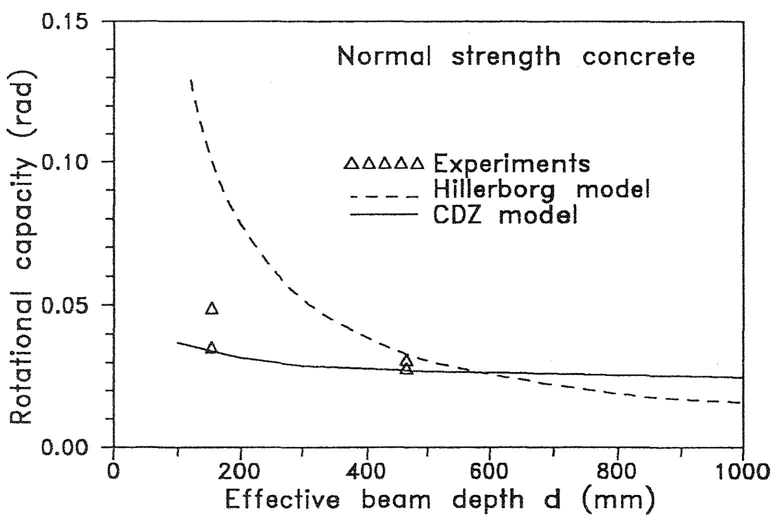


Fig. 10. Size effect on rotational capacity, Markeset (1993)

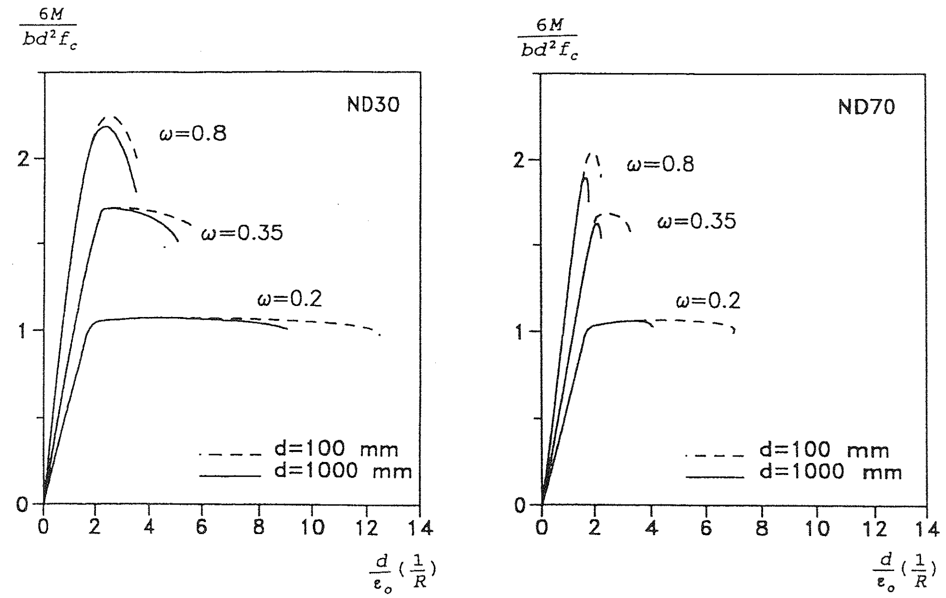


Fig. 11. Calculated moment-curvature diagram (dimensionless numbers) for effective beam depth d=100 and 1000 mm, Markeset (1993)

to the beam of lower strength concrete (ND30) is a result of the increased size dependence of the descending branch of the stress-strain curve. The stronger size effect of the high strength concrete is due to the fact that the G_F/f_c ratio decreases with increasing concrete strength and the localized deformation term in Eq (4) becomes dominant.

5 Conclusions

Localization occurs in compression failure of concrete. Under centric compression the failure is limited to a damage zone which is about 2 to 3 times the width of the specimen. Within this zone the failure mode is a combination of axial splitting and sliding in localized shear bands. It is shown that the size effect on the stress-strain curve has a clear influence on the structural behaviour of reinforced beams. It is of some importance for the moment capacity of over-reinforced beams and of great importance for ductility of under-reinforced beams. Further, high strength concrete seems to give a stronger size effect than normal strength concrete for beams of small/moderate size. This should be studied further, since existing design codes do not take such size dependence into account.

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