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LOAD DEFLECTION DIAGRAM OF OVER-REINFORCED CONCRETE BEAMS

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Abstract

The load - deflection behaviour of reinforced concrete beams is calculated using a specifically designed Finite Element method. The beams are discretized into multi - layered short elements. The Moment - Curvature diagram of each element is calculated applying the assumptions that plane sections remain plane and that the strain in the reinforcement is the same as that in the surrounding concrete. Any stress - strain diagram can be used for the compressed concrete and for steel. It may be defined either according to experimental curves or using analytical laws.

The calculation method consists of defining successive equilibrium states, increasing step by step the maximum concrete strain. This process allows us to get the entire moment - curvature diagram, including the softening branch. Up to now, the effects of shear are not taken into account. The global deformations such as rotations and deflections are deduced from a numerical integration of curvatures.

Key words: Numerical simulation, Navier's hypothesis, moment - curvature relationship, integration of curvature.

1 Introduction

This contribution to the Round Robin analysis on modelling of Over-Reinforced Concrete Beams consists of presenting a numerical method of predicting the behaviour of any reinforced concrete beam in a first section and of giving the results of its application in a second section.
The experimental data concerning the behaviour laws of the materials are shared by all the contributors so as it is not necessary to present them again.

2 Numerical method

Beams are composed of n multi - layer elements, (Fig. 1a). The equilibrium of each element is calculated assuming that Navier's hypothesis apply.

2.1 Equilibrium and moment - curvature diagram for one element

The elements are composed of m layers (50 layers in this application). Each one is considered to be subjected to uniaxial states of stress and strain, (Fig. 1b). For a given value of the curvature, the compressive concrete strain ε_c of the upper layer is calculated by iteration in such a way that the axial resulting force N_{int} tends toward zero ($|N_{int}| < 10$ daN). N_{int} is calculated through relation (1):

$$N_{int} = N_c + N_s$$

$$N_{int} = \sum_1^m A_{ci} . \sigma_c(\varepsilon_i) + \sum_1^{m_s} A_{si} . \sigma_s(\varepsilon_i) \qquad (1)$$

ε_i : strain of the layer i,
A_{ci} : section of the concrete layer i,
$\sigma_c(\varepsilon_i)$: stress of the concrete layer i, according to the strain - stress relationship of concrete,
A_{si} : section of the steel layer i,
$\sigma_s(\varepsilon_i)$: stress of the steel layer i, according to the strain - stress relationship of steel.

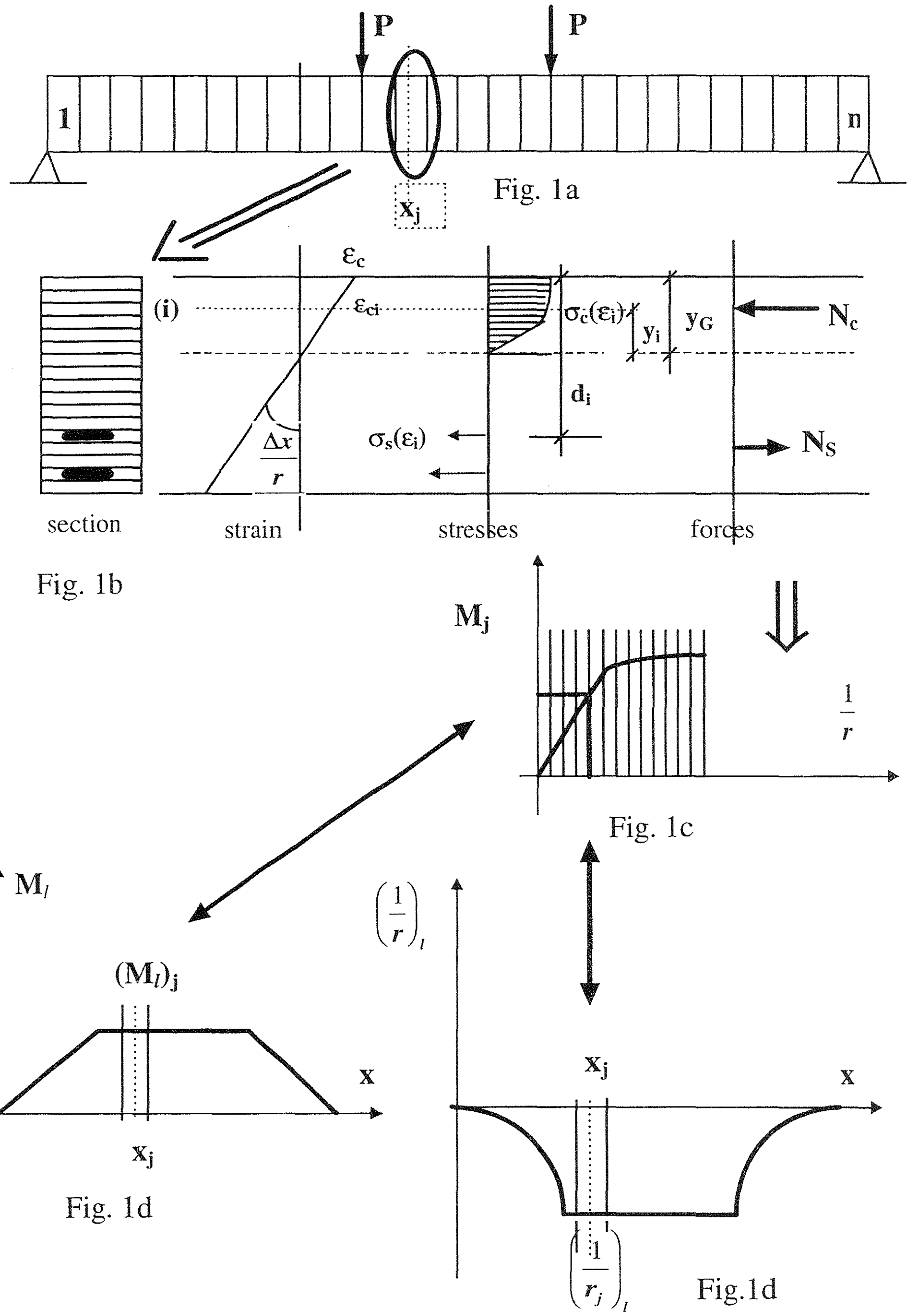

Fig.1. Principle of the numerical method

Then the bending moment can be deduced from relation (2) :

$$M_{int} = \sum_{i=1}^{m} A_{ci}.\sigma_c(\varepsilon_i).(y_G - y_i) + \sum_{1}^{m_s} A_{si}.\sigma_s(\varepsilon_i).(y_G - d_i) \qquad (2)$$

y_G : distance of the upper concrete fibre to the neutral axis,
y_i : distance of the concrete layer i to the neutral axis,
d_i : distance of the steel layer i to the upper fibre.

Finally, the moment - curvature diagram of an element j is calculated by incrementing step by step the curvature $\frac{1}{r}$, (Fig. 1c).
The tension stiffening effect is taken into account by modifying the curvature in relation to CEB-FIP 1990 recommendations.

2.2 Curvature diagram of the beam

The loading of the beam is controlled by the curvature of the central element. The moment curvature diagram of this element is calculated as described above. For a given step l, the curvature $\left(\frac{1}{r}\right)_l$ determines the bending moment in the central element. Thus, both the corresponding load P_l and the moment diagram of the beam can be calculated (Fig. 1d). Using the moment - curvature diagram of each element, the curvature diagram of the beam can be drawn (Fig. 1e). However, beyond the peak load, some elements may be subjected to decreasing curvature. In this case, it is assumed that the moment - curvature relationship is linear down to zero.

2.3 Load - deflection diagram

Rotations and deflections are obtained by numerical integration of the well known Bresse formulae :

$$\theta_K = \theta_0 + \sum_{j=1}^{K} \left(\frac{1}{r_j}\right)_l \Delta x_j \qquad (3)$$

$$\delta_K = \theta_0 x_K + \sum_{j=1}^{K} \left(\frac{1}{r_j}\right)_l .\Delta x_j.(x_K - x_j) \qquad (4)$$

θ_K : rotation of the element K,
δ_K : deflection of the element K,
Δx_j : length of the element j,

$\left(\frac{1}{r_j}\right)_l$: curvature of the element j at a given load level l.

The final position of the beam is obtained by a dichotomic approximation of the rotation θ_0 which induces a deflection δ_n close to zero ($|\delta_n| < \mathbf{5.10^{-2} mm}$).

3 Application

3.1 Modelling of the beams

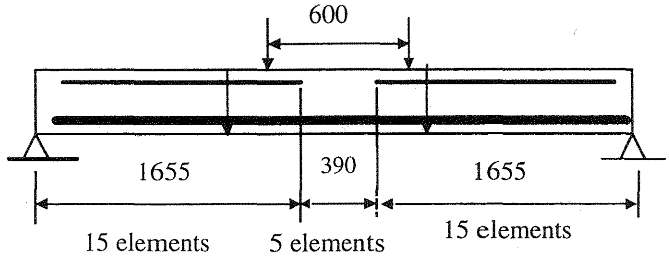

Fig. 2a. Small beam

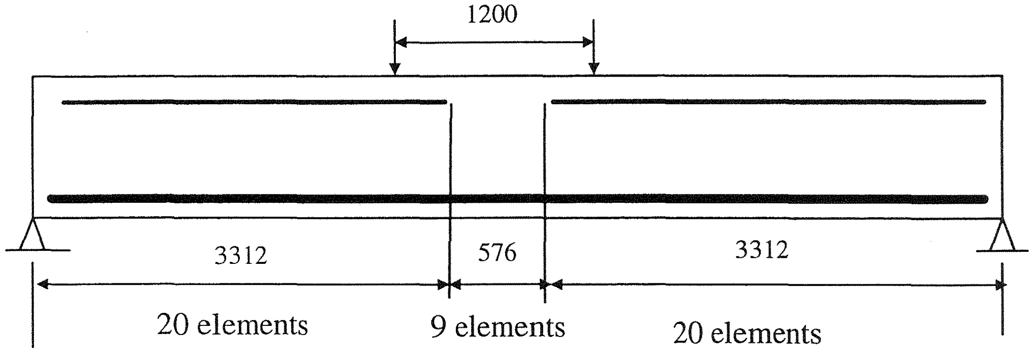

Fig.2b. Large beam

3.2 Results

Results are presented on one page for one type of beam and concrete with both boundary conditions of concrete compression tests (low fiction - LF, high friction - HF) :

- load - deflection diagrams (load = P, according to Fig. 1),
- variations of the strains at the upper fibre and at the lower fibre.

In addition, load - support rotation diagrams are given in the four last figures.

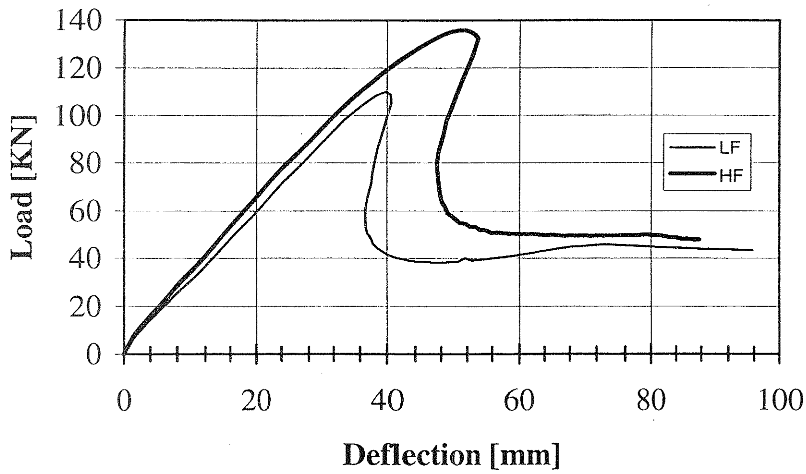

Fig. 3a. Load - deflection diagrams for large beams and normal concrete

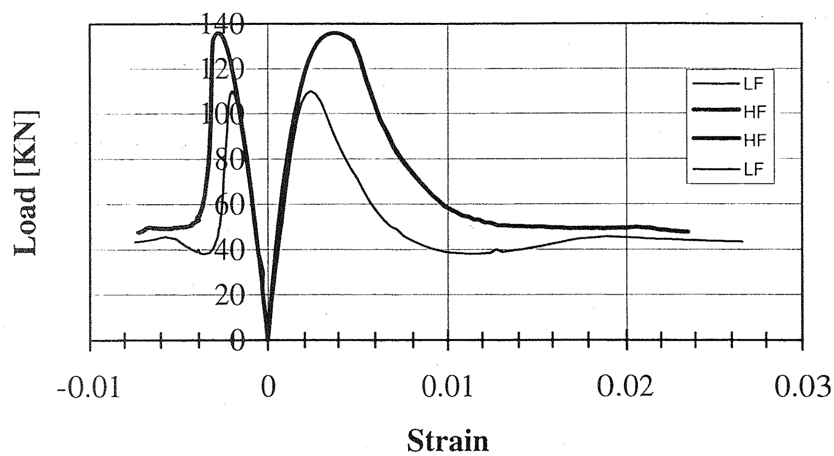

Fig. 3b. Strains at upper and lower fibres for large beams and normal concrete

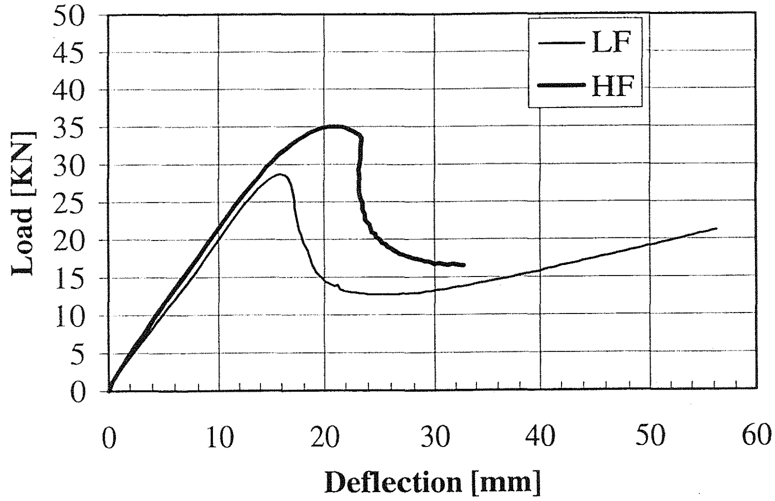

Fig. 4a. Load - deflection diagrams for small beams and normal concrete

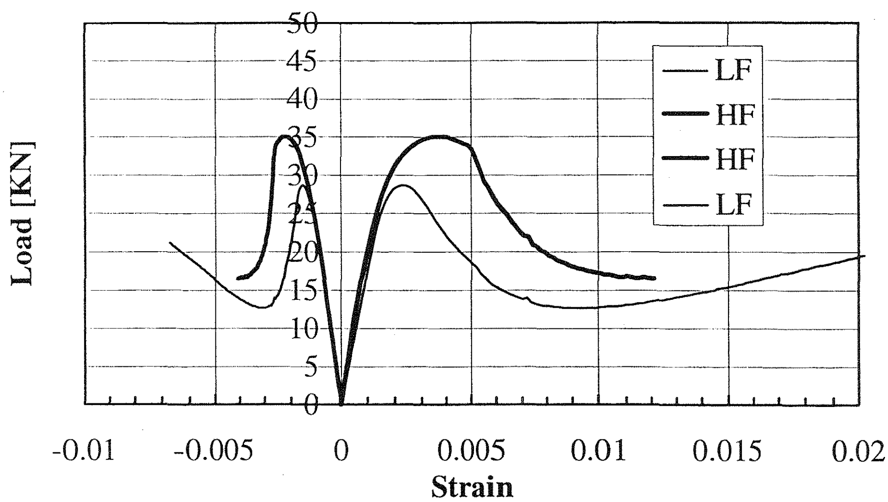

Fig. 4b. Strains at upper and lower fibres for small beams and normal concrete

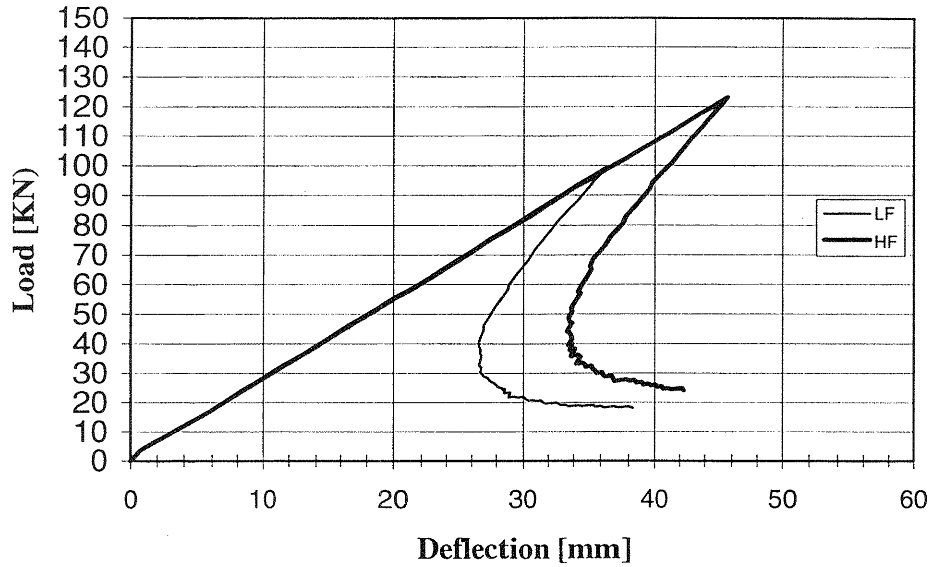

Fig. 5a. Load deflection diagrams for small beams and high strength concrete

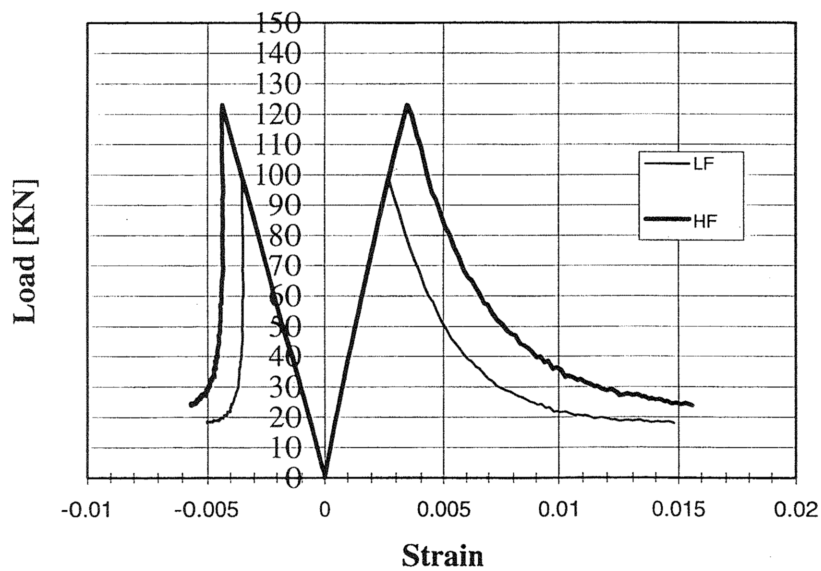

Fig. 5b. Strains at upper and lower fibres for small beams and high strength concrete

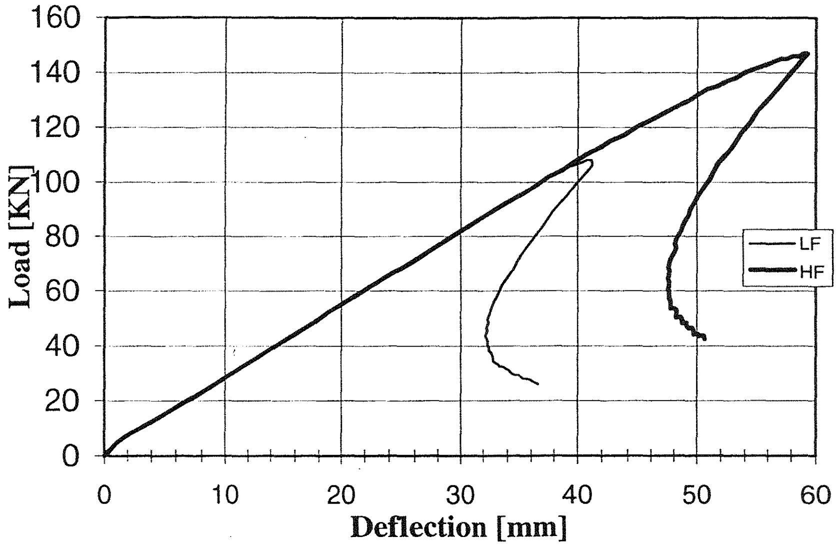

Fig. 6a. Load - deflection diagrams for small beams and fibre concrete

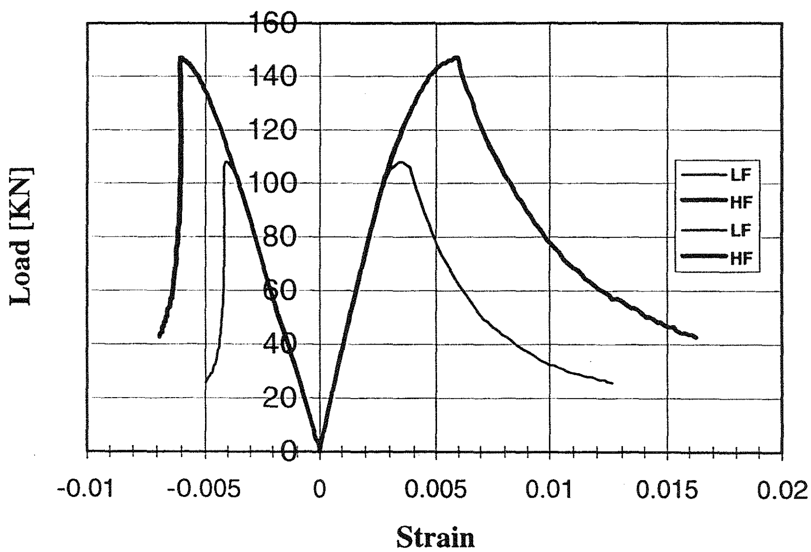

Fig. 6b. Strains at upper and lower fibres for small beams and fibre concrete

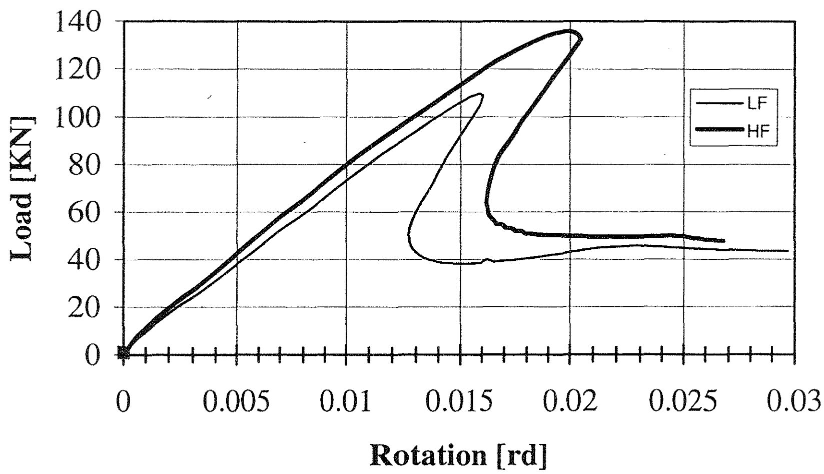

Fig. 7. Load - support rotation diagrams for large beams and normal concrete

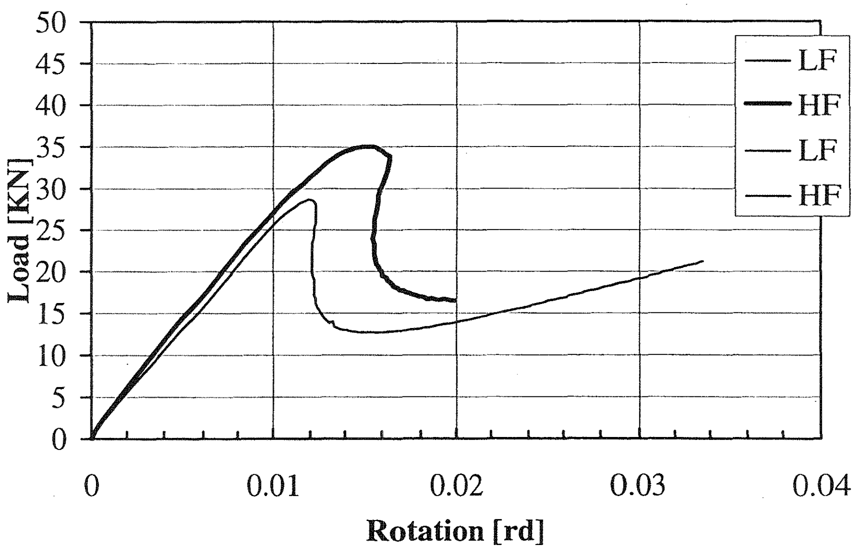

Fig. 8. Load - support rotation diagrams for small beams and normal concrete

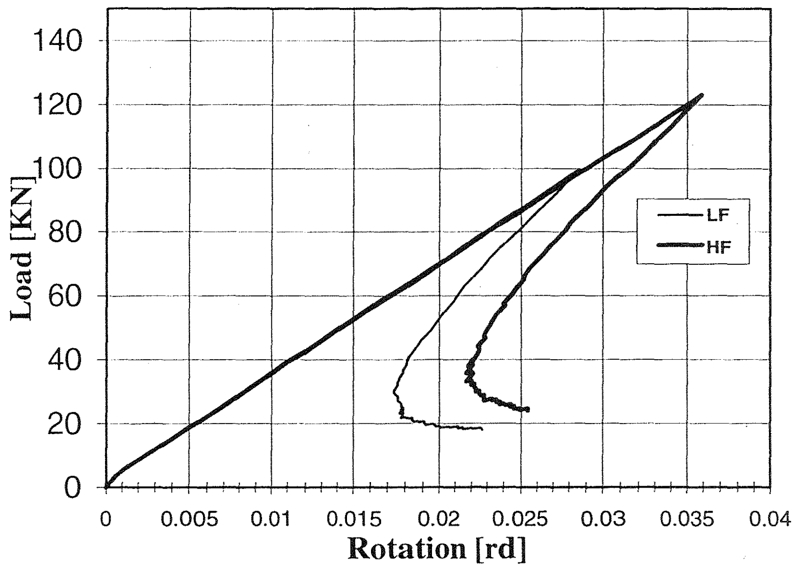

Fig. 9. Load - support rotation diagrams fors mall beams and high strength concrete

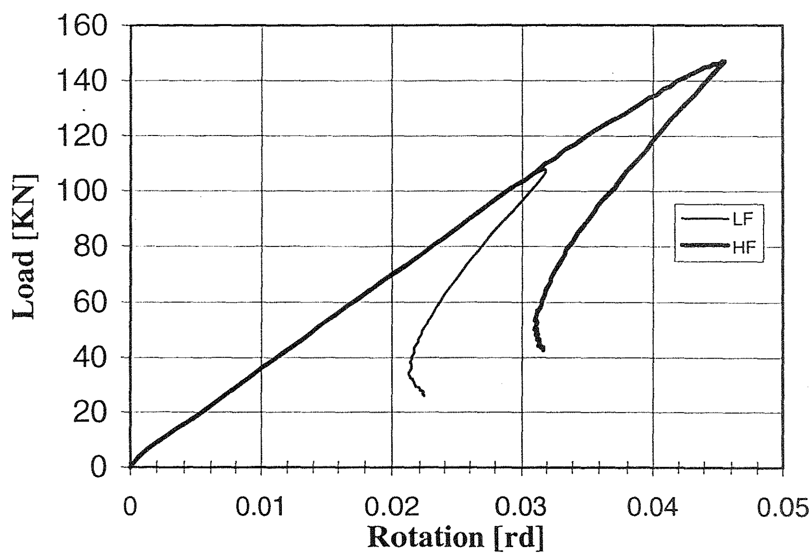

Fig. 10. Load - support rotation diagrams for small beams and fibre concrete

Table 1. Peak load characterisation

Beam	Concrete	Friction	F_{max} KN	Deflection mm	Comp. Strain	Tens. Strain	Sup. Rot. (rd)
Large	Normal	LF	110	40	0.00236	0.00204	0.016
Large	Normal	HF	136	51.6	0.00391	0.00289	0.02
Small	Normal	LF	28.7	15.9	0.00227	0.00153	0.012
Small	Normal	HF	35	21.6	0.00393	0.00227	0.015
Small	High. Str	LF	99.7	36.5	0.00271	0.00349	0.029
Small	High. Str	HF	123	45.7	0.00345	0.00435	0.036
Small	Fibre	LF	108	41.1	0.00352	0.00408	0.032
Small	Fibre	HF	147	59.3	0.00595	0.00605	0.045

4 Conclusion

Looking forward to the experimental results of the tests on beams, we just give a summary of our numerical simulations by reporting above, in table 1, the peak load and the corresponding parameters for each type of beam. These values will be easy to compare at first glance with the tests carried out at Aalborg University.