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PREDICTION OF THE BEHAVIOR OF OVER-REINFORCED CONCRETE BEAMS WITH TWO LEVELS OF SIMPLIFIED APPROACH

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Abstract

This paper presents the prediction of 12 concrete beams proposed for the competition on Modelling of over-reinforced concrete beams. Two methods were used, both adapted to engineering pratice: a curvature integration method (CIM) and a simplified finite element method (SFEM). The constitutive laws were identified on material tests and a stability approach was used to determine post-peak response.
Key words: damage mechanics, finite element method, multilayer beam elements, strain softening, concrete, high-strength concrete.

1. Introduction

It is commonly believed that behavior of reinforced concrete structures cannot be predicted adequately using only constituve laws identified on material tests. Structural effects may explain part of this behavior. It is the case of the so called scale effect and softening of the concrete compressive behavior. Hence, modelling of reinforced structures is a complex problem where it is important to understand the effect of the structure and of the intrinsic material behavior. The normal-strength concrete (NSC), high-strength concrete (HSC) and fiber high-strength concrete (FHSC) over-reinforced beams tested and proposed as a benchmark were a good means to evaluate this interaction. This type of structures are uncommon in practice; however, the role of concrete is predomminant in the global beam behavior and should be adequately modelled if a proper prediction of overall behavior is desired.

In this context, it is interesting to know if common procedures and tools of practical engineers are sufficient to predict adequately the behavior of this kind of extreme structure. Therefore, the objective of this paper is to use two levels of modelling: one very simple, and another more sophisticated, but both used in practice to evaluate the behavior of real structures. The two procedures are:

- The Curvature Integration Method (CIM): for a certain moment diagram the curvature in each section is computed and integrated along the member with the moment area theorem.
- The Simplified Finite Element Method (SFEM): a finite element program using multilayered beam elements and allows to compute directly the response of the structure from the constitutive laws. For concrete, an unidirectionnal damage model is used.

2. Curvature Integration Method - CIM

The curvature-moment relation is determined with the MNPHI program (Paultre, 1998). The concrete section is divided into layers and steel is condensed into steel layers. A sectional analysis is performed: for a certain curvature, the neutral axis depth is computed by equilibrium considerations; the resultant moment is calculated and the curvature-moment relation is determined. In each concrete and steel layer, constituve laws are used to link the strain in the layer to the stress. For concrete, the stress-strain curve proposed by Cusson and Paultre (1995) (thereafter called the C.P. curve) is used for compression loading. In the ascending part, for each strain, ε_c, the concrete stress, f_c, is:

$$f_c = f'_c \left[\frac{n(\varepsilon_c/\varepsilon'_c)}{n - 1 + (\varepsilon_c/\varepsilon'_c)^n} \right] \quad , \quad \varepsilon_c \leq \varepsilon'_c \tag{1}$$

where n controls the curvature of the ascending part (Cusson and Paultre 1995). The descending part of the curve is (Fafitis and Shah, 1985):

$$f_c = f'_c \cdot exp\left[k_1(\varepsilon_c - \varepsilon'_c)^{1.5}\right] \quad , \quad \varepsilon_c \geq \varepsilon'_c \tag{2}$$

where k_1 affects the general slope of the curve. Parameters f'_c, ε'_c and ε_{c50c} (obtained from the experimental stress-strain curve) are respectively the maximum stress, the corresponding strain, and the postpeak strain where sustained stress is 50% of f'_c.

Tensile behavior accounts for tension-stiffening effect (Collins and Mitchell, 1993) but cannot be identified on experimental results. The steel behavior is bilinear. However the plastic behavior of steel is not important since only the elastic range is reached in over-reinforced beams.

The complete load displacement curvature is computed by integration of the curvature profile along the member with the moment-area theorem for each moment diagram from zero-load up to failure. A specific program named DISP96 is used for this purpose (Légeron, 1998).

The CIM method is common practice for engineers, specifically in the seismic design field. It does not involves high computational costs and high level computer hardware and software. Sectional analysis program is a standard tool in engineering offices and integration of curvature represents a program of approximatively 200 lines.

3. Simplified Finite Element Method - (SFEM)

It is now common practice to use the finite element method to compute the non linear behavior of complete structures subjected to various loadings such as earthquakes, blasting etc.. However, this particular method requires highly sophistigated software and high computing costs. The LMT (*Laboratoire de Mécanique et Technologie*) developed a simplified approach with the computer program EFICOS, that gave good results in various benchmarks on the prediction of walls (Dubé, 1994; Ghavamian and Mazars 1996) and bridge structures (Légeron, 1998) under seismic loading. It was also used on other simple structures such as columns (LaBorderie, 1991; Légeron, 1998) and fiber-reinforced concrete beams (LaBorderie, 1991). EFICOS uses multilayer beam elements (Fig. 1). Each element is constituted of superposed layers. Each layer is either a concrete layer or a homogenized steel-concrete layer. The kinematics is simplified as plane sections remain plane (Bernouilli hypothesis). It limits the number of degrees of freedom, but enables to account for realistic material behavior. In each layer, a damage model is used to describe the behavior of concrete (LaBorderie, 1991):

$$\varepsilon_c = \frac{\sigma^+}{E_c(1-D_1)} + \frac{\sigma^-}{E_c(1-D_2)} + \frac{\beta_1 D_1}{E_c(1-D_1)} f'(\sigma) + \frac{\beta_2 D_2}{E_c(1-D_2)} \quad (3)$$

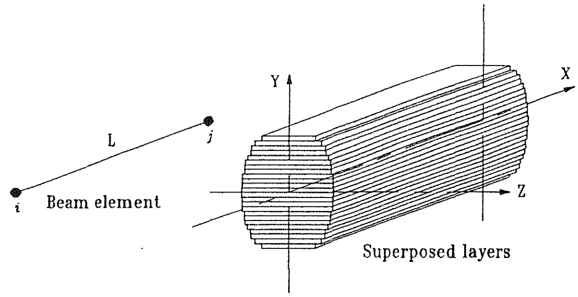

Fig. 1. EFICOS program: Superposed layer beam elements

where E_c is the initial Young modulus, σ^+ and σ^- are the positive and the negative stress (for tension stress, $\sigma^+ = \sigma$ and $\sigma^- = 0$. For compression $\sigma^+ = 0$ and $\sigma^- = \sigma$). D_1 and D_2 are damage variables in traction and compression, respectively; β_1 and β_2 are parameters defining inelastic behavior ; f is a function that allows to account for the closure mechanism of cracks. The model is unilateral which means that tension and compression behaviors are disconnected and managed by two separate damage variables D_1 and D_2. The evolution of damage variables is controled by energy restitution rates defined as:

$$Y_1 = \frac{\sigma^{+2} + 2\beta_1 f(\sigma)}{2E_0(1 - D_1)^2} \tag{4}$$

$$Y_2 = \frac{\sigma^{-2} + 2\beta_2 \sigma}{2E_0(1 - D_2)^2} \tag{5}$$

Six parameters completely define the behavior: Y_{01}, A_1 and B_1 for tensile behavior, and Y_{02}, A_2 and B_2 for compression behavior. All parameters are identified on material tests.

For softening structures, the size of the elements is generally an important parameter. However, in this case, a preliminary study showed that it was not a very sensitive parameter due to the constant moment zone between the two applied loads. This is partly due to the high reinforcement steel ratio on the tension side which dictates the behavior in tension. Also, the stability approach that will be discussed later limits softening in compression. For small beams, 50-mm elements were used and 100-mm elements for large beams. The beam tests are simulated in displacement control.

In composite steel-concrete layer, the behavior is homegenized considering the compatibility of strain between steel and concrete. In this layers, the behavior of steel is represented by a bilinear elastic plastic model.

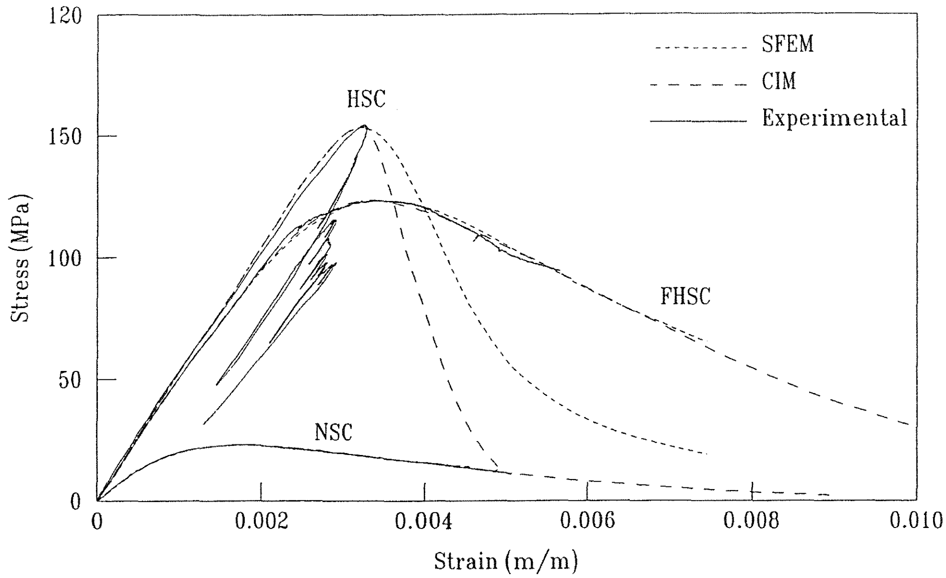

Fig. 2. Predicted and experimental compressive responses

4. Identification of parameters

Compression parameters were obtained from high-friction experimental tests. A 0.95 correction factor was applied to account for the small size of the specimens tested. The complete strain-stress curve was scaled. For the damage model an identification procedure was followed as described by Légeron (1998). For the C.P. curve, the parameters were directly read on the experimental curves. In the case of high-strength concrete (HSC), the postpeak curve was not reported. Hence, only the ascending part was accounted for the damage model, and for the C.P. curve, a typical value was used for ε_{c50c} (Légeron, 1998). Results of the identification procedure are reported in Tab. 1. The predicted strain-stress curves are reported in Fig. 2. The two models predict very well the behavior of tested concretes.

The parameters defining the tensile behavior of concrete, according to the damage model, were identified on tensile tests provided in the report. Parameters obtained are shown in Tab. 2.

5. Stability approach for failure

It is believed that post-peak softening is not only related to material response but also to structure in which it is used in. Hence, a structural behavior that is not stable even if statical equilibrium is

Tab. 1. Parameters for compressive behavior

		SFEM				CIM		
Beam	E_c	Y_{02}	A_2	B_2	β_2	ε_c'	f_c'	ε_{c50c}
NSC	27 GPa	0.01E6	0.805E-5	1.47	-0.343E8	0.0018	23.1 MPa	0.0050
HSC	52 GPa	0.10E6	0.180E-5	5.00	-0.507E8	0.0033	154.6 MPa	0.0040
FHSC	50 GPa	0.08E6	0.090E-5	2.60	-1.300E8	0.0034	123.6 MPa	0.0075

Tab. 2. Identification of parameters for tensile behavior

	SFEM				CIM
Beam	Y_{01}	A_1	B_1	β_1	f_t'
NSC	145	0.007	1.00	1.13E6	1.9
HSC	422	0.002	1.50	2.16E6	4.8
FHSC	471	0.005	1.55	2.22E6	5.0

satisfied might not be possible during tests.

A stability approach (Bažant, 1977; Légeron, 1998) was developed to evaluate if the post-peak behavior of the beams was stable or not. For this specific experimental set-up, a stability fonction is determined. Only the small beam made of normal strength concrete was stable after peak. It is accounted for through a strain limit in the damage model as well as in the C.P. curve. When strain is greater than this limit strain in each layer (in EFICOS as well as in MNPHI), the corresponding stress is equal to zero. This limit strains range from 0.0043 for HSC short beams to 0.007 for FHSC short beams.

6. Prediction of sectional behavior

Sectional analysis enable to predict the sectional behavior of the mid-span section. This section was heavily instrumented and it is believed that experimental sectional behavior can be plotted. The predictions are shown on Fig. 3 and 4 for small and large specimens.

7. Prediction of mid-span deflection

The prediction of applied loads as a function of mid-span deflection is performed by the two methods. These are shown in Fig. 5 and 6 for small and large beams respectively. In those curves, the applied load represents only the force at one point of application. Thus, the total load is obtained by multiplying it by two. The two methods give similar results. For small NSC beams, the two responses are superposed. The ascending parts of the response are slighly different

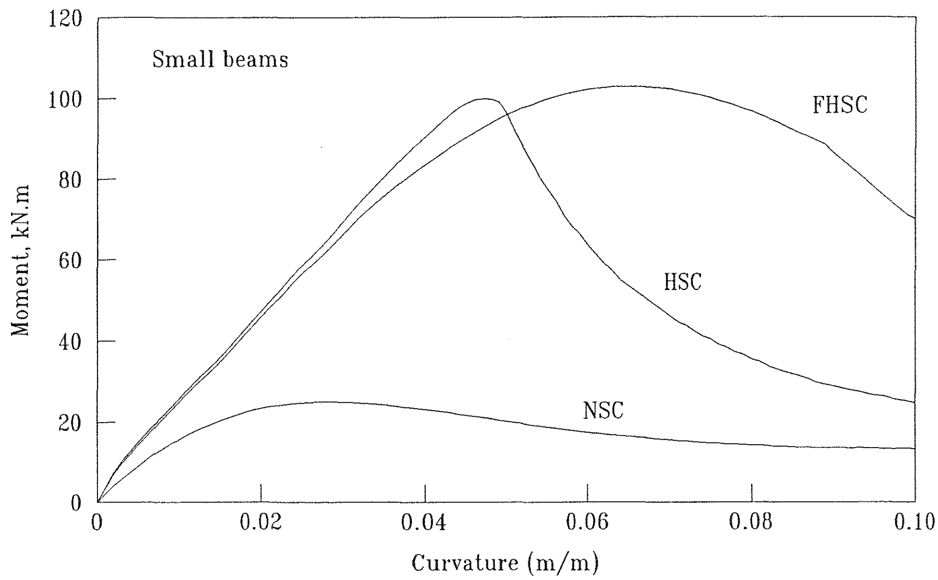

Fig. 3. Predicted curvature-moment response for small beams

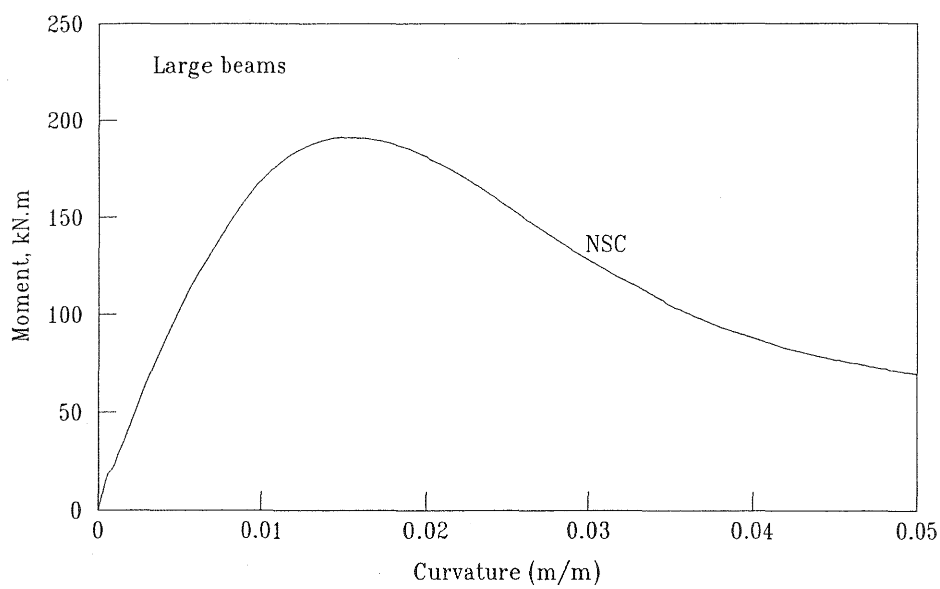

Fig. 4. Predicted curvature-moment response for large beams

Tab. 3. Predicted maximum load capacity and displacement, u_{max}

Beam	CIM		SFEM	
	P_{max} kN	u_{max} m	P_{max} kN	u_{max} m
Small NSC	16.6	0.0222	16.6	0.0214
Small HSC	66.5	0.0524	69.6	0.0563
Small FHSC	68.5	0.0645	69.4	0.0607
Large NSC	63.7	0.0511	64.2	0.0505

Tab. 4. Area of interest for the two proposed methods

	CIM	SFEM
Type of structures	isostatic	plane structures
Type of loading	monotonic	monotonic or cyclic
Computation cost	1 min	1 h
Pretreatment	identification of parameters	
	5 parameters	9 parameters
Post-treatment:		
Local	sectionnal (M-ϕ)	stress, strain and damage in each material
Global	load-displacement	

for the two methods. It can be attributes to the difference in the procedures as well as the difference in the tensile description of the behavior. The peaks are very similar for the two methods (Tab. 3). The post-peak curves, when existant, are similar.

8. Conclusion

Predictions of the behavior of over-reinforced beams are presented in this paper. The two simple "engineer's" methods used were selected to determine if they are precise enough to predict this kind of structures. Each method has a restricted area of interest which is summed up at Tab. 4. For isolated members, the CIM is interesting. For complete undetermined plane structures, computation of efforts in members is not possible directly and it is necessary to obtain distribution of load. In those cases, it is preferable to use the SFEM. EFICOS was primeraly developed for seismic analysis. Recent developments concerned large-displacement effects and material hysteretic dissipation (incorporated in the damage model).

Even if not common in real structures, over-reinforced beams predictions are very sensitive to the model of concrete used. It could underline failure mechanisms that should be included in a proper modelling of reinforced concrete structures. Therefore, comparison

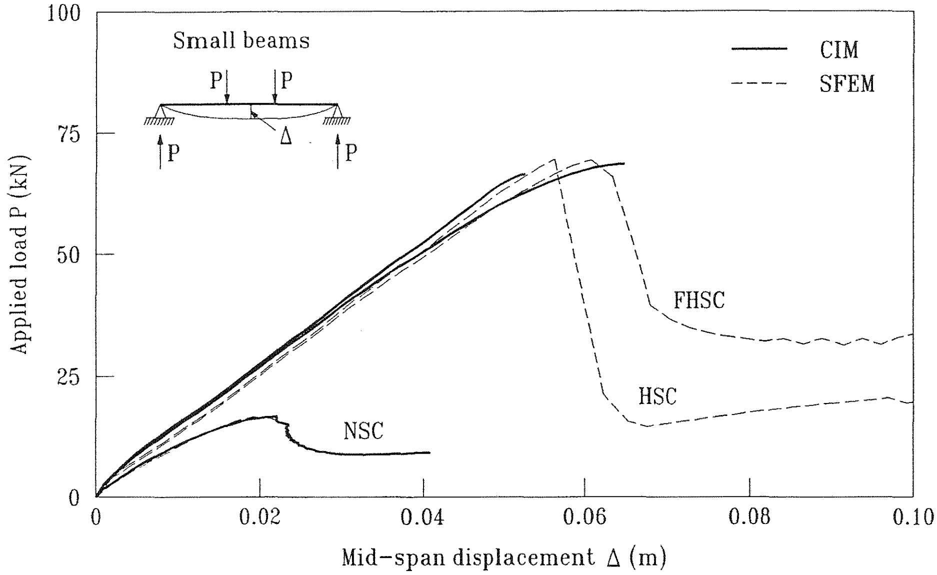

Fig. 5. Predicted mid-span displacement-applied load, small beams

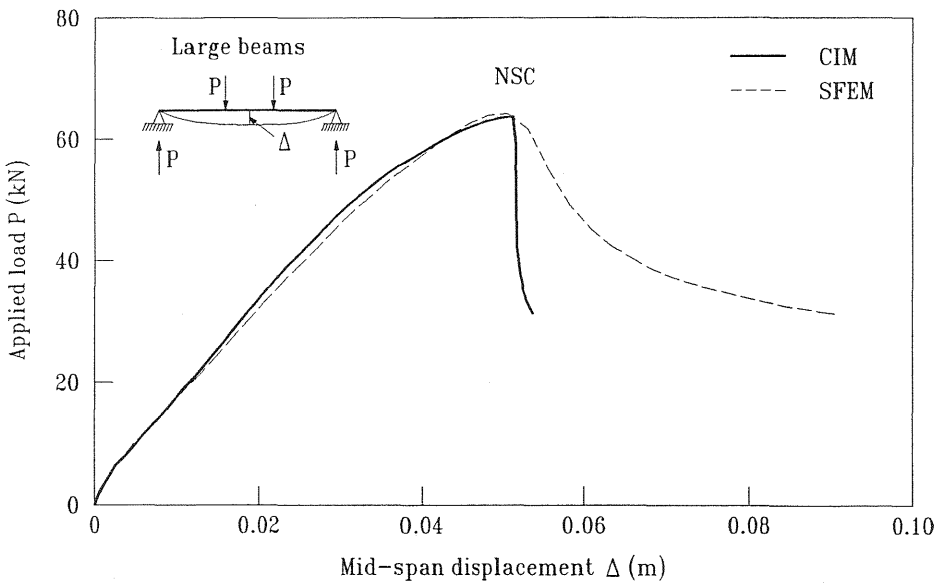

Fig. 6. Predicted mid-span displacement-applied load, large beams

with experience will provide interesting data on concrete modelling. However, the simple approaches proposed, which considered a concrete compression failure, are not able to predict all the phenomena that can influence the real behavior. Among them, delamination of concrete compressive strut from the longitudinal reinforcement which could be prominent because of the great number of longitudinal bars might be a early failure mechanism.

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