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FINITE ELEMENT ANALYSIS OF SLENDER CONCRETE COLUMNS SUBJECTED TO ECCENTRIC LOADING

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Abstract
The results of a non-linear finite element analysis on the behavior of reinforced concrete columns that were tested under eccentric monotonic loading are presented. The results of the finite element analysis correlate well with the results of the tests. A parametric study was made to examine the difference in failure modes for the different concrete strengths, length-to-width ratios and loading eccentricities. The higher compressive concrete strength was especially advantageous when the load eccentricity was small. When the eccentricity was increased, the strength of the high-strength concrete columns decreased more rapidly than that of the normal-strength columns. However, the high-strength concrete columns still exhibited a greater load capacity than the normal-strength columns.
Key words: High-strength concrete, slender columns, eccentricity, non-linear finite element analysis

1 Introduction

The introduction of high-strength concrete has made it possible to design columns more slenderly and, thereby, facilitate not only new architectural ideas but also economic benefits. An increase in compressive strength allows smaller cross sections which require less concrete and permit more rentable floor space; however, although it can reduce column size, the columns become less ductile due to the brittleness of high-strength concrete. The performance of structural elements made of high-strength concrete has recently become a major concern for design engineers. There are many aspects, such as ductility, amount of reinforcement, effect of slenderness, and eccentricity of the applied load, that have to be investigated in order to understand the behavior of the columns completely.

At the Division of Concrete Structures, Chalmers University of Technology, a research program on reinforced columns made of high-strength concrete is being carried out; it combines experiments and numerical simulations. In this study, 12 full-scale slender columns with square sections have been tested to failure under eccentric axial loading. The four parameters varied in this investigation were the concrete strength, stirrup spacing, slenderness and eccentricity of the applied compressive load. In addition, the mechanical properties, such as the compressive and the tensile concrete and steel strengths, the modulus of elasticity and the fracture energy, were measured in the experimental investigation. These material properties were incorporated into a model in which the material model for concrete was based on non-linear fracture mechanics. This model was, in turn, used in a non-linear finite element program in order to predict the responses of the slender concrete columns.

This paper describes the numerical aspect of the research program. The finite element model used in the analysis is presented, followed by verification of the experimental results. Observations of the failure mechanisms during the tests and the results of the analysis, as well as some reasons for the failure of the columns under eccentric compressive loading, are presented. Some effects of slenderness that were investigated are reported.

2 Finite element analysis

2.1 General

One of the aims of the study was to develop non-linear finite element models that could simulate the failure mechanism of the columns and, together with the experiments, make possible a better understanding of the mechanical behavior until final failure. These models were generated in the non-linear finite element program ABAQUS (1995). A model based on three dimensional 3-node hybrid beam elements was established. The advantage of a beam element model is that it can be run in a reasonable amount of time. However, the amount of information obtained is more limited than with a solid element model. Nevertheless, the beam element model was chosen for this study, as it was believed to give the required information. Furthermore, a previous study had shown good agreement between the results of an analysis using beam elements and one using solid elements, see Claeson (1995). Approximately 50 elements were used for all lengths. The model included the four vertical reinforcement bars, however the stirrups could not be modeled. The reason this was not considered to be a major disadvantage is that the main effect of the stirrups, with a spacing of 130 mm or 240 mm, in columns subjected to eccentric loading is to prevent the reinforcement bars from buckling and not, as in the centric compressive case, to produce a triaxial stress state. In the model the reinforcement bars cannot buckle; in other words, the model simulates a column where this is not a problem. Although this somewhat limits the study, the model does simulate the structural behavior accurately up to maximum load, as well as to the point when the reinforcement bars would buckle in reality. For columns tested with an initial eccentricity of 20 mm, this happened at a very late stage in the normal-strength concrete columns. However, the behavior of the high-strength concrete columns may be looked upon as a model simulating a column with close stirrup spacing. As the eccentricity increases, the tendency of the reinforcement bars to buckle decreases; also, any buckling that occurs is at a later stage of the post-peak curve. This conclusion was also reached by Lloyd and Rangan (1996) who stated that no buckling of the reinforcement bars in their tests was observed.

To model reinforced concrete, the program ABAQUS combines standard elements of plain concrete with a special option, called rebar. This option strengthens the concrete in the direction chosen, thereby simulating the behavior of a reinforcement bar. By this approach, the material behavior of the plain concrete is taken into account independently of the reinforcement.

2.2 Description of columns

Table 1 shows the details of the columns that were analyzed in this study. All of the columns had a 200 × 200 mm cross-section and were reinforced in an identical manner. The reinforcement was made up of four deformed bars with diameters 16 mm and 8 mm stirrups spaced at either 130 mm or 240 mm. The clear concrete cover to the stirrups was 15 mm. Sufficient additional reinforcement was provided to prevent failure in the end zones. The load was applied with an eccentricity of 20 mm. For further details about the columns and the tests, see Claeson (1995).

Table 1. Details of columns including material parameters at day of testing

Column	Concrete[1]	Length [mm]	Buckling length [mm]	Stirrup spacing [mm]	$f_{c,cube}$ [MPa]	$f_{c,cyl}$ [MPa]	$f_{t,split}$ [MPa]	E_c [GPa]
1	NSC	3000	3196	130	43	33	3.7	25.0
2	NSC	3000	3196	240	43	33	3.7	25.0
3	HSC	3000	3196	130	116	91	7.4	41.5
4	HSC	3000	3196	240	112	92	7.4	40.0
5	NSC	4000	4196	130	49	37	3.8	28.5
6	NSC	4000	4196	240	49	37	3.7	29.0
7	HSC	4000	4196	130	118	93	6.6	43.0
8	HSC	4000	4196	240	119	93	6.2	42.5

[1] NSC denotes normal-strength concrete, HSC high-strength concrete

2.3 A constitutive model for concrete and reinforcement

The material model for concrete provided in ABAQUS was used in the analysis. When the principal stress components are compressive, the response of the concrete is modeled by an elastic-plastic model. The uniaxial compressive stress-strain relations used in the finite element (FE) analysis were based on cylinder tests.

The smeared crack approach has been chosen to model cracked reinforced concrete. According to the smeared crack concept, a cracked solid is imagined to be a continuum for stress and strain. This means that the behavior of cracked concrete can be described in terms of stress-strain relations. Prior to cracking, the concrete is modeled sufficiently accurately in tension as an isotropic, linear elastic material. The fracture energy was determined from tests on three-point bending beams, RILEM 50-FMC Committee (1985), and, together with the tensile strength and the crack spacing from the tests, was used to calculate the tensile softening relation. While the fracture energy of the concrete strength did not differ much, the crack spacing of the normal strength concrete columns was approximately half of that of the high strength concrete columns. This, in combination with the higher tensile strength, lead to a steeper descending slope of the high-strength concrete than that of the normal strength concrete.

The longitudinal reinforcement bars were modeled by an elastic-plastic material model. Tension tests were performed on at least three steel samples of each bar diameter for each batch of steel bars. The average properties based on the nominal bar areas of 201 and 56 mm^2 respectively were as follows: (1) yield strength – 636 MPa and 466 MPa; (2) ultimate strength – 721 MPa and 607 MPa; and (3) modulus of elasticity – 207 GPa and 221 GPa. The Poisson ratio was approximated to be 0.3.

2.4 Results of the finite element analysis

A comparison of the load deflection curves from tests and the analysis shows that the finite element beam model does capture the structural behavior of the columns satisfactorily, see Fig. 1. The load-deflection curves of the columns subjected to an initial eccentricity of 20 mm were compared with the results of the tests to validate the accuracy of the model. In all of the cases analyzed, the accuracy was found to be satisfactory. However, it was observed that the results from the FE analysis gave a stiffer initial behavior than that of the tests. This discrepancy is primarily attributed to the formulation of the beam element and the differences between the measured and modeled geometric and material imperfections. The FE analysis was terminated in all cases due to numerical problems.

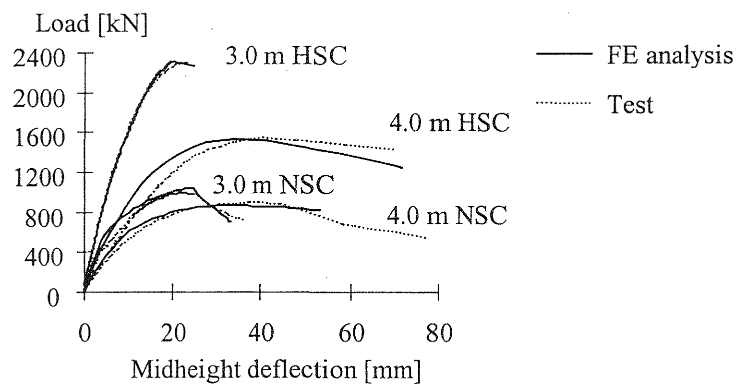

Fig. 1 Comparison between the results from the tests and those of the FE analysis

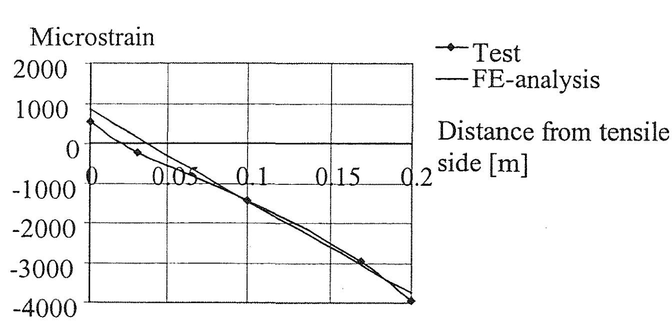

Fig. 2 Strain distribution at maximum load for a 3m high-strength concrete column with stirrup spacing 130 mm

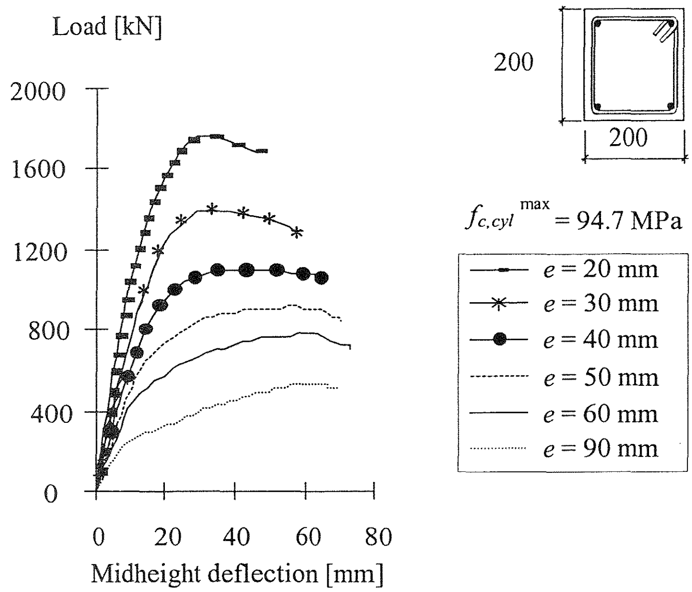

Fig. 3 Load-deflection curve of a 4 m high-strength concrete column subjected to different initial load eccentricities.

In addition to the comparison of the load-deflection curves, the strain distribution at maximum load at midheight was studied. Figure 2 shows one such a curve. From the figure it can be seen that the behavior of the column is captured rather well by the FE-model.

To enable a comparative study of the influence of different eccentricities on structural behavior, for the two different length-to-width ratios and concrete strengths, one high-strength concrete and one normal-strength concrete were chosen (the 3.0 m values). Figure 3 shows the results of the simulations of a column subjected to different initial load eccentricities.

3 Discussion

The length-to-width ratio, defined as the ratio of the column length, L, to the cross-section dimension, h, was either 15 or 20. The structural behaviors of these two length-to-width ratios, with a 200 × 200 mm cross section, subjected to different eccentricities were studied. The high-strength concrete columns with a length-to-width ratio of 20 exhibited a less brittle behavior than the high-strength concrete columns with a length-to-width ratio of 15; the deflection was greater and the maximum load capacity less. The same observations apply to the normal-strength columns. From the results of the FE analysis, it was observed that the midheight deflection, for the columns with a length-to-width ratio of 15, was almost the same for the same initial eccentricity regardless the concrete strength. It was when the eccentricity was 90 mm that the deflection of the two strengths started to differ. For a length-to-width ratio of 20, the midheight deflection was approximately the same for the two types of concrete when the eccentricity was 20 mm. This was also observed by Lahoud (1991). However, for the other eccentricities, the high-strength concrete columns had a midheight deflection which was approximately 10 mm less than the deflection of the normal-strength concrete columns. The same midheight deflection of the columns subjected to small eccentricities may be explained by the fact that the two types of concrete have almost the same deformation capacity, *i.e.*, the maximum load divided by the secant modulus for high-strength concrete columns is approximately the same to that of the normal-strength concrete columns.

Figure 4 presents the influence of eccentricity, *e*, on the maximum strength of columns for length-to-width ratios, L / h, of 15 and 20; P_{max} denotes the maximum load capacity of the columns. It is clear that column capacity is strongly affected by the amount of eccentricity. However, although the same trend is observed for both normal and high-strength concrete columns as *e* increases, the strength of the high-strength concrete columns decreases more rapidly than that of the normal-strength concrete columns.

Figure 4 also illustrates the different failure modes. The failure modes differed with varying eccentricity. When the eccentricity was small, the shape of the compression curve and the maximum compressive strength were the determining factors. However, when the eccentricity increased, the yielding of the reinforcement bars determined the maximum load. This trend was the same for both length-to-width ratios and concrete strengths, although the behavior was more prominent for the columns with a length-to-width ratio of 20.

In addition, Fig. 4 shows that when the concrete strength is increased, the load capacity increases. This was the case for all of the eccentricities in this study. The advantage of using high-strength concrete is the greatest when the eccentricity is small. However, gains in strength are achieved even when the eccentricity is quite large.

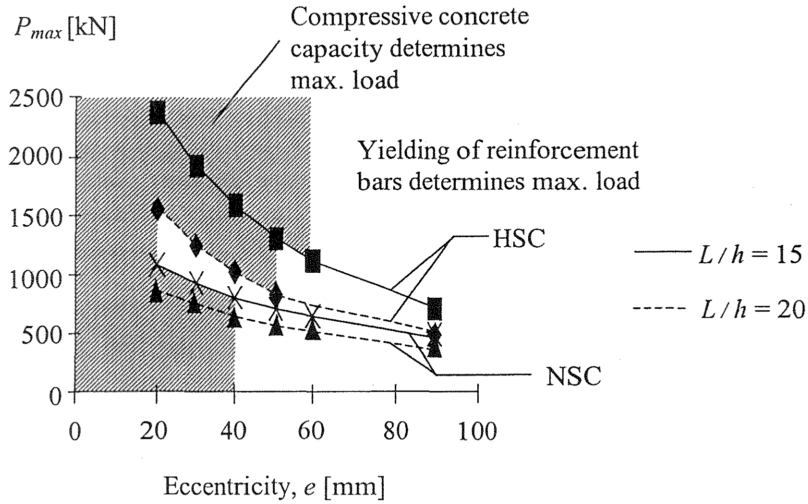

Fig. 4 The effect of eccentricity on the maximum load of different slenderness ratios and concrete strengths

To study the slenderness effect, two additional length-to-width ratios were added: 30 and 40. Although these length-to-width ratios have not been verified by testing they are included in this study to demonstrate the capacity of the FE model. The columns with length-to-width ratios of 30 and 40 failed due to instability. Figure 5 shows the relation between the load and the midheight deflection for four high-strength concrete columns subjected to an initial eccentricity of 20 mm. When the length-to-width ratio increases, the load bearing capacity and the advantage of using a higher compressive concrete strength decrease.

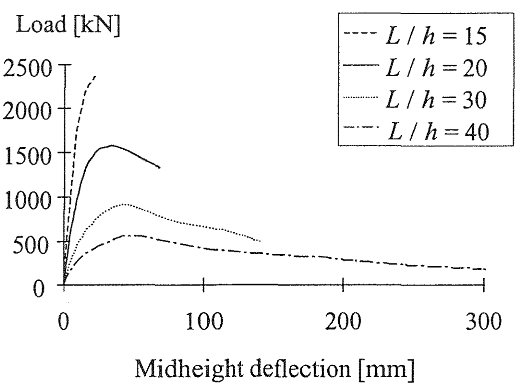

Fig. 5 Effect of slenderness on high-strength concrete columns subjected to an eccentricity of 20 mm.

4 Conclusions

The results of the finite element analysis on slender square reinforced concrete columns presented here allow the following conclusions to be drawn.

Four failure modes were observed. When the load eccentricity was small, the compressive strength of concrete played a dominant role; when the eccentricity was increased, yielding of the compressive reinforcement bars determined the maximum load and, when the eccentricity was large, the yielding of the tensile reinforcement bars determined the maximum load. This was the case for both the high and the normal-strength concrete columns with a length-to-width ratio of 15 or 20. However, the first failure mode dominated for the high-strength concrete columns, while the two other modes did so for the normal-strength concrete columns. The high-strength concrete columns with a length-to-width ratio of 30 or 40 failed due to instability.

The midheight deflections at maximum load were, for a length-to-width ratio of 15, with the same initial load eccentricities and cross-sections, almost the same regardless of concrete strength. For a length-to-width ratio of 20, the deflections of the normal-strength concrete columns were slightly greater than the deflections of the high-strength concrete columns.

The influence of eccentricity on column strength for two different length-to-width ratios and concrete strengths was studied. The column strength is strongly affected by the amount of the eccentricity. Although the same trend is observed for both normal and high-strength concrete columns, it was found that, when the eccentricity increased, the strength of the high-strength concrete columns decreased more rapidly than that of the normal-strength columns. In addition, the high-strength concrete columns with a length-to-width ratio of 15 obtained higher load capacities than either those with a length-to-width ratio of 20 or the normal-strength concrete columns with length-to-width ratios 15 and 20.

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6 References

ABAQUS theory manual, version 5.5. (1995), Hibbit, Karlsson & Sorensen Inc., Providence, R.I.

Claeson, C. (1995) **Behavior of reinforced high strength concrete columns**. Licentiate thesis, Publication 95:1, Division of Concrete Structures, Chalmers University of Technology, Sweden, 54 p.

Lahoud, A. E. (1991) Slenderness effects in high-strength concrete columns. **Can. J. Civ. Eng.,** 18, 765-771.

Lloyd, N. A. and Rangan, B. V. (1996) Studies on high-strength concrete columns under eccentric compression. **ACI Struct. J.**, 93(6), 631-638.

RILEM 50-FMC Committee (1985) Determination of the fracture energy of mortar and concrete by means of three-point bend tests on notched beams. **Mat. and Struct.,** 18, 285-290.