Fracture Mechanics of Concrete Structures
Proceedings FRAMCOS-3
AEDIFICATIO Publishers, D-79104 Freiburg, Germany

COUPLED BOND AND BRIDGING STRESS TRANSFER IN CRACKED REINFORCED CONCRETE

H. M. Salem, and K. Maekawa
Department of Civil Engineering, The University of Tokyo, Tokyo, Japan

Abstract
The aim of this study is to get the spatial average stress- average strain relationships of both reinforcing bars and cracked concrete in RC members based on the local bond characteristics between concrete and reinforcing bars and tension-fracture of plain concrete. The computations are based upon the versatile local bond stress-slip-strain model. The local stress and strain profiles of both reinforcing bars and concrete between two adjacent cracks are explicitly evaluated with the softening bridging stress at individual cracks. From these profiles, the spatial average constitutive model in tension is derived. The computation is also capable of predicting the average crack spacing.
Key Words: Bond stress-slip-strain, tension stiffening, crack spacing.

1 Introduction

The tension stiffening effect represents the capacity of concrete to carry the internal tensile force developing between adjacent cracks. At particular cracked sections, the local tensile force is carried by both steel reinforcing bars and concrete residual softened tension. The force developing in reinforcement is partly transferred to concrete between adjacent cracks through bond stress transfer between reinforcing bars and concrete, while the residual tensile stresses at crack sections are applied directly to the fracturing planes.

The tension stiffening effect is usually treated by assuming a relationship between the average concrete tensile stress and the average concrete tensile strain over a long-gauge length in the direction normal to cracks surfaces. At the same time, the stress-strain relationship of reinforcement has to be on average basis. As the stress distribution in reinforcement embedded in concrete varies along bars, the average stress-average strain relationship of reinforcement is significantly different from pointwise behavior of bare bar (Shima et al. (1987)). The bar begins to yield at concrete cracks prior to the remaining parts. Thus, the average yield stress generally becomes lower than the yield stress of bare bar as clearly pointed out by Okamura et al. (1990). After yielding, some parts of reinforcement close to cracks come into the strain hardening zone, whereas the remaining parts are still in the elastic zone. Therefore, the average response has stiffness between the elastic and the hardening one. Usually, a bilinear model is assumed for the average response of bars.

The smeared reinforcement model of Shima assumes a sinusoidal distribution of the steel stress along the reinforcement. This assumption is acceptable for heavily reinforced concrete, where the spacing of cracks is relatively small. However, for lightly reinforced concrete, the crack spacing is larger and localized like plain concrete. Therefore, Shima's stress distribution is no longer valid and the local steel stress has to be computed from micro-bond characteristics. The importance of considering tension softening arises in lightly reinforced concrete, as its contribution becomes high compared to the contribution of bond stress transfer. In this study, both microscopic bond stress transfer and tension fracturing of plain concrete are considered. The local stresses and strains of steel and concrete are computed, and hence the macroscopic behavior is evaluated.

The present tension stiffening model proposed by Shima et al (1987) is an empirical one and does not take into consideration the effect of the amount of reinforcement. However, the amount of reinforcement has a considerable effect on the tension stiffness of lightly reinforced members.

In the present study, the tension stiffness is analytically computed and the effect of amount of reinforcement is taken into account. The aim of this study is to get a versatile smeared tension stiffening model for reinforcing bars and concrete in heavily and lightly reinforced concrete.

2 Spatial averaged constitutive laws in tension

2.1 Bond-slip-strain model:

Shima et al. (1987) proposed a universal bond stress-axial slip-steel strain model for RC. The model offers unique relationship that expresses the bond characteristics derived from both pull-out and axial tension tests. The authors adopt this model for the local stress interaction between concrete and reinforcement,

$$\tau(\varepsilon, s) = \tau_0(s) g(\varepsilon) \tag{1}$$

$\tau(\varepsilon, s)$: Bond stress, $\tau_0(s)$: Bond stress when strain is zero

$$\tau_0(s) = f_c' \, k \left[\ln(1+5s)\right]^c \tag{2}$$

$$g(\varepsilon) = \frac{1}{1+10^5 \varepsilon} \tag{3}$$

f_c': Compressive strength of concrete; k: Constant=0.73; c: Constant=3; s: Non dimensional slip =1000S/d ; S: Slip; d, ε : Diameter, strain of bar.

2.2 Bond deterioration model:

Shima's model can not be applied to the bond deterioration zone where the "near crack surface effect" is predominant. In fact, the localization of plastic steel yielding is initiated from the bond deterioration zone. Thus, the modeling of bond close to cracks plays an important role for post-yield behavior of RC in tension. Qureshi et al. (1993) assumed in the RC joint model that the bond stress is linearly decreasing to zero at a distance 5 d from the crack surface, and that the bond stress drops suddenly to zero at a distance 2.5 d from the crack surface due to splitting and crushing of concrete around the bar beside the crack surface. Fig.1 shows a schematic drawing of bond deterioration formulated by Qureshi et al. (1993) as,

$$\tau(x) = \tau_{max} - \frac{\tau_{max}}{L_b}\left[x - \left(\frac{L_c}{2} - L_b\right)\right], \qquad \frac{L_c}{2} - L_b \le x \le \frac{L_c}{2} - \frac{L_b}{2} \tag{4}$$

$$\tau(x) = 0 \qquad , \qquad \frac{L_c}{2} - \frac{L_b}{2} \le x \le \frac{L_c}{2} \tag{5}$$

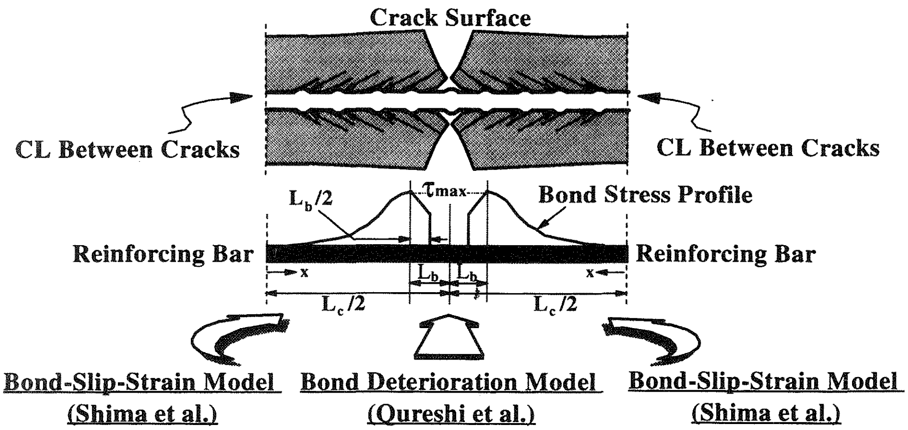

Fig.1 Bond deterioration zone by Qureshi et al. (1993)

2.3 Tension softening at crack surface

When concrete is cracked, the stress carried by concrete at the crack surface does not drop to zero suddenly. The bridging of the two faces of the crack causes a transfer of some residual stresses. This phenomenon is known as tension softening of plain concrete. Regarding reinforced concrete members with ordinary reinforcement ratios, this softening can be neglected compared to the force carried by bond stress transfer. However, in case of small reinforcement ratio this softening cannot be neglected. Usually the tension softening is expressed as a relationship between the residual tensile stress and the crack width. The surface crack width can be considered being compatible with the reinforcement slip at the crack. Thus, surface crack width is equal to the sum of the bar slip on both sides of the crack. In other words, the surface crack width is equal to twice the reinforcement slip, from one side, at the crack location. The average crack width used by Qureshi et al. (1993) is adopted here as,

$$w = C(2S_{max}), \quad S_{max} = S|_{x=L_c/2} \tag{6}$$

where C is equal to (1/1.3). The tension softening model adopted in the analysis (Uchida et al. 1991) is defined as,

$$\sigma_{br} = f_t\left[1 + 0.5\left(\frac{f_t}{G_f}\right)w\right]^{-3} \tag{7}$$

where, σ_{br} is the bridging stress across crack, f_t is the tensile strength, w is the crack width and G_f is the fracture energy ranging from 0.1 to 0.15 kgf/cm for plain concrete.

3 Analysis

In order to get the steel stress profile, other governing equations have to be solved simultaneously. By dividing the reinforcing bar between two adjacent cracks into infinitely small strips and satisfying the static equilibrium of all elements, we get the following continuum equilibrium equation as,

$$\frac{d\sigma}{dx}=\frac{\pi d}{A_s}\tau \qquad (8)$$

where, A_s: Reinforcing bar cross sectional area, τ: bond stress.

The second equation is the bond-slip-strain model (Eqs. 1~3), together with the bond model in the bond-deterioration zone (Eqs. 4,5). The third equation is derived from the slip compatibility. The slip is computed by integrating the strain over the length of the reinforcing bar starting from the midway between adjacent cracks as shown in Fig.2, i.e. the slip at the midway between cracks is zero. Thus, we have,

$$S(x)=\int_0^x \varepsilon dx \qquad (9)$$

The fourth one is the constitutive equation for bare bar which represents the pointwise relationship between the steel stress and strain at each bar section as,

$$\sigma=\sigma(\varepsilon) \qquad (10)$$

The overall scheme of computation is summarized in Fig.2. Firstly, the crack spacing is equal to the total length of the specimen, and during analysis the local concrete tensile stresses are checked and a new crack is introduced whenever the stress reaches f_t and a new average crack spacing is computed. Starting from the midway between two adjacent cracks, a finite segment with length Δx is studied. The boundary conditions for this segment are set by equating both the slip and the bond stress at the middle section to zero, and assuming an arbitrary value to the strain at the middle. This arbitrary strain value represents the loading level. The four equations are simultaneously solved using an iterative procedure. Finishing the computation of this segment, the boundary conditions of the next division are defined and a similar computation procedure is followed. Hence, the strain and stress profiles of the steel reinforcement can be drawn. It results in the steel average stress and average strain as,

$$\bar{\varepsilon}=\frac{2}{L_c}\int_0^{\frac{L_c}{2}}\varepsilon(x)dx\cong\frac{2}{L_c}\sum_0^{\frac{L_c}{2}}\varepsilon(x)\Delta x \tag{11}$$

$$\bar{\sigma}_s=\frac{2}{L_c}\int_0^{\frac{L_c}{2}}\sigma_s(x)\,dx\cong\frac{2}{L_c}\sum_0^{\frac{L_c}{2}}\sigma_s(x)\Delta x \tag{12}$$

By computing the stress profile of reinforcement, the stress profile of concrete is obtained by subtracting reinforcement force profile from the reinforcing bar force at crack. Adding the bridging stress, the average stress of concrete is mathematically defined as,

$$\bar{\sigma}_c=\sigma_{br}+\frac{2}{L_c}\int_0^{\frac{L_c}{2}}\sigma_c(x)\,dx\cong\sigma_{br}+\frac{2}{L_c}\sum_0^{\frac{L_c}{2}}\sigma_c(x)\,\Delta x \tag{13}$$

Fig. 3 illustrates the analysis of two cases, one with heavy reinforcement (2%) and the other with very low reinforcement ratio (0.01%). From that figure it can be concluded that, for ordinary RC, tension softening can be disregarded in analysis and only bond stress transfer mechanism can be considered, whereas in very small reinforcement ratio or plain concrete, the contribution of the tension softening at cracks is the predominant and has to be considered.

Fig. 4 shows the gauge length effect on the average response of concrete. Comparing two cases, one with heavy reinforcement and the other with light reinforcement, it can be observed that the tension stiffening is more stable and independent on gauge length for the heavy reinforcement, while it depends very much on the gauge length in the case of low reinforcement or plain concrete. In fact, this is due to the

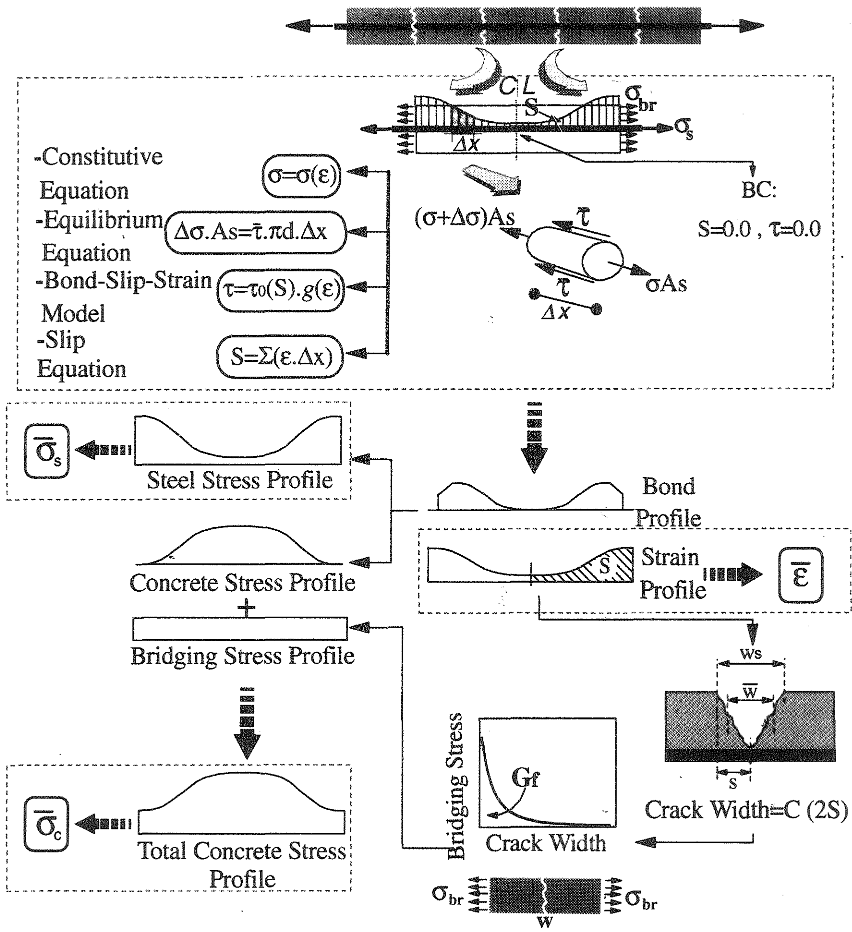

Fig.2. Scheme of solving bond governing equations with finite discretization

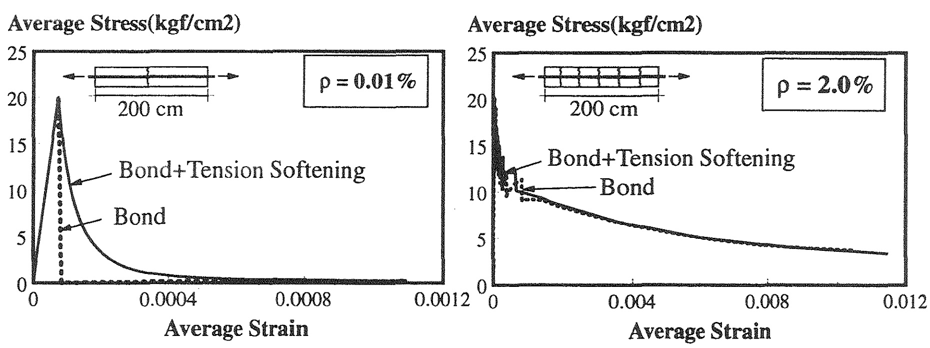

Fig.3. Tension softening effect on tension stiffness of RC

localization of cracks in lightly reinforced concrete, causing the behavior to be similar to plain concrete and the average response becomes dependent on the gauge length. On the other hand, for reinforced concrete with high reinforcement ratio, the cracking is controlled by the reinforcement through bond mechanism resulting in an almost equal crack spacing for different gauge lengths.

A comparison with the experiments by Shima et al. (1987) is shown in Fig. 5 and 6 for various cases with different reinforcement ratio and material strengths. The analysis agrees well with the reality.

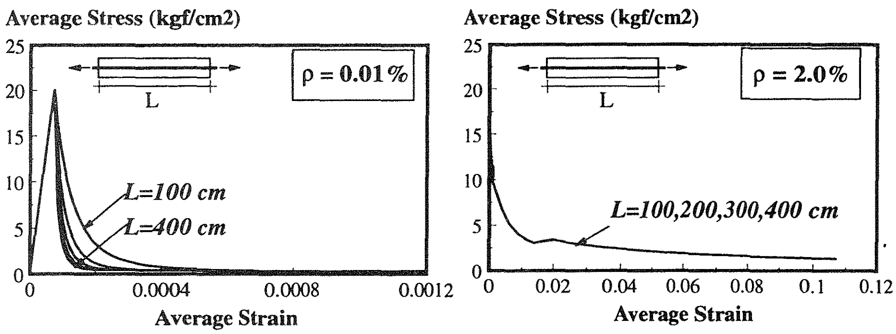

Fig.4. Size effect on average response of concrete

4 Conclusions

The conclusions of this paper can be summarized as follows:

1. Based on the microscopic bond-slip-strain model, bond deterioration, and tension softening at crack surface, the stress profile as well as the strain profile of reinforcing bars embedded in concrete can be computed. Hence, the macro average stress- average strain relationship of reinforcing bars as well as the tension stiffening of concrete can be computed. From the microscopic behavior of reinforced concrete, the macroscopic behavior can be detected.
2. In reinforced concrete members with ordinary reinforcement ratio, the tension softening at the crack surface can be neglected without considerable influence on the average response of concrete and reinforcement. However, in very lightly reinforced concrete or plain concrete, tension softening at fractured crack planes is predominant and has to be taken into consideration.

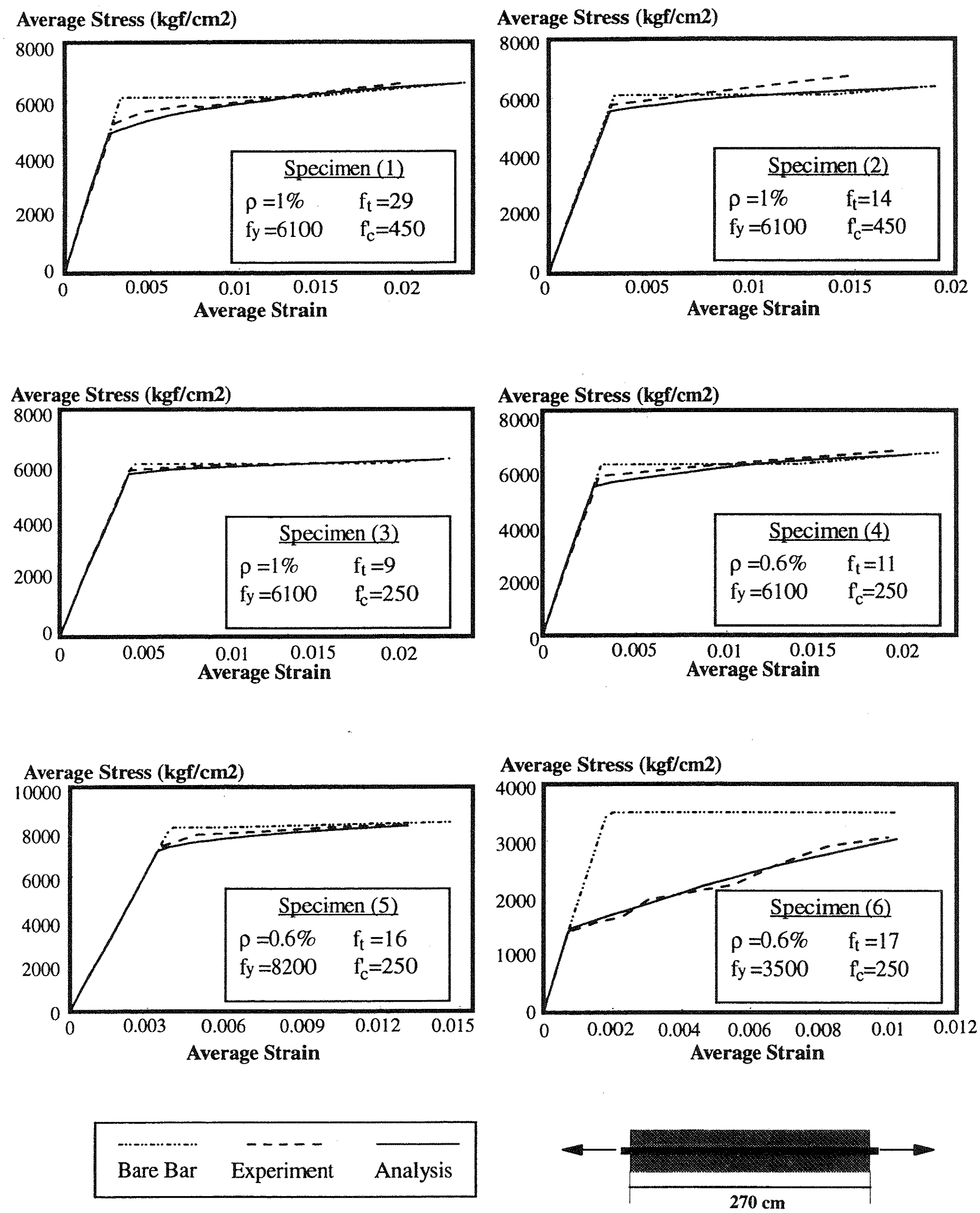

Fig.6. Average response of reinforcement: A comparison with the experimental work of Shima et al. (1987)

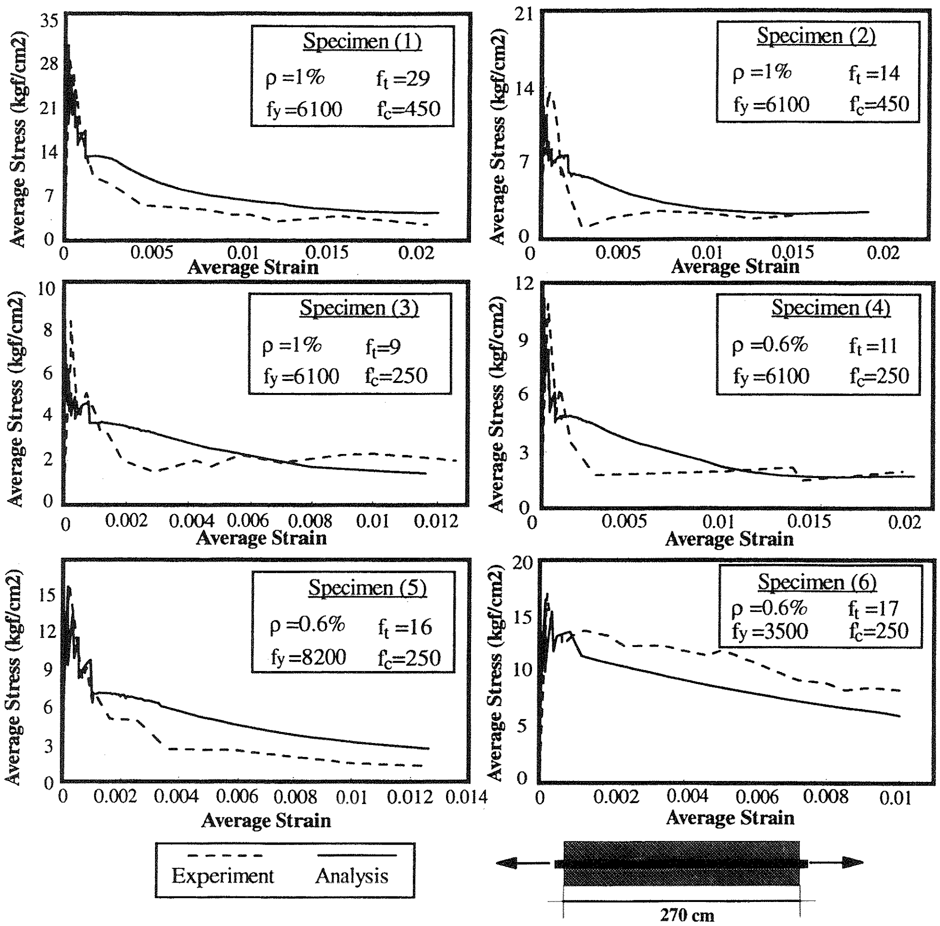

Fig.5. Average response of concrete: A comparison with the experimental work of Shima et al. (1987)

5 References

Okamura, H. and Maekawa K. (1990) **Nonlinear Analysis and Constitutive Models of Reinforced Concrete**, Gihodo, Tokyo.

Qureshi, J. and Maekawa, K. (1993) Computational Model for Steel Bar Embedded in Concrete under Combined Axial Pullout and Transverse Shear Displacement, **Proc. of JCI**, 15, 2, 1249-1254.

Shima H., Chou L., and Okamura, H. (1987) Micro and Macro Models for Bond in Reinforced Concrete, **Journal of The Faculty of Engineering, The University of Tokyo (B)**, 39, 2, 133-194.

Uchida, Y., Rokugo, K., and Koyanagi, W. (1991) Determination of Tension Softening Diagrams of Concrete by Means of Bending Tests, **Proceedings of JSCE**, 14, 426, 203-212.