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GEOMETRIC NON-LINEAR EFFECT IN THE BEHAVIOUR OF DAMAGEABLE STRUCTURES: PREDICTION OF THE ULTIMATE CAPACITY USING A SIMPLIFIED APPROACH

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Abstract

The ability of most commercial FE codes in performing complete powerful analysis is usually accompanied by great needs in computing power and time. By focusing the efforts on specific mechanical phenomena, it is then possible to elaborate simplified numerical tools capable to provide results of good quality with reasonable cost. The aim of this paper is to demonstrate this argument around the code EFiCoS (LMT ENS Cachan). For this purpose, an experimental study has been chosen and comparisons are made. Analyses performed by a commercial code, ABAQUS, at Chalmers University are also presented for the purpose of a code/code comparison underlining the ability of simplified algorithms as a substitute to sophisticated ones.

Key words: simplified method, geometrical nonlinearity, damage concept, RC columns.

1 Introduction

Even though the behaviour of reinforced concrete columns have been studied for decades, even centuries, the strive to cut costs puts new demands on the design tools. To fully understand the structural behaviour of concrete columns extensive test series would be desired. However, in view of the economical limitations that prevent complete test series being carried out, it would be of great benefit to establish suitable numerical models that reflect the structural behaviour accurately.

To model reinforced concrete, several different approaches have been proposed such as discrete or smeared plasticity approaches, damage theory, etc. (Bazant et al., 1994; Lemaitre & Chaboche, 1990; Dragon, 1994; Mazars 1986; Chen and Buyukozturk, 1985). The aim of this paper is to analyse and describe the simulation of major phenomena which may contribute to the behaviour, up to failure, of RC structures by validating a *simplified approach.*

The ability of this simplified FE program, EFiCoS, has been examined by simulating the behaviour of several RC columns. These were not only involved in an experimental program but also studied by a commercial code, ABAQUS, which incorporates *sophisticated algorithms* capable of describing major mechanical needs (3D description, full description of geometrical nonlinearities, etc.).

The FE models used in the analyses are presented followed by a verification against experimental results. Observations of the failure mechanisms as well as some reasons for the failure of columns under eccentric compressive loading are presented.

2 Experimental program and test results

2.1 The specimens

From a test series carried out at Chalmers University of Technology, three test specimens were chosen for the analyses. The details of these specimens are shown in Figure 1 and Table 1. The columns had a 200 x 200 mm cross section and a length of 3000 or 4000 mm.

The thickness of the clear concrete cover, measured to the outer edge of the stirrup, was 15 mm. Deformed bars with 8 mm diameter and yield strength of 466 MPa and a modulus of elasticity of 221 GPa were used for lateral reinforcement while deformed bars with 16 mm diameter and yield strength of 636 MPa and modulus of elasticity of 207 GPa were used for longitudinal reinforcement. The columns were hinged at both ends and the load was applied with an initial eccentricity of 20 mm at both ends. Further details about the specimen are found in Claeson and Gylltoft (1998).

Table 1. Characteristics of each specimen

	Specimen 1	Specimen 2	Specimen 3
Cross section (mm)	200 x 200	200 x 200	200 x 200
Length (m)	3	3	4
Concrete compressive strength (MPa)	33	91	37
Steel yield stress (long. bars) (MPa)	636	636	636
Steel E-modulus (long. bars) (MPa)	207 000	207 000	207 000
Steel plastic tangent stiffness (MPa)	12 275	12 275	12 275
Axial load eccentricity (mm)	20	20	20
Stirrups spacing (mm)	130	130	130

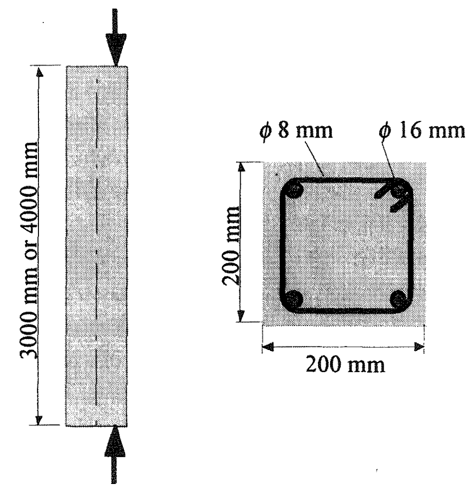

Figure 1. Test specimens

2.2 Test results

The failure of specimen 1 and 3 made of normal-strength concrete (NSC) was gradual and, unlike the high strength concrete columns (specimen 2), the post-peak curves could be captured. The columns displayed significant softening on the compressive side including gradual spalling of the concrete cover before reaching the maximum load. Furthermore, tensile cracks were visible on the tensile side. All of the columns failed due to crushing of the concrete and the failure zone was located at or near mid-height. None of the reinforcement bars had reached yielding at maximum load. The measured maximum load was 990 kN, with a corresponding mid-height displacement of 22 mm for specimen 1, 900 kN and 40 mm for specimen 2, and 1530 kN and 39 mm for specimen 3. The product of these maximum deflexions by the vertical loads indicates the presence of significant second order moments necessary to be included in numerical analyses.

3 FE models

3.1 General

The aim of the study was to evaluate the capacity of the finite-element program EFiCoS to capture the structural behaviour of these concrete columns where geometrical nonlinearity may constitute an important aspect of the behaviour as well as material nonlinearities. The finite element program was developed at the LMT Cachan. In principle, the code provides two-node beam elements constituted from multiple layers. In it's original form, the formulation of the beam elements was based on Navier-Bernoulli assumptions, and it's ability to reproduce the behaviour of experimental specimens has been extensively demonstrated. However, the Navier-Bernoulli theory is no longer sufficient if non-linearity due to geometrical deformations need to be considered. Thus, an improvement was needed to extend the ability of this simplified code to study high slender elements with significant axial loadings.

A description of this technique is given bellow as well as a brief review of the code. Details on the ABAQUS analyses are also provided, indicating the type of element used, the description of the material nonlinear behaviour and other considerations concerning the geometrical non-linearities.

3.2 Theoretical basis of EFiCoS

EFiCoS is a finite element program based on a multilayered beam formulation and capable of nonlinear dynamic and static analysis. The beam elements have two nodes, and are built by superimposing layers with nonlinear stress-strain laws, whether they contain concrete or concrete+steel materials. Plane section hypothesis allows the use of uniaxial stress-strain laws in the layers. The kinematic assumption for layers containing both concrete and steel is that both materials undergo the same strain. Therefore, no bond slip is included in the analysis. Reinforcement steel is modelled by an elastoplastic, linear kinematic hardening law and the behaviour of concrete is treated by a constitutive law based on damage mechanics (La Borderie, 1991). Key features of this law are as following :

- two scalar damage variables govern the tension and compression behaviour,
- opening and re-closure of cracks, allowing a gradual restoration of stiffness under compression,
- inelastic strains created as damage progresses.

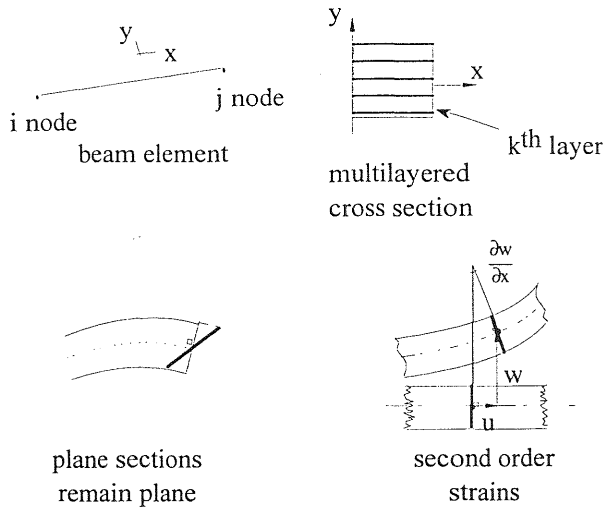

Figure 2. Original and modified kinematic formulations

Navier-Bernouilli	Large displacements
$\varepsilon_{xx}\ \sigma_{xx}$	$\varepsilon_{xx}\ \sigma_{xx}$
$u(x,y) = u(x) - y.\theta$	$u(x,y) = u(x) - y.\theta$
$\theta = \frac{\partial w}{\partial x}$	$\theta = \frac{\partial w}{\partial x}$
$\varepsilon_{xx} = \frac{\partial u(x)}{\partial x}_{y=0} + y\frac{\partial^2 w}{\partial x^2}$	$\varepsilon_{xx} = \frac{\partial u(x)}{\partial x}_{y=0} + y\frac{\partial^2 w}{\partial x^2} + \underbrace{\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)}_{\text{second order terms}}$

The beam formulation is of sufficient accuracy for the modelling of structures of typical slenderness and the stiffness matrix is formulated in accordance with the kinematical hypothesis.

In case the shearing cannot be neglected, an enrichment has been provided at the cost of the calculation of the shear deformations (Dubé, 1997). This has allowed the use of the program in simulating the nonlinear behaviour of wall-type structures treated in several experimental studies (CASSBA, 1994; Ghavamian, 1996).

In the case where geometrical non-linearities may no longer be neglected, a solution has been proposed which consists in considering the geometrical stiffness matrix for beams (Przemieniecki, 1963; Ghavamian, 1998). While this method has been derived for the elastic linear theory of beams, it was interesting to evaluate it's ability within a nonlinear damage behaviour. Figure 2 describes the theoretical basis of this method, as compared to the initial classical beam theory implemented in this FE program.

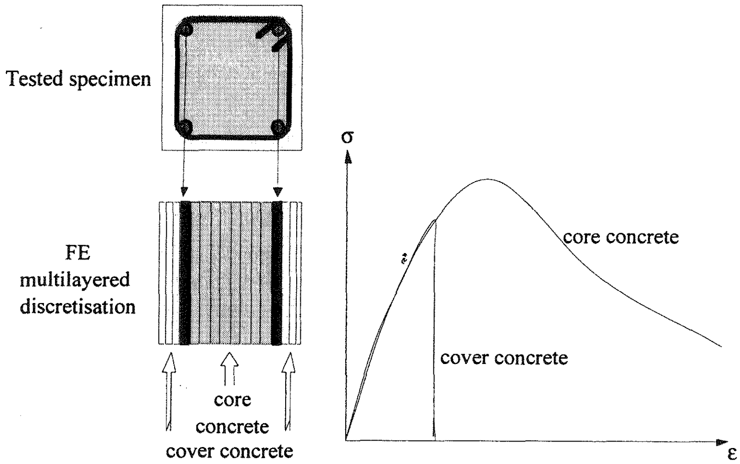

Figure 3. Multilayered beam constitution including spalling mechanism

Considering second-order axial strains, the construction of the elementary stiffness matrix would result in additional terms linearly dependant on the length of the element and the axial load level. This matrix may be divided into two parts : $\boldsymbol{K}_{tot} = \boldsymbol{K}_{mat.} + \boldsymbol{K}_{geom.}$ Changes in material properties imply a reactualisation of the $\boldsymbol{K}_{mat.}$ matrix. Whereas $\boldsymbol{K}_{geom.}$ needs no updating unless the axial force varies significantly.

Using multilayered beam elements, the spalling phenomena observed in the tests, were also simulated by conferring to cross sectional unconfined layers the capacity to undergo crushing at specific strains (Figure 3).

Ten beam elements were used to model the columns tested. Each beam was subdivided in 22 layers of concrete and two including reinforcement. When treating the softening behaviour of materials, special care must be taken in order to avoid problems such as *localisation*, Bazant and Pijaudier-Cabot (1988). While this may be achieved by using a nonlocal form of the constitutive law or by considering rate effects, here we have adopted another strategy: the smallest size of elements is set equal to the length of the plastic hinge, as described in paragraph 3.3.

3.3 ABAQUS

A model based on three-dimensional three-node hybrid beam elements was established. Taking into account the plastic hinge of 450 mm at mid-height, a total of 10 elements were used. The length of this plastic hinge was first obtained by applying an empirical formula proposed by Paulay and Priestley (1992). This was then found to be consistent with the damage zone observed at the end of the tests. The cross section had 5x15 integration points. The model included the four vertical reinforcement bars; however, the stirrups could not be modelled. The latter implies that the vertical reinforcement bars cannot buckle either.

To model reinforced concrete, the program ABAQUS combines standard elements of plain concrete with a special option called *rebar*. This option strengthens the concrete in the direction chosen, thereby simulating the behaviour of a reinforcement bar.

The material model provided in ABAQUS was used in the analysis. When the principal stress components are compressive, the response of the concrete is modelled by an elastoplastic model. The smeared crack approach has been chosen to model cracked reinforced concrete. According to the smeared crack concept, and similarly to the damage mechanics concept, a cracked solid is supposed to be a continuum for stress and strain computation. This means that the behaviour of cracked concrete can be described in terms of stress-strain relations. Prior to cracking, the concrete is modelled in tension as an isotropic, linear elastic material. The fracture energy was determined from tests on three-point bending beams (RILEM50-FMC Committee, 1985) and, together with the tensile strength and crack spacing from the tests, was used to calculate the tensile softening relation.

The longitudinal reinforcement bars were modelled by a linear elastic-plastic material model. The modulus of elasticity, the yield strength and the ultimate strength of the reinforcement bars were determined through tension tests (Table 1), and the Poisson ratio was approximated to be 0.3.

4 Results of the FE-analyses

Figures 4 to 6 show comparisons of the load-deflection curves from the tests and the analyses of the columns. This comparison shows that both FE-models do capture the structural behaviour satisfactorily.

Due to the small eccentricity (20 mm), the maximum concrete strength determined the maximum load, and consequently, the failure was compressive. Evaluating the maximum strength capacity of these columns is quite independent of the softening considerations of its material constituents thanks to the reinforcement ratio. However this determination is strongly conditioned by the consideration of nonlinear geometrical effects (Figure 7).

Simulating the post-peak behaviour is more problematic since determining the exact material parameters, governing their softening behaviour is difficult. Indeed, the overall responses are very sensitive to these parameters and special care must be taken to represent accurately this part. Also, the length of plastic-hinge and the spalling mechanism do influence the results by amplifying the localisation phenomenon. To demonstrate this fact, Figure 8 illustrates results from ABAQUS for different plastic-hinge lengths (from 60 to 20 mm) for specimen 3.

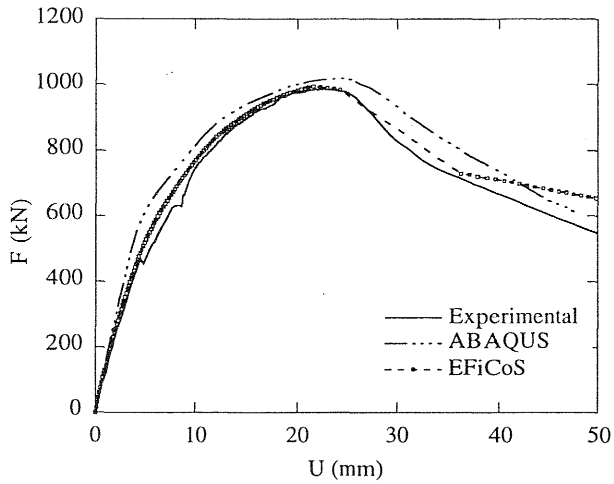

Figure 4. Load-deflection curves for specimen 1

Figures 4 and 6 indicate that both analytical methods provide good results up to the peak value of the strength. However, the importance of the capacity to model the spalling of the concrete is observed since this is not captured in the analysis using ABAQUS (Figure 4), as the cross section is modelled as one material and not as a concrete core with one material and a concrete cover with another.

This is even more crucial for the high-strength concrete. Due to the sudden spalling of large parts of the concrete cover at mid-height, little data, after the maximum load had been reached, were recorded in the test of specimen 3. Therefore, no comparisons with FE results have been carried out.

Concerning the geometrical nonlinearity, to demonstrate its importance using EFiCoS, an analysis with no geometrical nonlinearity effect has been performed and the response is included in Figure 8.

From this comparison two conclusions may be drawn : one is the confirmation of the importance of the geometric nonlinear effect on the overall behaviour of the structure, the other is the ability and the efficiency of the code in simulating geometrical nonlinear effects.

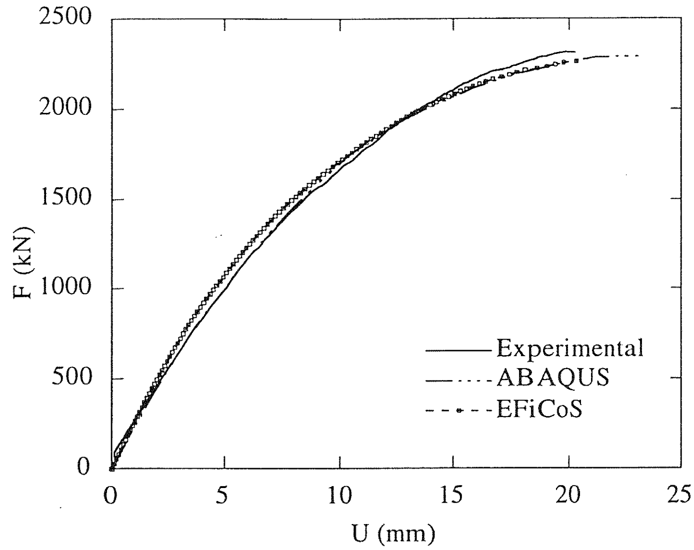

Figure 5. Load-deflection curves for specimen 2

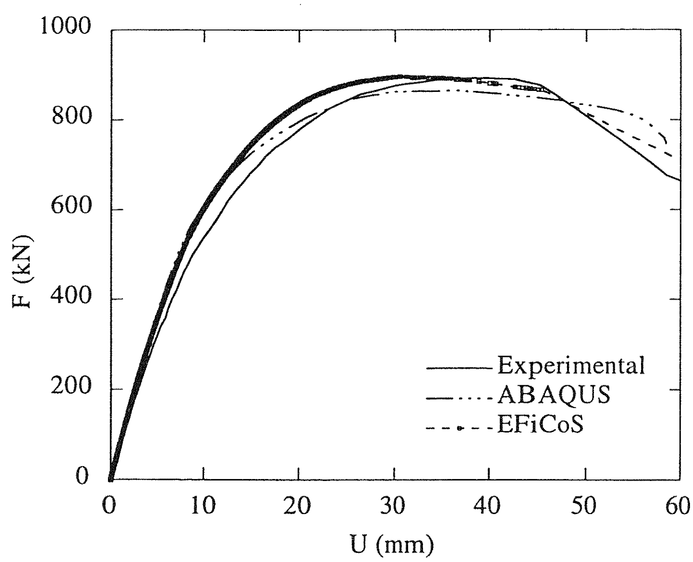

Figure 6. Load-deflection curves for specimen 3

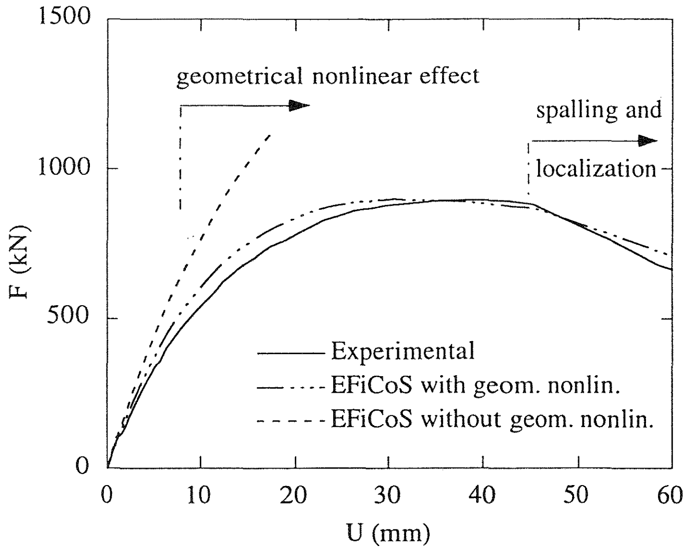

Figure 7. Quantification of geometrical nonlinear effect

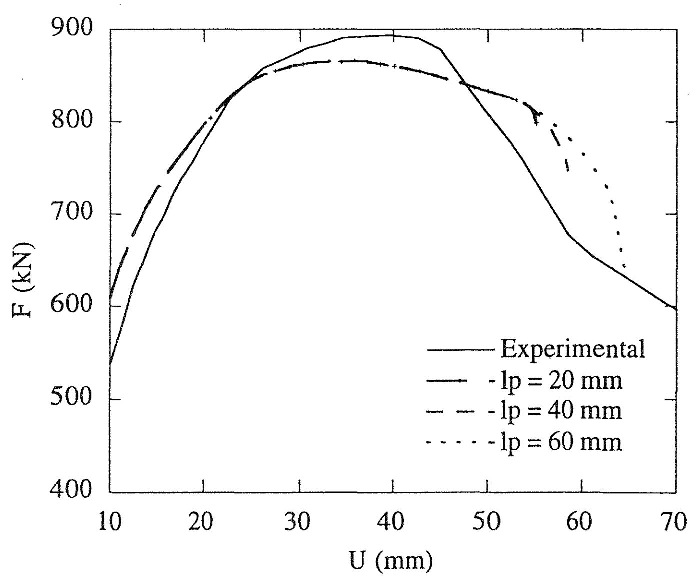

Figure 8. Load-deflection curves for specimen 3.
Influence of the plastic-hinge length on post-peak phase

7 Conclusions

Regarding the experimental tests invoked here, it was essential to detect the most important mechanical phenomena (geometrical and material nonlinearity with spalling) needed for a numerical simulation of the behaviour. These were then integrated in the FE program by implementing simplified and robust algorithms, and from these comparisons, one can conclude that the efficiency of EFiCoS in simulating the behaviour of slender columns with high axial loading is achievable.

Compared to the commercial code ABAQUS, which uses more sophisticated algorithms (Updated Lagragian formulation), the technique implemented in EFiCoS provides the advantage of performing analyses as fast and accurate as ABAQUS. However, this is valid in static calculations; in dynamic, the efficiency of EFiCoS may be more significant when compared to most commercial codes by performing calculations with more reasonable time costs which allow to perform parametric studies. The simplicity of the code and the use of a multilayered beam description of elements make the program an adequate research tool allowing changes and modifications with a reasonable amount of effort. This was the case when considering the mechanism of spalling. If necessary, other mechanical phenomena may also be integrated (ex.: buckling of rebars, bond slip, local failure criteria necessary to determine the maximum capacity of structures).

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