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3D ANALYSIS USING UNIFIED CONCRETE PLASTICITY MODEL WITH REINFORCEMENT UNDER COMPRESSION MODELLED AS BEAM ELEMENT

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Abstract
Recently, simulation of bending failure of RC members was successfully done where concrete was modelled by a classical plasticity model with 8 nodded cubic element and the reinforcement was modelled as 2 nodded rod element. Though the analysis was successful, unrealistic confinement effect was noticed near the stirrup in the compression zone. In this paper, a simplified beam element is proposed for modelling of reinforcement in compression region. Sectional properties are calculated at gauss points and integration is done to calculate the stiffness matrix. Though the results looks promising, the model is still in the developing stage.
Keywords: Reinforcement, beam, Unified Concrete Plasticity Model.

1 Introduction

Many of the structures in recent years have undergone severe damages due to earthquakes. In most of the cases, the failure phenomenon are quite three dimensional in nature. Though two dimensional analysis of RC members is enough in the initial stage, three dimensional(3D) analysis is important in the simulation of the behaviour in the failure stage. Discrete crack approach has rarely been applied to 3D analysis. Most of the classical plasticity models take different criteria for tension and compression and their application in general conditions are difficult. Moreover, the classical plasticity models face the problems related to mesh sensitivity. Recently, various non-local approaches are being proposed to take care of the mesh sensitivity problems. However, these non-local approaches are still in the formative stages. It was realised that a combination of a good plasticity model with some practical implementation of the non-local formulation would be a practical way of analysing reinforced concrete structures. Hence the authors are at present engaged in the development of a classical plasticity model, named Unified Concrete Plasticity Model, in 3D stress space (Tanabe et. al.(1994), Gupta and Tanabe(1997,1998), Gupta(1997) and Gupta et al.(1998)) with the hope of implementation non-local approach with this model in future.

3D analysis of various bending and shear failure problems of concrete structures were done using this model successfully. After these analysis, it was realised that *proper modelling of the reinforcement* is also an important factor and requires further attention.

In the analysis of RC members failing in bending mode, the modelling of reinforcement in tension for RC beam under two point loading and cantilever column with lateral loading required different implementation of tension stiffening effect(Gupta and Tanabe(1998) and Gupta(1997)). In this problem, unrealistic high confinement was noticed near the stirrup (modelled as 2 nodded rod element) in the compression zone.

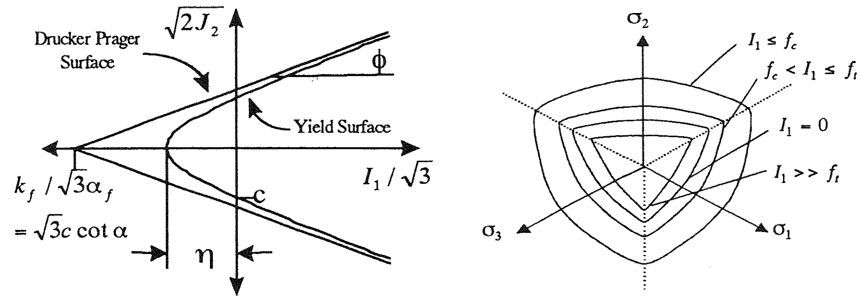
In this paper, this problem is analysed by modelling the reinforcements in compression zone using a proposed beam element to take care of the effect of decrease of compressive strength due to lateral deflection. This implementation is done for the main reinforcement and the stirrup in central critical compressive zone only.

Pallewatta(1993), Pallewatta et. al(1995) have used beam element for column under axial compression. Nakamura et. al(1997) have used beam element to separately analyse the reinforcement element under compression to derive the appropriate stress-strain analysis of reinforcement for direct implementation in finite element analysis. Though both these methods are effective, in these cases the sectional moment curvature relationship are calculated and the stiffness term EI is obtained from the slope of the sectional M-ϕ characteristics.

A new beam element modelling for the reinforcement is proposed in this paper. Sectional properties are calculated at gauss points and the stiffness matrix is directly calculated by gauss point integration technique.

In this paper, first a short review of the Unified Concrete Plasticity Model used for modelling of concrete is presented, followed by the formulation, verification and application of the beam element as reinforcement in compression zone.

2 Unified Concrete Plasticity Model



a) Meridian Plane b) Diviatoric Plane

Fig.1. The initial yield surface for Unified Concrete Plasticity Model

Fig. 1 shows the initial shape of the yield surface of the Unified Concrete Plasticity Model. This model is a modified Drucker-Prager model where a parameter y is introduced in the denominator of Eq. 2, which is dependent on I_1 and θ (Eq.4) to get triangular shape in tensile region and more circular shape in compressive region.

$$g(\sigma,\omega)=\sqrt{J_2+k_f(1-AA^*/\eta_0)^2}-(k-\alpha_f I_1)=0 \qquad (1)$$

$$k_f=\frac{6C\cos\phi}{\sqrt{3}\left(3+y\sin\phi_1\right)},\alpha_f=\frac{2\sin\phi}{\sqrt{3}\left(3+y\sin\phi_1\right)} \qquad (2)$$

$$AA^*=\sqrt{3}c_o\cot\phi_o/\eta_o \qquad (3)$$

$$y=\sqrt{a(\cos 3\theta+1.00)+0.01}-1.10$$

$$a=0.5r^2+2.1r+2.2$$

$$r=\begin{cases}3.14 & I\le f_c' \\ 6.07-2.93\cos\left(\dfrac{f_t-I_1}{f_t-f_c'}\pi\right) & f_c'<I\le f_t \\ 9.0 & I>f_t\end{cases} \qquad (4)$$

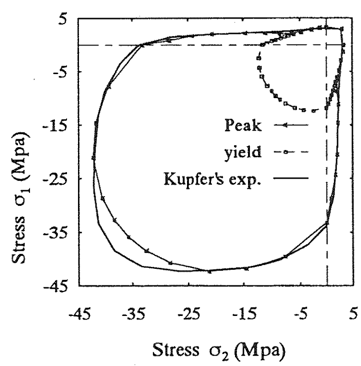
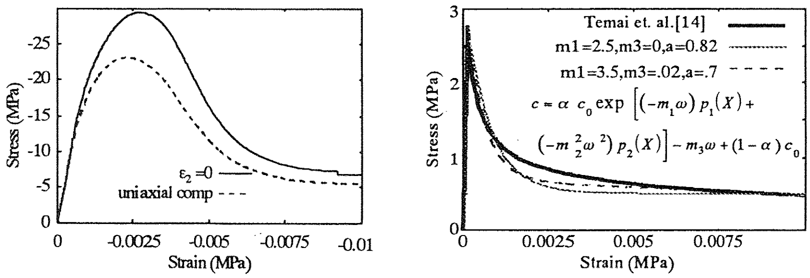


Fig. 2. Kupfer's stress envelop

where I_1, J_2 and J_3 are stress invariants; $\cos 3\theta=(3\sqrt{3}J_3)/(2J_2^{1.5})$; ϕ_1=14° is a material constant. Cohesion c and friction angle ϕ are the most important parameters depending on the damage ω. The change of frictional angle ϕ and cohesion c is assumed to be different in case of tension and compression zone. An appropriate smooth variation between the two zones was proposed based on $X(=I_1/\sqrt{3J_2})$. The material parameters are such chosen that biaxial peak strength match the Kupfer's peak strength envelop



a) Uniaxial compression b) Uniaxial tension

Fig. 3. Stress strain behaviour

(Fig. 2) and proper stress strain curve uniaxial tensional and compression (Fig. 3). The tension stiffening or fracture energy for tension can be easily taken care by modifying the c-ω relation as is shown by Gupta et. al(1998).

3 Reinforcement in Compression as Beam Element

Reinforcement in the central top compression zone (Fig. 7) are modelled as beam element(Fig. 4) with the following assumptions:

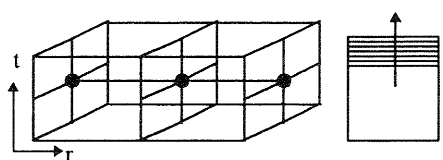


Fig. 4 3 nodded beam element

a) Displacement and strain are small and the beam is initially straight.
b) The reinforcement is parallel to the either of the x or y axis and the problem of co-ordinate transformation is avoided. Hence, it is only sufficient to consider bending effect due to vertical deflection.
c) The section is considered square as the first step of implementation.

Here (N_i, u_i), (V_i, w_i), (M_i, θ_i) are the nodal variables. The element has 3 nodes. 2 gauss points are used for integration as the first step. The relation between the general co-ordinates and displacement with nodal co-ordinates and displacement in terms of r (the local co-ordinate) are

$$x_r = \sum_{i=1}^{3} N_i(r) x_i \tag{5a}$$

$$\{u \quad w \quad \theta\}_r^T = \sum_{i=1}^{3} N_i(r) \{u_i \quad w_i \quad \theta_i\}^T \tag{5b}$$

where N(r) is the cubic polynomial shape function and the calculations for Jacobean (a scalar quantity here) is given as

$$[J] = [\partial x / \partial r] = \left[\sum_{i=1}^{3} \left(\partial N_i(r) / \partial r\right) x_i\right] \tag{6}$$

$$\frac{\partial}{\partial x}\{u \quad w \quad \theta\}_r^T = J^{-1}\frac{\partial}{\partial r}\{u \quad w \quad \theta\}_r^T = J^{-1}\sum_{i=1}^{3}\frac{\partial N_i}{\partial r}\{u_i \quad w_i \quad \theta_i\}^T \tag{7}$$

The strain components are given as

$$\begin{Bmatrix} \varepsilon_x \\ \gamma \\ \phi \end{Bmatrix}_r^T = \begin{Bmatrix} \partial u / \partial x \\ \partial w / \partial x + \theta \\ \partial \theta / \partial x \end{Bmatrix}^T = \sum_{i=1}^{3} \begin{bmatrix} J^{-1}\partial N_i / \partial r & 0 & 0 \\ 0 & J^{-1}\partial N_i / \partial r & N_i \\ 0 & 0 & J^{-1}\partial N_i / \partial r \end{bmatrix} \begin{Bmatrix} u_i \\ w_i \\ \theta_i \end{Bmatrix} = \mathrm{B}\boldsymbol{u} \quad (8)$$

The stress-strain at each strip can be calculated at each gauss points as

$$\varepsilon_x(r,t_j) = \varepsilon_x + (h/2 - t_j)\phi \quad (9)$$

$$\sigma_x(r,t_j) = f(\varepsilon_x(r,t_j)) = f(\varepsilon_x,\phi) \quad (10)$$

where n is number of sections, and t_j is the distance of the centre of jth section from the top. Axial, bending forces, shear terms are calculated as

$$N = \int_A \sigma_x dA \qquad \text{(function of } \varepsilon_x \text{ and } \phi) \quad (11a)$$

$$M = \int_A \sigma_x (h/2 - t_j) dA \qquad \text{(function of } \varepsilon_x \text{ and } \phi) \quad (11b)$$

$$V = \alpha G \gamma A \qquad \text{(function of } \gamma) \quad (11c)$$

where $G = E/2(1+\mu)$, μ=0.3 for steel. N, M and V can be assumed as the stress tensor at the gauss points related to the strain at the gauss points as

$$\sigma(r,t_j) = \{N \quad V \quad M\}_r^T = f(\varepsilon_x,\gamma,\phi) \quad (12)$$

The stress tensor in the incremental form can be written as

$$d\sigma = d\begin{Bmatrix} N \\ V \\ M \end{Bmatrix} = \begin{bmatrix} \partial N / \partial \varepsilon_x & 0 & \partial N / \partial \phi \\ 0 & \alpha GA & 0 \\ \partial M / \partial \varepsilon_x & 0 & \partial M_1 / \partial \phi \end{bmatrix} d\begin{Bmatrix} \varepsilon_x \\ \gamma \\ \phi \end{Bmatrix} = D d\varepsilon \quad (13)$$

where

$$\frac{\partial N(\varepsilon_x,\phi)}{\partial \varepsilon_x} = \sum_{j=1}^{N} \frac{\partial \sigma_j}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial \varepsilon_x} dA_j = \sum_{j=1}^{N} \frac{\partial \sigma_j}{\partial \varepsilon} dA_j \quad (14a)$$

$$\frac{\partial N(\varepsilon_x,\phi)}{\partial \phi} = \sum_{j=1}^{N} \frac{\partial \sigma_j}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial \phi} dA_j = \sum_{j=1}^{N} \frac{\partial \sigma_j}{\partial \varepsilon} \left(h/2 - t_j\right) dA_j \quad (14b)$$

$$\frac{\partial M_1(\varepsilon_x,\phi)}{\partial \varepsilon_x} = \sum_{j=1}^{N} \frac{\partial \sigma_j}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial \varepsilon_x} (h/2 - t_j) A_j = \sum_{j=1}^{N} \frac{\partial \sigma_j}{\partial \varepsilon} (h/2 - t_j) dA_j \quad (14c)$$

$$\frac{\partial M_1(\varepsilon_x,\phi)}{\partial \phi} = \sum_{j=1}^{N} \frac{\partial \sigma_j}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial \phi} \left(h_y/2 - t_j\right) dA_j = \sum_{j=1}^{N} \frac{\partial \sigma_j}{\partial \varepsilon} \left(h/2 - t_j\right)^2 dA_j \quad (14d)$$

$$\partial V / \partial \gamma = \alpha GA \quad (14e)$$

The stiffness matrix and the nodal force vectors can be calculated as

$$K = \int_l B^T D B \, dl \quad (15a)$$

$$F = \int_l B^T \sigma \, dl \quad (15b)$$

To verify the validity of this formulation, the reinforcement between stirrups is modelled as a simply supported beam with two 3-nodded beam element(Fig. 6). Fig. 7a shows the axial stress-strain behaviour when vertical force is first applied to a particular level and then axial load is applied keeping the vertical load constant. Fig.7b shows the behaviour when axial force is applied to a particular level and vertical load is applied keeping the axial load constant. When the beam element is pulled without central vertical force, it resembled the stress-strain curve. In these calculations, increase of application of either vertical force at the centre or the axial force gradually decreases the capacity of the axial force or vertical load respectively and results looks logical.

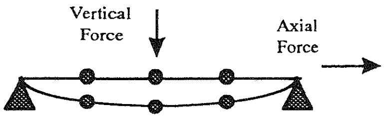
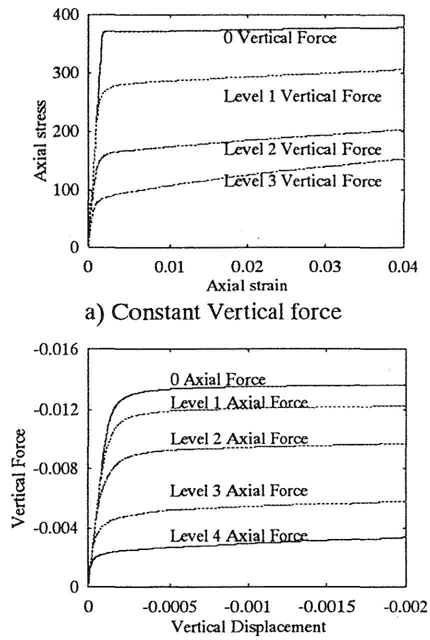


Fig.6: Beam Element Formulation



a) Constant Vertical force

b) Constant Axial force

Fig. 7. Parametric Study of the beam element behaviour using two elements

4 Analysis and Discussion

The concrete is modelled as 8 nodded cubic element and the reinforcement is modelled as two nodded rod element or as three nodded beam element. A RC beam under two point loading is adopted (Fig. 7a). Only a quarter section of the beam is taken for analysis. Fig. 7b shows the mesh adopted.

Tanabe and Gupta(1998) and Gupta et al.(1998) explained that tension stiffening effect and fracture energy concepts should be implemented for RC members if the member is expected to fail in bending or shear mode due to sufficient or insufficient reinforcement respectively. This matter is still being investigated. Hence here tension stiffening effect is implemented. The parameters for concrete are E_c=34600 MPa, μ=0.22, f_t=3.15 MPa, f_c' = 33.75 MPa, c_o = 22.58 MPa , $\phi_1 = 14^\circ$, $\phi_o = 5^\circ$, $\phi_f = 36^\circ$, m_1=4.0, m_2=0.83, η_o = 7.0 MPa, k = 1.0 x 10^{-3}, ω_f = 1.0, β' = 35, γ = 0.92, a_1= -1.0 and a_2 = -0.15(refer Tanabe and Gupta(1998) for further details).

4.1 Reinforcement modelled as rod element

Table 1: Parameters for Reinforcements

Case	f_y (MPa)	slope (x Es)	f_{y1} (MPa)	slope (x Es)
A	370.0	0.0001	-	-
B	323.4	0.00785	-	-
C	323.4	0.00785	370.0	0.0001

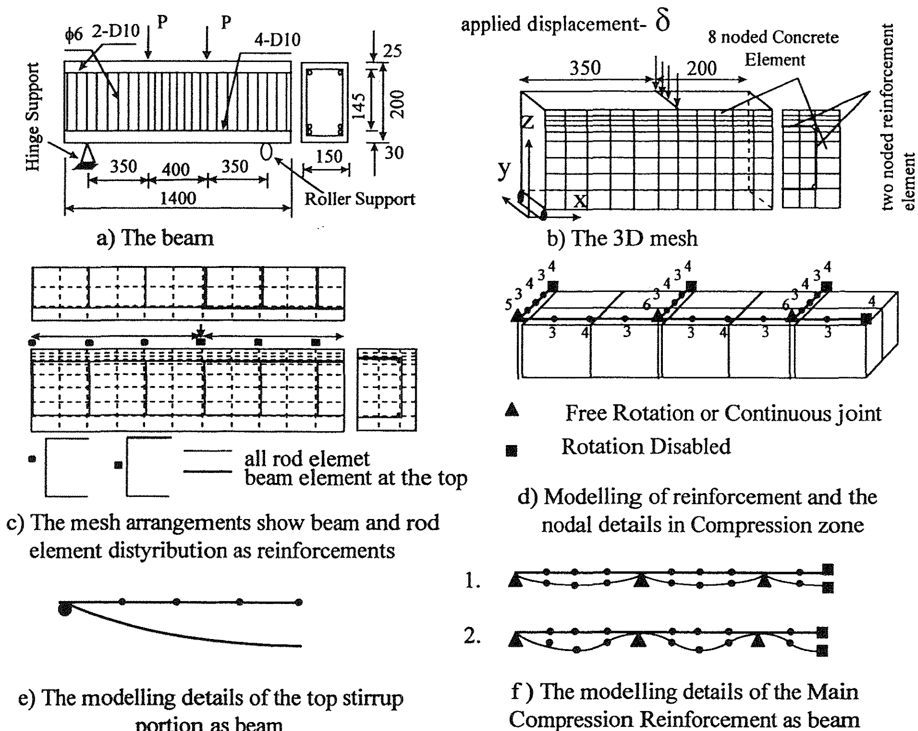
Gupta and Tanabe(1998) modelled the reinforcement with two nodded rod element. Comparison was done to find the proper implementation of tension stiffening effect for the reinforcement element in tension as shown in Fig. 8. In case A, a bilinear elastic-perfectly plastic curve with post yield slope = 0.01% of E_s was taken for stability of the analysis. In case B, a bilinear curve with approximate apparent yield stress and higher slope to simulate tension stiffening effect was taken. In case C, a trilinear curve with case B type bilinear curve in the beginning and change to case A type curve (with yield stress of f_{y1} and post yield slope of 0.01% of E_s) at the intersection point was taken. Fig. 10a shows the comparison with experimental results. It was realised that proper implementation of tension stiffening effect was very important to simulate the results at the yielding point of the reinforcement.

Though the results matched well when tension stiffening effect was implemented, final failure phenomenon could not be captured. Unrealistic confinement effect was noticed near the stirrup in compression zone(Fig.9).

4.2 Reinforcement in compression modelled as beam element

In the present analysis, the reinforcement (stirrup and main reinforcement) are modelled as two nodded rod element except for the reinforcement in compression zone as shown by the dark lines in Fig. 7c. Fig. 7d,e,f show the details of the beam element adopted as reinforcement in compression region.

In this case, the 8 nodded element need u, v and w displacements and the beam element need u or v, w and θ_x or θ_y for main reinforcement and stirrups respectively. It is assumed that the outer nodes of the beam element is connected with the nodes of the cubic element(no slip) and the central node of the beam element is free and not connected with the cubic element. In other words, force is transferred between the beam element(reinforcement) and the cubic element(concrete) only through the outer nodes of the cube. Hence, u or v and w displacements are assumed as common variables between the beam element and the cubic element. The rotational degree of freedom of the outer nodes and the degrees of freedom of the central unconnected node of the beam element are considered



a) The beam

b) The 3D mesh

c) The mesh arrangements show beam and rod element distyribution as reinforcements

d) Modelling of reinforcement and the nodal details in Compression zone

e) The modelling details of the top stirrup portion as beam

f) The modelling details of the Main Compression Reinforcement as beam (Separate and Continuous Formulation)

Fig. 7. Modelling of the RC beam under two point loading

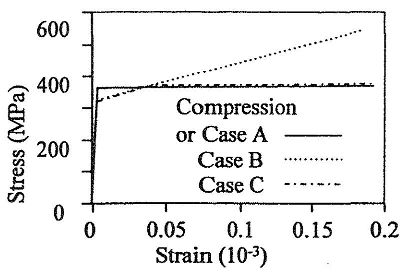


Fig. 8. Reinforcement stress-strain

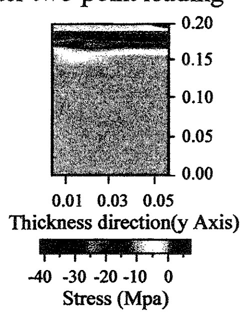
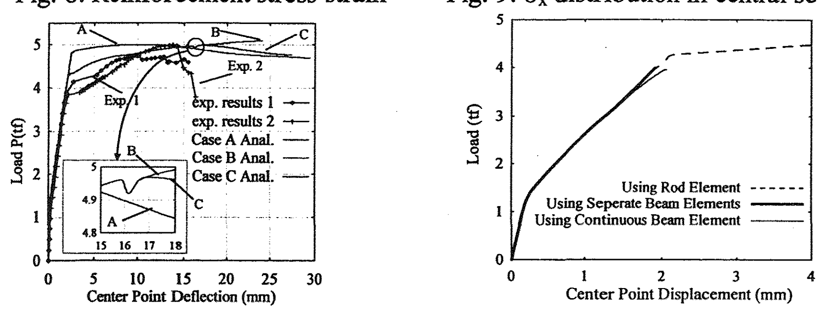


Fig. 9. σ_x distribution in central section



a) Use only rod elements

b) Use beam element in compression region

Fig. 10 Analysis with different type of elements as reinforcement independent of the degrees of freedom of the cubic element. Fig. 7d shows the number of degrees of freedom and possible boundary conditions.

Fig 7e shows the modelling for the stirrups. Fig. 7f(1) and (2) shows the formulation of the main reinforcement modelled as beam elements that are unconnected and connected with each other at stirrup location respectively. The reality should be something in between as the beams will undergo plastic deformation near stirrup location. Hence, the outer nodes of the beam element connected with the cubic element would have 3 translation(u, v and w) and 1 or 2 rotational degrees of for connected and unconnected beam element respectively. The central node of the beam element needs 3 degrees of freedom. The rotation degrees of freedom and the required degrees of freedom of translation is restrained at the symmetry section.

For the beam as reinforcement element in compression, tension stiffening effect in not implemented as it is part of compression element.

The results of the analysis are shown in Fig. 10b. Before the apparent yielding of tensile reinforcement, from where the load deflection diagram becomes more ductile and increase in load with increasing deflection is less, we can see that the load-deflection diagram matched well between the different cases. It implies, there is not much additional advantage of implementation of beam element at this stage.

However, based on the fact that results matched well in this region, it can be said that the beam element modelling is correct. After this region, convergence problems were faced and the reasons are being investigated.

5 Conclusions

Three dimensional analysis of reinforced concrete beam under two point analysis showed (Gupta and Tanabe (1998)) unusually high confinement effect near the compression stirrups. Implementation of beam element in place of the rod element is a usual solution. Most of the researchers use non-linear beam element by calculating the stiffness parameter EI from the slope of the M-ϕ relationship.

In this paper, a simplified formulation for modelling of reinforcement as beam element in compression zone is proposed where the non-linear behaviour is implemented at the gauss points. For implementation in the actual FEM analysis, a number of assumptions were made.

When beam element was implemented as reinforcement in compression region, the load displacement behaviour matched with the previous analytical results where rod element was used. However, convergence problems were noticed at the yielding stage. Hence we can say that, this formulation looks simple and promising. Further work is being done to investigate the reasons for the convergence problems.

6. Acknowledgement

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