Fracture Mechanics of Concrete Structures
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A NEW SIMPLIFIED AND EFFICIENT TECHNIQUE FOR FRACTURE BEHAVIOR ANALYSIS OF CONCRETE STRUCTURES

K. Meguro and H. Tagel-Din
International Center for Disaster Mitigation Engineering,
Institute of Industrial Science, The University of Tokyo,
Japan

Abstract
A new method for fracture analysis of reinforced concrete structures is proposed. The concrete is modeled as an assembly of distinct elements made by dividing the concrete virtually. These elements are connected by distributed springs in both normal and tangential directions. The reinforcement bars are modeled as continuos springs connecting elements together. A brief comparison between the proposed method and other numerical techniques like FEM, RBSM and EDEM is carried out to show the strong points of our proposed method.
Key words: nonlinear analysis, fracture analysis, large deformation, reinforced concrete.

1 Introduction

Failure analysis of reinforced concrete structures has been mainly carried out using the Finite Element Method (FEM). However, the FEM assumes that elements are connected by nodes and these nodes are not permitted to separate during the analysis. Moreover, separation of elements at node location results in stress singularity at the crack tip location. The FE analysis is appropriate mainly before the generation of extensive cracking.

On the other hand, many techniques were developed to deal with cracks. These techniques, such as Smeared Crack approach, Okamura et al. (1991), can not be adopted in zones where separation occurs between adjacent structural elements. While, Discrete Crack Methods, Okamura et al. (1991), assume that the location and direction of crack propagation are predefined before the analysis. To deal with these problems, many other methods were developed. The Rigid Body and Spring Method, RBSM, Kawai (1980), is one of them. The main advantage of this method is that it simulates the cracking process with relatively simple technique compared to the FEM, while the main disadvantages is that crack propagation depends mainly on the element shape, size and arrangement, Kikuchi et al. (1992) and Ueda (1993). One of the recent methods to deal with fracture analysis of concrete is the Modified or Extended Distinct Element Method (MDEM or EDEM), Meguro et al. (1989,1994). This method can follow the highly non-linear geometric changes of the structure during failure, however, the main disadvantages of this method are that in some cases accuracy is not enough for quantitative discussion and it needs relatively long CPU time compared with the FEM and RBSM.

The major advantages of the proposed method are the simplicity in modeling and accuracy of the results in short CPU time. Using the method, highly nonlinear behavior, i.e. crack initiation, crack propagation and totally collapse process of the structure can be followed with high accuracy.

2 Element Formulation

The two elements shown in Fig. 1 are assumed to be connected by pairs of normal and shear springs located at contact points which are distributed around the element edges. Each pair of springs totally represent stresses and deformations of a certain area of the studied elements. The total stiffness matrix is determined by summing up the stiffness matrices of individual spring around each element. Failure of springs is modeled by assuming zero stiffness for the spring being considered. Consequently, the developed stiffness matrix is an average stiffness matrix for the element according to the stress situation around the element. In the 2-dimensional model, three degrees of freedom are considered for each element. This leads to a relatively small stiffness matrix (size: 6x6). Stiffness matrix is developed for an arbitrary contact point with one pair of normal and shear springs as shown in Fig. 2. In this formulation, the element stiffness matrix depends on the contact point location and the stiffness of normal and shear springs.

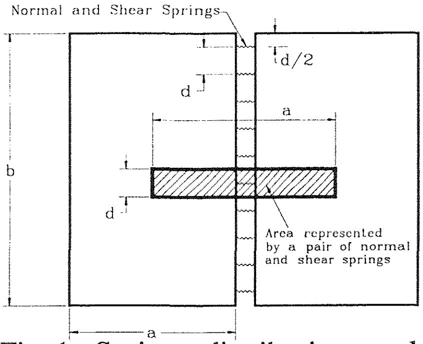


Fig. 1 Spring distribution and area of influence of each pair of springs

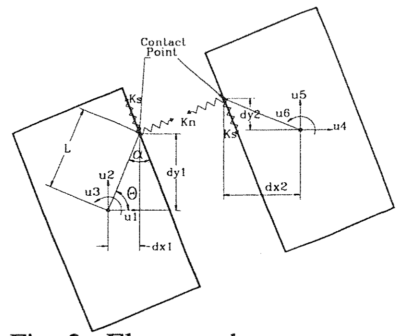


Fig. 2 Element shape, contact point and degrees of freedom

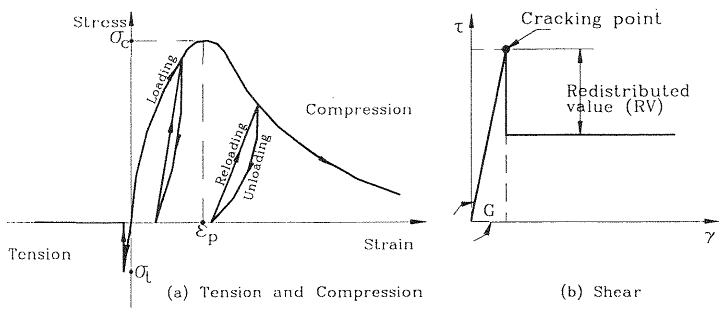


Fig. 3 Tension, compression and shear models for concrete

2 Material Modeling

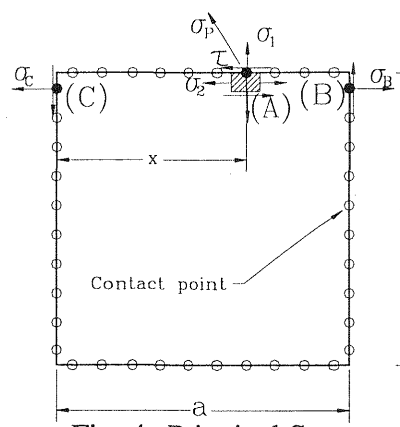


Fig. 4 Principal Stress determination

Spring stiffness is calculated for each spring according to the stress situation and material type of each spring. Material models used are shown in Fig. 3. One of the main problems associated with the use of rigid elements for representation of reinforced concrete is the modeling of diagonal cracking. Applying Mohr-Coloumb's failure criteria calculated from normal and shear springs, not based on principal stresses, is not correct. This assumption leads to increasing the resistance of a structure and inaccurate fracture behavior of the structure. To determine the principal stresses at each spring location, the following technique is used. Referring to Fig. 4, the shear and normal stress components (τ and σ_1) at the point (A) are determined from the deformation of normal and shear springs attached at the contact point.

The secondary stress (σ_2) is calculated from normal stresses in points (B) and (C). This value of principal stress, (σ_p) is compared with the tension resistance of concrete. When σ_p exceeds the critical value of tension resistance, the normal and shear spring forces are redistributed in the next increment by applying the shear and normal spring forces in the reverse direction. The redistributed forces are transferred to the element centroid as a force and a moment in the next increment. The redistribution of spring forces at the crack location is very important in following the proper crack propagation. For the normal spring, the whole force is redistributed to have zero tension stress at the crack faces as shown in Fig. 3(a). To consider the effects of friction and aggregate interlocking, a redistributed value (RV), shown in Fig. 3(b), is adopted.

3 Effects of Element Size and The Number of Springs

To illustrate the effects of element size, a series of analyses was made for the laterally loaded cantilever structure using the models as shown in Fig. 5. Elastic analyses were performed by the proposed method for the different cases. The results were compared with those obtained from elastic theory. The ratio of error in maximum displacement and the CPU time (CPU: DEC ALPHA 300 MHz) are shown in the figure. To study the effects of the number of connecting springs, two different analyses were performed using 20 and 10 springs connecting each pair of adjacent element faces. From the figure, it is evident that increasing of the number of base elements leads to decreasing of the error but increasing of the CPU time. The error becomes less than 1% when the number of elements at the base increases to 5 or more. Although the CPU time in case of 10 springs is almost half of that in case of 20 spring, its results congruent with those of 20 springs.

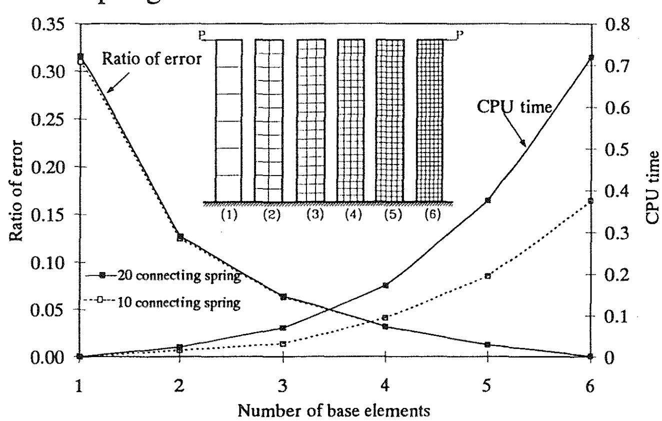


Fig. 5 Relation between the number of base elements, ratio of error and CPU time

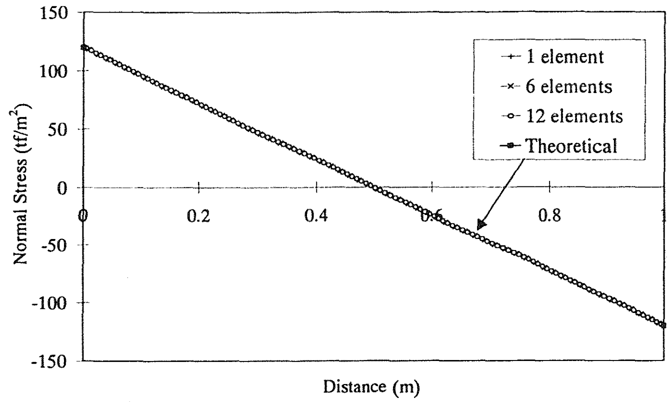


Fig. 6 Normal stress distribution at the column base

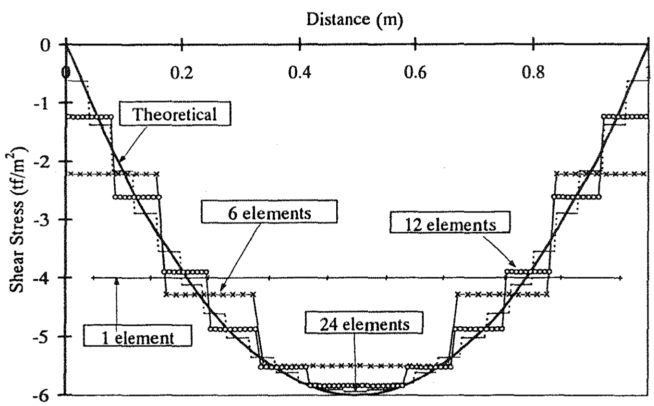


Fig. 7 Shear stress distribution at the column base

Figs. 6 and 7 show the normal and shear stress distribution at the base of the studied columns for different number of base elements. From these figures, the followings can be noticed:

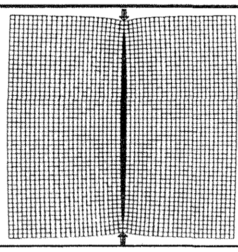
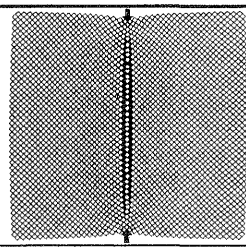
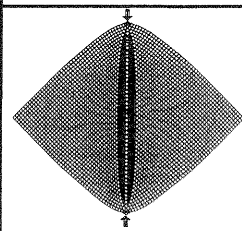
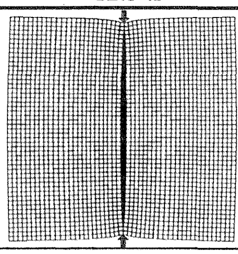
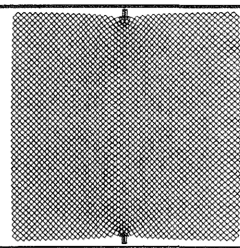
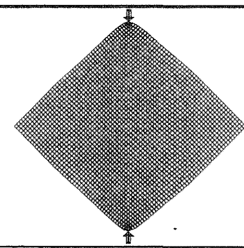
1 Calculated normal stresses are very near to the theoretical values even in case of smaller number of elements at the base.
2 Shear stress values are constant for the same element.
3 Shear stress values are far from the theoretical values in case of smaller number of elements and the error decreases when the number of elements increases.

This means that the elements of relatively large size can be used to simulate the behavior for cases where the effects of shear stresses are minor, like case of slender frame structures. On the other hand, in case of walls and deep beams, elements of small size should be used to follow the fracture behavior in the shear dominant zone.

4 Effect of Element Arrangement

To check the accuracy of our model in comparison with other numerical techniques using rigid elements, such as RBSM and DEM, Brazilian test simulation was performed on concrete cubes subjected to concentrated loads. Three different mesh configurations were used. The distance between loading points is 20 cm in all cases and 10 springs were set between each two adjacent faces. The results and configuration of the problem are summarized in Table 1. Theoretical failure load is 12.5 tf in cases (1) and (2). In this simulation, compression failure under the applied load is not permitted.

Table 1. Brazilian test results of concrete cubes

Fracture Criterion	Case (1)	Case (2)	Case (3)
Proposed Method (Principal Stress)			
Failure Load	12.5 tf	12.5 tf	11.7 tf
RBSM or DEM type (Normal and Shear Stresses)			
Failure Load	12.5tf	Not failed	Not failed

From the results, it can be noticed easily that the obtained failure load in our simulation did not change for different mesh arrangement, while failure load could not be calculated using material models used by RBSM or DEM for 45° discretization mesh of case (2). This means that results obtained by RBSM or DEM depend mainly on the element discretization, Ueda et al. (1993). This is mainly due to the use of Mohr-Coloumb's failure criterion based on two components of stresses (not based on principal stresses), the spring stiffness which is not determined in a proper way to simulate the element deformation, Kikuchi et al. (1992), the use of relatively large sized elements, and the use of relatively small number of springs between edges which leads to an inaccurate failure mechanism.

5 Simulation of Two-Storied RC Wall

To verify the accuracy of the model, the simulation results are compared with the experimental results of a two-storied RC wall. The size and shape of the wall, reinforcement and loading location are shown in Fig. 8. For more details about the columns, beams and wall reinforcement, or the material properties, refer to Hajime et al. (1976). The wall is modeled using 1,845 square elements. The number of springs between each two adjacent faces is 10. Reinforcement locations are defined by their nearest spring coordinates.

Figure 8 shows a comparison between measured and calculated load-rotation relations. First, to discuss the effects of load increment in failure process, three models of different load increments, calculated by dividing the estimated failure load by 50, 250 and 500, with the constant number (10) of springs were used. Next, to study the effects of the number of connecting springs between faces, additional simulations were carried out using the case of 250 load increments with 5 and 2 springs between faces and the results were compared with those obtained with 10 springs. The failure loads calculated in all cases were within the range of 64 to 70 tf while the measured one was 67 tf. The calculated failure load using the FEM was 64 tf, Okamura et al. (1991). In general, the calculated failure loads are very close to the measured ones. The results of 50, 250 and 500 increments are almost congruent up to at least 95% of failure load. As the CPU time is proportional to the number of increments, the CPU time in the case of 500 increments is 10 times higher than that of 50 increments. To avoid long CPU time, relatively large value of load increment can be used till about 90% of expected failure load. From Fig. 8, the agreement between experimental and numerical results is fairly good for 250 increments with 10 or 5 connecting springs.

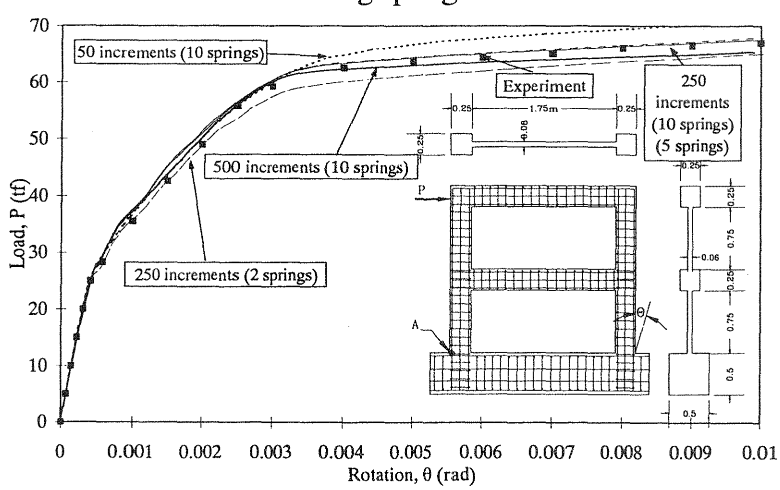


Fig. 8 Relation between load and wall rotation for 2-storied RC wall

Surprisingly, for the case of 250 increments with only 2 springs connecting each two adjacent faces, the results are also reliable till reaching failure of the structure. It is also noted that using large sized load increments leads to the results in slightly higher failure load (70 tf) while using a few number of connecting springs gives slightly lower one (64 tf). This means that our model gives reliable results even when using a few number of connecting springs or relatively large sized load increments.

Although increasing the number of springs leads to increasing the calculation time required for assembling the global stiffness matrix, the time required for the solution of equations, which is dominant when the number of elements is large, does not change because the number of degrees of freedom is independent of the number of springs used. This means that we can use larger number of springs between edges without significant change of the CPU time. On the other hand, as the total number of connecting springs used is generally large, it is necessary to use the computer with large memory capacity.

Figure 9 shows the relation between load and the number of failed springs for each increment. Cumulative curves also show the total number of failed springs till that increment. It can be noted that although the number of increments in both cases is different, both of the cumulative curves are close to each other. This gives good indication that the solution is generally stable. Excessive cracking begins to appear when the applied load is about 28 tf. At the same load, behavior of the structure begins to be highly nonlinear.

Figure 10 shows the deformed shape during the application of load in case of 500 load increments with 10 springs. The location of cracks and crack propagation can be easily observed and they are very similar to those obtained from the experiment. This means that the proposed model can be applied for fracture behavior of RC structures, such as, failure load, deformations, crack generation, crack location and crack propagation, etc.

It should be emphasized that although the shape of elements used in the simulations are squares, it does not affect the crack generation or crack propagation in the material. Diagonal cracks, as shown in Fig. 10, coincide well with those obtained from the experiment. In the simulation using rigid elements, like RBSM (Kawai, 1980), shapes and distributions of elements were decided based on the assumption that cracks were generated and propagated in previously expected locations and directions.

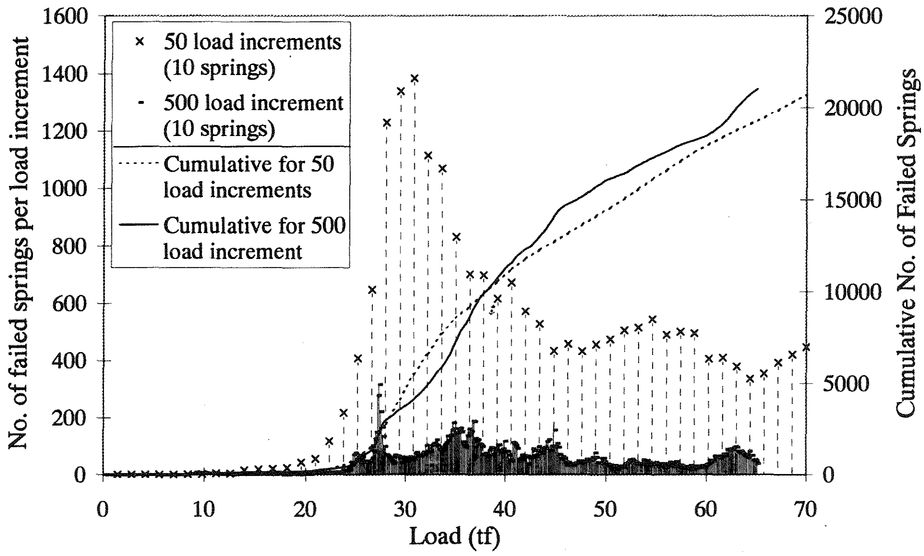


Fig. 9 Relation between load and the number of failed springs

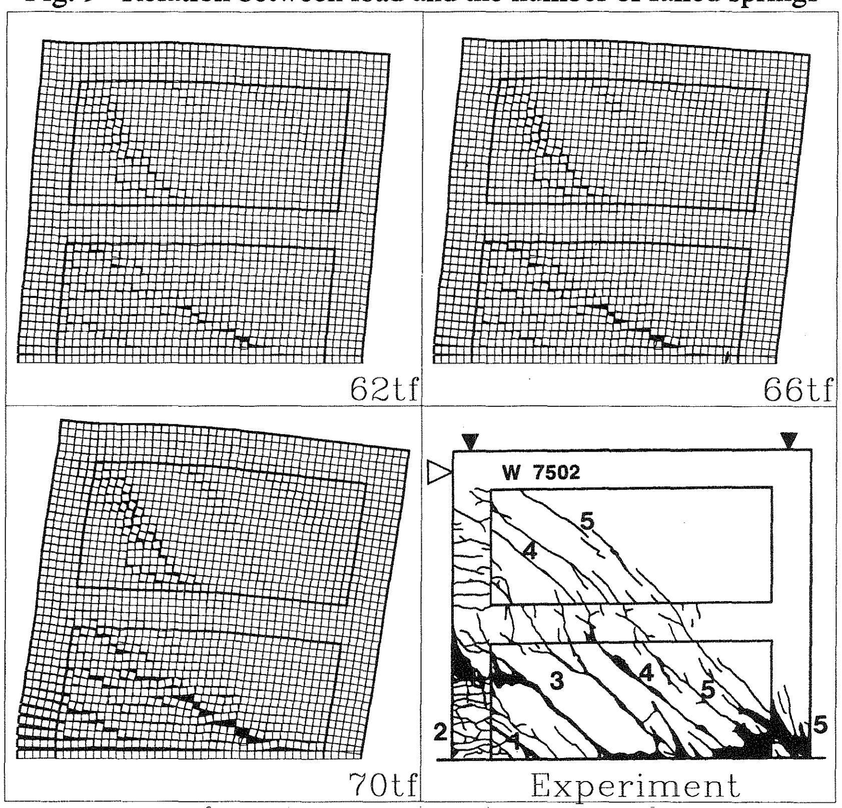


Fig. 10 Deformed shape and crack locations of the 2-storied RC wall structure (in case of 500 increments with 10 springs between each two adjacent faces, Simulation Scale Factor=30)

6 Conclusion

A new technique to simulate nonlinear and fracture behavior of structures was proposed. Through the comparison of obtained results with experimental and numerical results by other methods, it was shown that this technique could follow elastic behavior, crack initiation and propagation, and mechanical behavior till collapse with reliable accuracy in reasonable time. By the use of this technique, cracking behavior can be followed without previous knowledge about the location of cracks and crack propagation direction. Because elements can be separated, this technique can be extended easily to follow the post failure behavior of structure and till total collapse.

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