

Fracture Mechanics of Concrete Structures  
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**SELF INDUCED STRESSES IN HIGH PERFORMANCE  
CONCRETE AT EARLY AGES: EXPERIMENTAL RESULTS AND  
MATERIALS MODELLING**

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**Abstract**

This paper presents results from a four-year research programme. The activities range from basic materials research to analysis and instrumentation of real concrete structures. The analytical part of the project, based on further development and application of the program system DIANA, is briefly described. Experimental results on mechanical properties for concrete subjected to various temperature histories, are compared to material models. Results from a "shrinkage rig", able to measure both thermal dilatation and autogenous shrinkage, are reported. Furthermore, results from the "temperature-stress-testing-machine" are presented, and compared to calculation based on the materials models implemented in the DIANA program. Analysis of a typical structural example is included.

Key words: High strength concrete, temperature, shrinkage, hardening, analysis.

## 1 Introduction

High performance concrete is more sensitive to early age cracking than normal strength concrete, both in the initial phase (before and during setting) and in the thermo phase (during the heat generation and the subsequent cooling). Several mechanisms are active: (1) plastic shrinkage may develop very early due to minimal bleeding, (2) chemical shrinkage due to hydration reactions results in autogenous shrinkage, and (3) high temperature rise due to high cement contents results in tensile stress development during subsequent cooling. The inability of temperature criteria alone to predict this type of cracking is known, and in practice, they are now gradually being replaced by stress-strain based criteria.

## 2 Project overview

The research project, «NOR-IPACS» (IPACS= Improved Production of Advanced Concrete Structures), started in 1995, and is supported by the Norwegian Research Council, a contractor and three material suppliers. Researchers at NTNU and SINTEF perform the major part of the work, but the industry partners are also active, both in laboratory work and field tests on selected structures. The work is co-ordinated with a European project, «IPACS», which started in 1997. The activities range from basic material research and development of material models to field investigations on real concrete structures. The main objective is to develop a more precise methodology for planning and control of the production of concrete structures to prevent early age cracking. The need for such a project is illustrated by Fig. 1, which shows stress development calculated with five different computer programmes by some of the participants in IPACS. The relatively large disagreement between the programmes is discouraging, especially because all participants used the same experimental background data on concrete properties. The deviations can probably be explained by differences between the material models and solution methods, simplifications of the structural system etc.

The NOR-IPACS project considers both the initial phase and the thermo phase, but in this paper only the latter is considered. In the laboratory, stress generation under full restraint is measured from about six hours after casting in a «temperature-stress-testing-machine» (TSTM), and in a separate «shrinkage rig» the free movement due to both thermal dilatation and autogenous shrinkage is measured. By means of a temperature control system, the temperature may be varied from isothermal to realistic

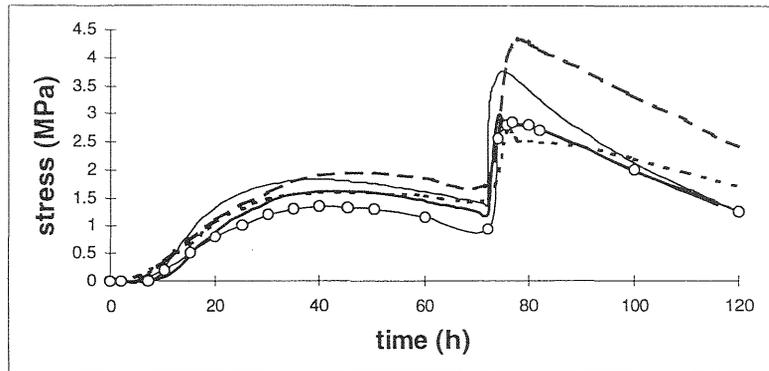


Fig. 1. Stresses calculated by five different computer programs

temperature histories.

The results from the shrinkage rig are used to identify material parameters for thermal dilation and autogenous shrinkage, and together with test results for the modulus of elasticity, creep and tensile strength, the results are being used to formulate and verify material models. The influence of the temperature history on the development of modulus of elasticity, creep strains, autogenous shrinkage, thermal dilation and tensile strength is an important task. A prototype of a tensile creep rig with a temperature control system is now completed, and as experimental data becomes available, special attention will be paid to creep models and the validity of the theory of linear viscoelasticity for aging materials. When creep tests are carried out, all model parameters can be identified without utilizing the TSTM-results. Consequently, the TSTM can serve as facilitator to the calculation methods, and hence serve as control of or be used to investigate the validity of solution methods for the typical stress-histories of young concrete.

Application and further development of the material models in the program-system DIANA is a main activity in the project. In addition, a method based on the age-adjusted effective modulus of elasticity is included. The purpose is to extend the general applicability of stress/strain based hardening technology.

SELMER ASA and NTNU are now carrying out a comprehensive field test on a culvert-system in Oslo. The instrumentation results in this structure demonstrate that the main crack problem is caused by restraint from previously cast structural elements. In the first part of the test, several types of strain gauges are being investigated. So far, the agreement between the different methods seems to be fairly good, and agreement within  $\pm 20 \times 10^{-6}$  can be achieved.

### 3 Early age concrete modelling in DIANA

#### 3.1 General

Early age analysis of concrete structures includes temperature- and stress analysis. In both cases two different principles can be used to describe the evolution of hydration heat and mechanical properties: the degree of hydration or equivalent age (in hardening technology frequently denoted maturity). From a practical point of view, the latter approach is most convenient, mainly because the determination of model parameters is simpler. In addition the degree of hydration is rarely measured in connection with experimental work.

The standard version of DIANA applies the first principle, with the degree of hydration (or degree of reaction) defined as the relative amount of hydration heat. The new developments carried out within this project apply the equivalent age principle.

#### 3.2 Heat of hydration

In the standard version of DIANA, the evolution of the heat of hydration is described by the following equations:

$$Q(t) = \int_0^t q \cdot dt \quad ; \quad q = \gamma \cdot q(r) \cdot q(T) \quad ; \quad q(T) = \exp\left(-\frac{b}{T + 273}\right) \quad (1)$$

in which  $q$  is the momentary heat production,  $\gamma$  a material parameter, and  $q(r)$  the normalized heat production function depending on the degree of reaction,  $r$ . The equation for  $q(T)$ , in which  $b$  is the Arrhenius' constant, describes effect of the temperature.

A simpler and more commonly used model is the model proposed by Freiesleben and Pedersen in 1977, based on equivalent age,  $t_{eq}$ :

$$Q(t_{eq}) = Q_{\infty} \cdot \exp\left[-\left(\frac{\tau}{t_{eq}}\right)^{\alpha}\right] \quad (2)$$

where  $Q_{\infty}$  is the total heat amount of hydration heat, and  $\tau$  and  $\alpha$  are material parameters. This latter equation is implemented in DIANA. The degree of hydration may then be replaced by the equivalent age, and consequently, all material properties can be dependent on equivalent age instead of degree of hydration.

### 3.3 Stress calculation

The stress calculation of early age concrete with DIANA is based on linear viscoelasticity with aging effects described as:

$$\varepsilon(t) = \int_0^t J(t, \tau) \mathbf{C} \cdot \dot{\sigma}(\tau) \cdot d\tau \quad , \quad (3)$$

where  $\mathbf{C}$  is the compliance matrix and  $J(t, \tau)$  is the creep function which for instance can be modelled by the double power law:

$$J(t, \tau) = \frac{1}{E(\tau)} (1 + q \cdot \tau^{-d} (t - \tau)^p) \quad , \quad (4)$$

where  $\tau$  is age at loading, and  $q$ ,  $p$ ,  $d$  are the material parameters.

The modulus of elasticity of hardening concrete can be modelled as a function of the degree of reaction or as a function of equivalent age. In this project a modified version of an equation from the CEB-FIP Model Code is used:

$$E(t_{eq}) = E_{28} \left( \exp \left[ s \left( 1 - \sqrt{28} / \sqrt{t_{eq} - t_0} \right) \right] \right)^n \quad (5)$$

In which  $E_{28}$  is Young's modulus after 28 days,  $s$  and  $n$  material parameters, and  $t_0$  the equivalent age corresponding to the time when E-modulus starts developing.

The general form of input for autogenous shrinkage is the discrete function of time or degree of hydration. After implementation of the equivalent time concept in DIANA it is possible to specify shrinkage as a discrete function of equivalent age, but there is presently little experimental evidence that such a description has general value.

## 4 Mechanical properties

The E-modulus and the tensile strength are important parameters for prediction of cracking risk. In addition both the compressive cylinder- and cube-strength are also included in the test program. Realizing that material models must be valid for general temperature histories, tests have been carried out at different concrete ages for two types of temperature histories: isothermal and realistic.

So far the E-modulus development in both compression and tension has been determined for nine different concrete mixes. Fig. 2 shows the results for one of the concretes, and the differences between results from the compressive and tensile tests seem to be insignificant. The main preliminary conclusion is that equation (5), based on the equivalent age principle, seems to be a reasonable simplification of reality.

Tensile strength tests have been carried out as uniaxial tests on 100x100x600 mm prisms and splitting tensile tests on 100-mm cubes and 100x200 and 150x300 mm cylinders. The results are presently being evaluated, and so far, general relations between the different types of tests have not been established. Fig. 3 shows the tensile strength development for the same concrete as in Fig. 2. The tensile strength development is described by the same equation as the modulus of elasticity.

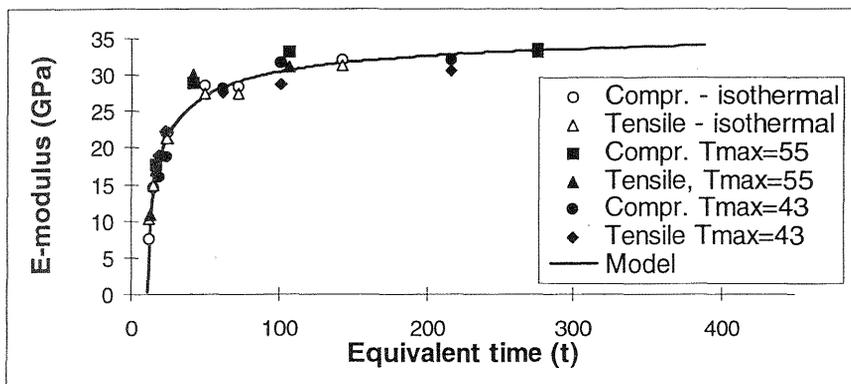


Fig. 2. E-modulus versus equivalent age

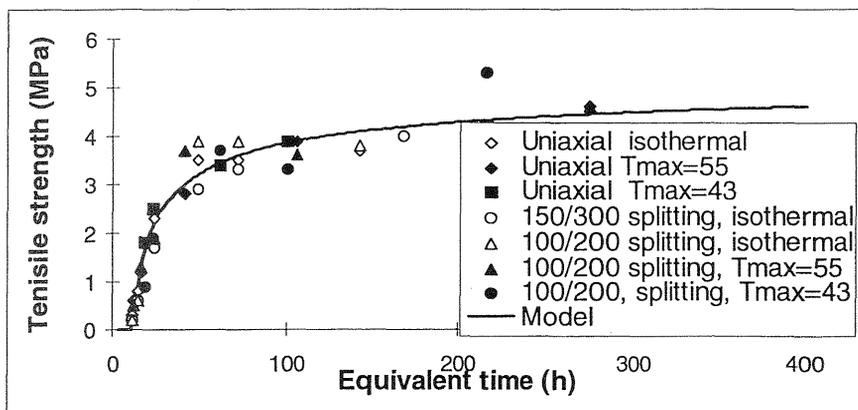


Fig. 3. Tensile strength versus equivalent age

Fig. 4 shows the relative time dependence for the material properties, according to the material models.

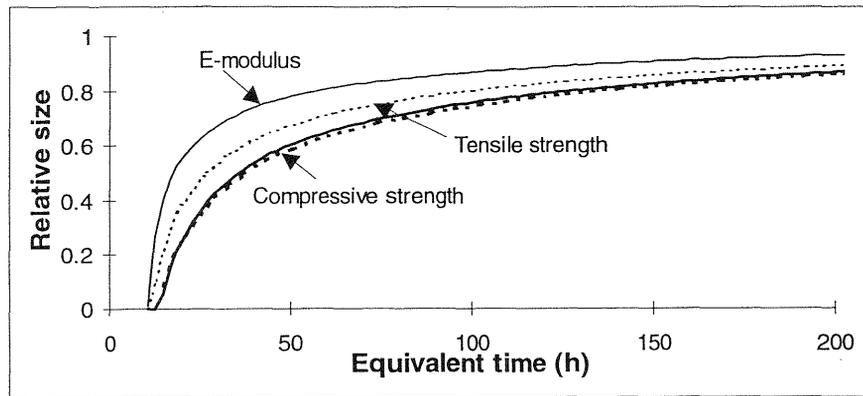


Fig. 4. Relative development of different model parameters

## 5 Results from the Temperature-Stress-Testing-Machine

Figure 5 shows total stress independent strain from the «shrinkage rig». The temperature history was realistic, i.e. the measured strains are the sum of thermal strain and autogenous shrinkage. Results so far indicate very complex relationships between the strain types and temperature effects. Presently, therefore, a comprehensive study is being carried out to develop general material models for both types of strain, Bjøntegård et al. (1998). In the subsequent calculations, the level of ambition is lowered, and it is assumed that the thermal dilatation coefficient is constant and equal to  $8.5 \times 10^{-6}$ . The autogenous shrinkage is then calculated to be the difference between the total measured strain and the assumed thermal strain.

Figure 6 shows comparisons between experimental and calculated results for two concretes. The creep parameters are determined from literature data since no creep data has yet been generated in the project.

Because the results from the shrinkage rig are directly used as input to the calculations, the results are only a verification of the calculation method regarding development of the E-modulus, the creep properties and the validity of linear viscoelasticity for aging materials.

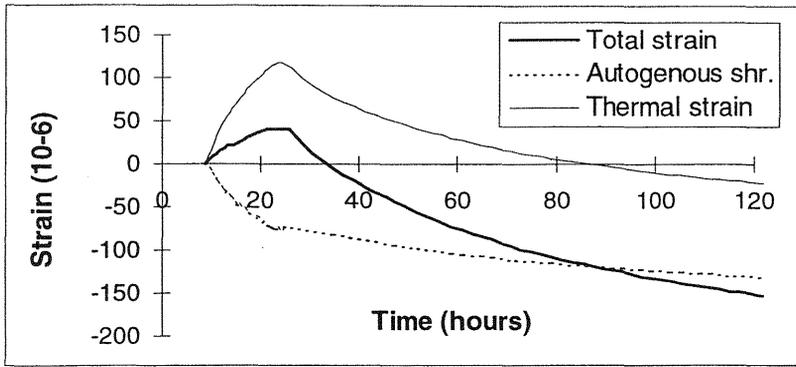
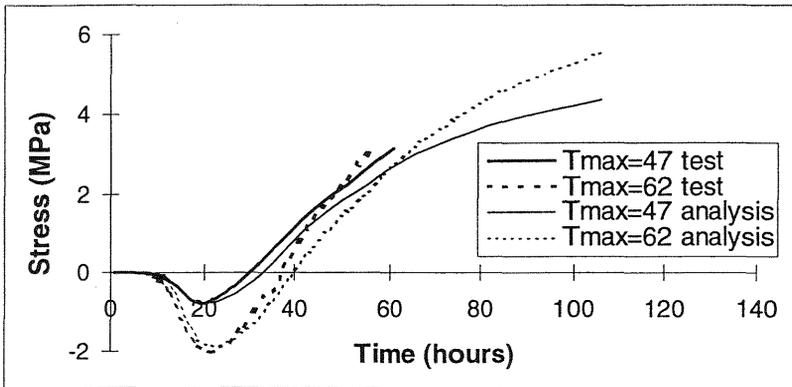


Fig. 5 Stress independent strain determined from the shrinkage rig, (concrete 1) under realistic temperature development

a)



b)

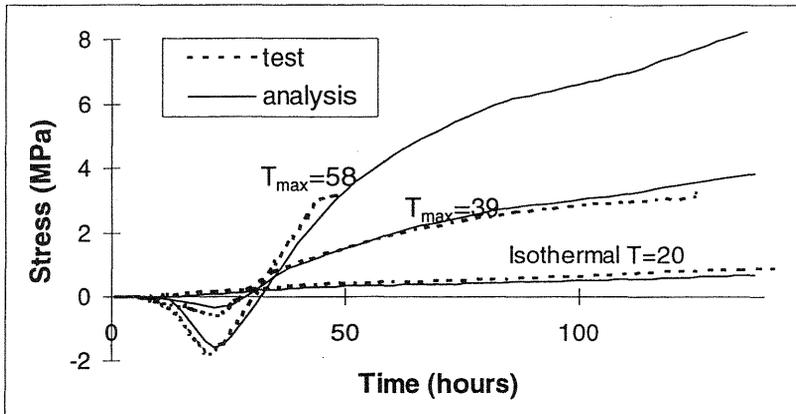


Fig. 6 Experimental and calculated results from the temperature-stress machine, (a) concrete 1, (b) concrete 2

## 6 Analyses of structures

New models for heat of hydration and E-modulus are evaluated by calculating temperature and stress development in a wall cast on previously cast base slab, Bosnjak et al. (1997). Temperature was measured in the wall, and calculation results based on equivalent age and degree of hydration are compared with measurements. (fig. 8. and 9.)

Since the wall is very long with uniform cross sections and boundary conditions in the longitudinal direction, the heat flow in that direction can be neglected. Consequently, the temperature and the stresses are almost constant from section to section, except for the end zones. That makes it possible to model only one slice of the wall assuming that plain cross sections remain plane and that curvature is prevented.

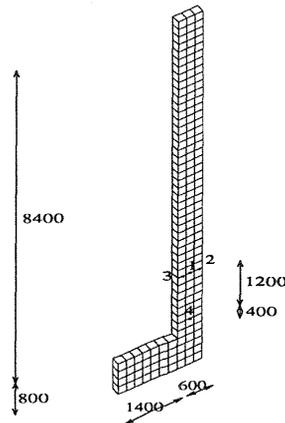


Fig. 7 Finite element model

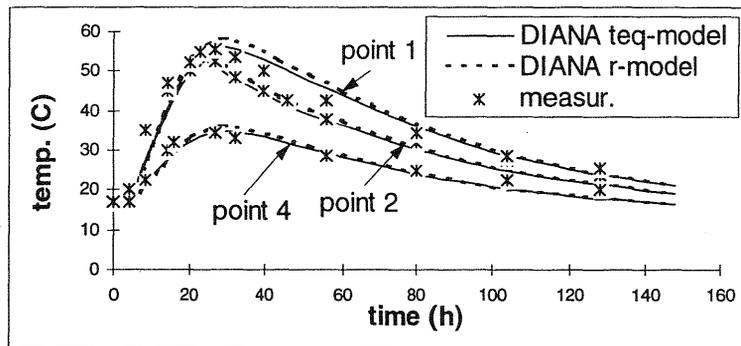


Fig. 8 Temperature development in the wall, calculations results based on equiv. age and degree of hydration compared to measurements

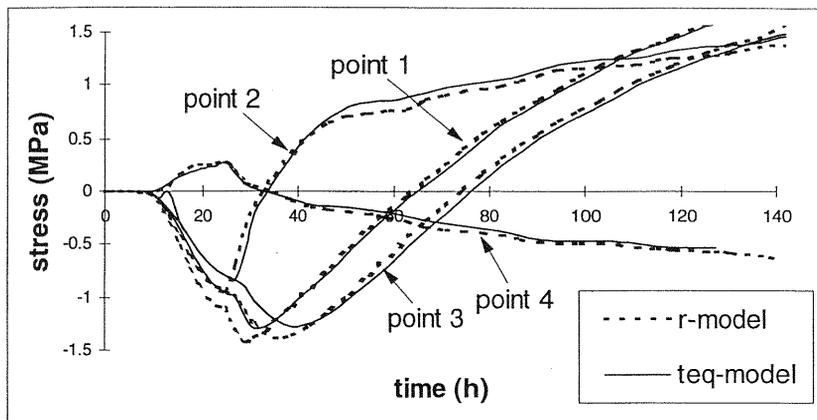


Fig. 9 Stress development in the wall, calculations results based on equivalent age and degree of hydration

## 7 Conclusions

- The theoretical framework described in this paper, based on simple equations and linear viscoelasticity for aging materials, gives results in reasonable agreement with experimental behavior.
- It is not possible to propose any refinements of the theory before more experimental data on creep, autogenous shrinkage and thermal dilation are available

## 8 Acknowledgement

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