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ADAPTIVE FINITE ELEMENT ANALYSIS OF CONCRETE FRACTURE BASED ON COHESIVE MODEL

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Abstract

Fracture analysis based on cohesive models is usually carried out by finite element method. For rigorous analysis of progressive cracking in concrete, discretization errors in the FE analysis are to be minimised. It leads to better understanding of the fracture models and the associated softening laws. To achieve this objective a computational framework for adaptive finite element analysis of concrete fracture based on discrete crack model is proposed. The procedure involves at various stages, fracture analysis, error estimation, and mesh refinement. Node patch based superconvergent error-estimation technique proposed by Zienkiewicz and Zhu was adopted for the elastic-softening problem. Hillerborg's fictitious crack model has been chosen to model cohesive fracture in the present study. The non-linear problem of crack propagation is solved by incremental-iterative procedure. Mesh refinement is automatically done at stages of crack propagation. The procedure is demonstrated on a plain concrete specimen under Mode I fracture condition. Mesh refinement is necessary at crack tip when no special crack tip elements are used. Results indicate the need for adaptive analysis procedure when local behaviour of crack is of interest.

Key words: Concrete, fracture, cohesive crack model, adaptive finite element analysis, local behaviour.

1 Introduction

Failure of plain concrete members takes place suddenly and catastrophically due to proliferation of flaws or microcracks. Such a problem can be modelled by means of a fracture criterion added to the classical field equations of continuum mechanics. Intensive research in the area of concrete fracture mechanics led to the development of non-linear fracture models that are able to describe progressive fracture of the material. The cohesive crack models simulate the fracture process zone(FPZ) by a closing pressure that reduces the stress singularity at the crack tip. The essence of this model is the description of non-linearity by means of a relationship between cohesive stresses and crack openings. These models require an analytical framework for fracture evaluation.

The finite element method has generally been found to be efficient and reliable and hitherto by far the most common method used to model concrete fracture. Although FEM is the most widely used tool for the simulation of fracture, the accuracy of the solution may always be questioned. From a global point of view, this inaccuracy may be attributed to the modelling drawbacks of FEM since it is practically impossible to characterise the infinite number of degrees of freedom of a real physical system by a discrete numerical model. This modelling deficiency usually results in a lower bound of the solution which is manifested by *stiffening* in structural mechanics problems. This overstiffness of the system produces discretization errors. Error in the analytical framework for fracture evaluation, is bound to reflect on the performance of the fracture model. This aspect is to be given attention when studying the local behaviour of the cracks using the fracture models in finite element analysis. The work reported here has been performed with the following objectives:

1. to arrive at a framework for adaptive FE analysis of fracture in concrete
2. to study the local behaviour of crack using a cohesive model for fracture, and
3. to study the influence of softening approximations in finite element analysis of concrete fracture.

We start in section two with a brief review on general properties of the cohesive cracks and their merits and demerits in capturing the fracture phenomenon. The third section describes with the aid of a flow diagram, the procedure for the adaptive finite element analysis, estimation of errors, and mesh refinement. Essentials of the fracture simulation code as was developed by the authors are briefly explained. The details of the constitutive relations that govern the FPZ are mentioned in section four. Details of the specimen studied, results of FE simulations are presented in section five. A summary of the essential conclusions closes the paper.

2 Cohesive models for concrete fracture

Cohesive crack model for concrete fracture is based on the Dudgale-Barrenblatt energy dissipation mechanism for metals. Hillerborg et al (1976), Bazant and Oh (1983) proposed cohesive models for progressive fracture in concrete. A cohesive model assumes that the energy required to create new surfaces is small compared to that required to separate them. It is assumed that the material away from the partially fractured zone behaves elastically. The energy release rate (G) during Mode I crack propagation can be expressed as

$$G = \int_0^{w_c} \sigma \, dw \qquad (1)$$

in which σ is the normal traction acting on the crack surface, and w is the crack opening displacement (COD) and w_c is the width of traction free crack. The advantage of any cohesive model is that its formulation and the FE modelling are simple and accurate results can be obtained in a straightforward manner.

2.1 Hillerborg's fictitious crack model (FCM)

The success of FCM in describing the progressive fracture in concrete is well documented. In this model the softening stress - separation curve is assumed to be a material property that is independent of structural geometry and size. The flexural tensile strength of concrete is also assumed to be size independent. The entire FPZ is considered to be in the wake of the fictitious crack tip. Based on experimental and numerical investigations, Elices et al (1994) demonstrated the ability of FCM in predicting reasonably the maximum loading capacity of concrete specimens in the practical range of sizes.

Parametric studies were done by many investigators and limitations of the model in predicting the fracture behaviour were also discussed. Nallathambi and Karihaloo (1986) incorporated the non-linear response of the material ahead of crack tip due to microcracking and reported better prediction of the global behaviour of beams. Considering the tortuous nature of the crack, a reduction in the value of *fracture energy*, G_f was suggested. Sundara Raja Iyengar et al (1996) observed that the existing softening relationships are able to predict global behaviour very well only in the pre-peak region. Miller et al (1991) reported disparities in comparing the predicted and observed local strain behaviour of a centre-cracked plate under tension.

2.2 Significance of present study

Fracture behaviour of concrete is a complex phenomenon. The theoretical models are based on certain assumptions, and as such may only catch some of the actual physical mechanisms. Modelling of FPZ mainly depends on the rate of consumption of the fracture energy G_f. Any inaccuracy in the modelling results in erroneous distribution of cohesive forces. As the distribution of cohesive forces is highly localised in nature, error in its modelling will affect the local behaviour. If local behaviour of crack is of interest, there is a need to reduce discretization errors of FE computations. The focus of the present work is on the local behaviour of the cracks using adaptive finite element analysis(AFEA). The predictions are compared with test results published by Du et al (1990). The experimental procedure reported therein involved the use of laser moiré interferometry to determine the location of micro-crack tip and to measure the COD within the FPZ under Mode I condition.

3 Adaptive finite element fracture analysis

Adaptive Finite Element techniques seek to construct reference solutions, define error norms and, in general, create a more accurate and reliable numerical solution by using a feedback strategy incorporating the reference solutions and error norms. A computational framework for adaptive analysis of progressive cracking using discrete crack model is developed. Mesh refinement strategies for linear elastic analysis, are used for the elastic-softening problem. Refined meshes are obtained as the crack advances with node patch based superconvergent error-estimator. The preceding philosophy has some implications for the generally accepted notion of convergence to the correct solution as the mesh is refined. In the present context the approach is meaningful as the objective is to study the performance of fracture model with refined FE meshes. The basic components in this procedure are shown in fig. 1.

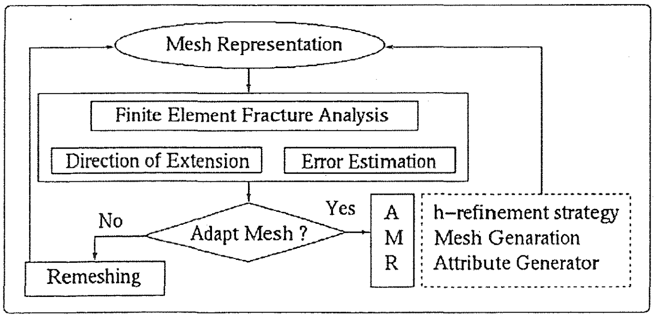


Fig. 1. Adaptive finite element fracture analysis

3.1 Simulation of Fracture

The principle of virtual work for the elastic-softening problem, with crack surface S_c can be written as,

$$\int_V d\varepsilon^T \sigma \, dV = \int_V du^T X \, dV + \int_{S-S_c} du^T p \, dS + \int_{S_c} du^T p_c \, dS \qquad (2)$$

The problem results non-linear as the tractions on the crack surface p_c, depend on the unknown COD. An iterative procedure based on verification of equilibrium condition and congruence condition has been formulated to solve the indeterminate problem. The equilibrium condition for the crack in a state of mobile equilibrium is governed by the crack tip stress intensity factor. In the present study as singular elements are not used at the fictitious crack tip, the convergence is verified by means of the congruence condition that compares the calculated reaction at the crack tip to the closing force in the crack line. The non-linear problem is solved by incremental-iterative procedure, based on Newton-Raphson scheme. The crack propagates along the inter element boundaries. The analysis determines a load factor associated with the incremental length of the crack. In the development of software for fracture simulation interactive graphics facilities are provided for visualisation of evolving geometry.

3.2 Refinement strategy

The h-version refinement strategies transfer the element error information into new element size information at the centroid of the old element. The errors in stresses are computed as a difference between the FE stresses and a reference stress field. The elementwise error in energy is computed from these errors in stresses. Zienkiewicz and Zhu (1992a, 1992b) proposed a best guessed stress type superconvergent estimate based on patch wise stress recovery. In this method, a patch of elements is selected around a node. The unknown stress field is assumed as a polynomial whose coefficients are evaluated from the minimization of a discrete least square functional which is fitted to the superconvergent stress values of the elements in the patch. The constraint conditions to equilibrate the elements within a patch is ensured by enforcing the internal element equilibrium and the Von Neumann boundary conditions for tractions.

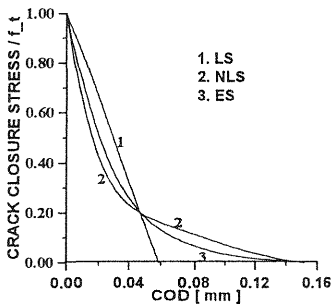
3.3 Adaptive mesh refinement (AMR)

The physical subdivision of mesh is to be done using adaptive mesh refinement procedure. Meshing by successive Superelement Decomposition (MSD), is an automatic unstructured quadrilateral mesh generator. MSD is essentially a two-step process; in the first, a skeletal curve (medial axis) based automatic shape interrogation process decomposes the problem domain into mappable, mutually non intersecting, quadrilateral and triangular superelements. Next, a splitting transfinite curve procedure is used to create a boundary based quadrilateral mesh generation method inside these superelements to create a fully unstructured grid. A detailed description of this process may be found in Krishnamoorthy et al (1995).

The attribute generator automatically imposes engineering analysis attributes (e.g.. loads, boundary condition information, material data) to the refined mesh. It also generates current process zone into new mesh.

4 Stress - crack width relation

The constitutive relation governing the FPZ is a non-linear crack closure stress versus COD relation, which is characterized by a strain softening behaviour as a result of aggregate bridging across the path. Many softening laws were proposed based on accurate experimental investigations. Solutions will appreciably differ from one another, depending on the rate of consumption of the fracture energy G_f. The softening approximations considered in the present study are shown in fig. 2. They are, 1. Linear softening (LS), Hillerborg et al (1976) 2. A non-linear softening (NLS), Reinhardt et al (1986) with $c_1 = 3.0$ and $c_2 = 6.93$ and 3. Exponential softening (ES). Once the softening function is decided, the crack propagation analysis may be performed in a conceptually straightforward manner using FEM.



1. $\sigma = f_t\left(1 - {}^{w}\!/_{w_c}\right)$

2.

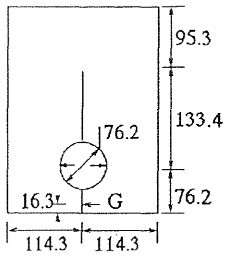
$$\sigma = f_t\left[\left\{1+\left(c_1\frac{w}{w_c}\right)^3\right\} e^{\left(-c_2\frac{w}{w_c}\right)} - \frac{w}{w_c}\left(1+c_1^3\right) e^{-c_2}\right]$$

3. $\sigma = f_t\, e^{-kw}$

Fig. 2 Crack closure stress Vs COD relationships

5 Case study

A double cantilever beam (DCB) specimen, was analyzed under Mode I fracture condition. The dimensions, loading etc. for the specimen have been described in Du et al (1990), and are shown in fig. 3. The member is analyzed up to failure. Analysis was also done with a fixed FE mesh



Thickness	50 mm.
Modulus of elasticity	29.6 GPa
Poisson's ratio	0.15
Compressive strength	35.4 MPa
Tensile strength	3.40 Mpa
Fracture Energy	100 N/m.

Length unit : mm.

Fig. 3. Details of concrete fracture specimen

(fig. 4) for comparison purpose. This mesh is refined twice (fig. 5) and taken as the starting mesh for adaptive analysis. The mesh is adapted for every 12 mm increment in the process zone length. Minimum size of element is restricted to 0.5 mm. The mesh configuration at various stages of progressive fracture is shown in fig. 6. Number of nodes and the global error computed based on energy norm are indicated there in.

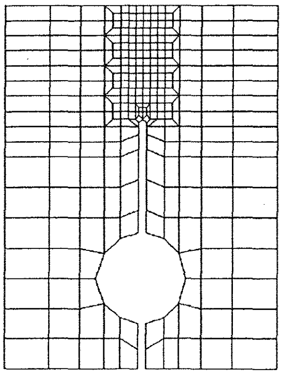


Fig. 4. Fixed mesh, 325 nodes (82%)

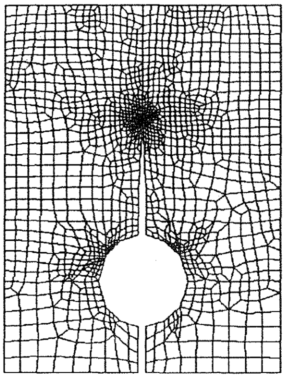
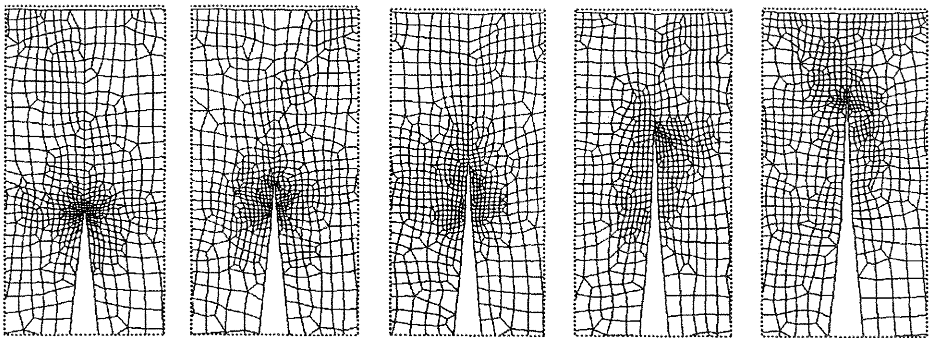
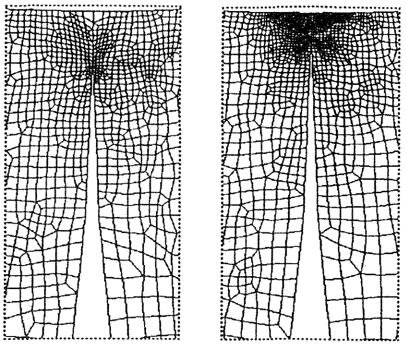


Fig. 5. Adapted mesh, 1651 nodes (48%)



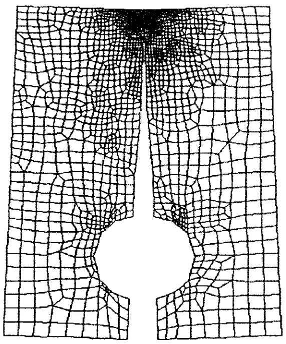
a) 1737 (43%) b) 1777 (37%) c) 1752 (37%) d) 1751 (36%) e) 1783 (36%)

Fig. 6. Progressive fracture and adaptive mesh refinement



f) 1908 (36%) g) 2285 (33%)

Fig. 6. Progressive fracture (contd.)

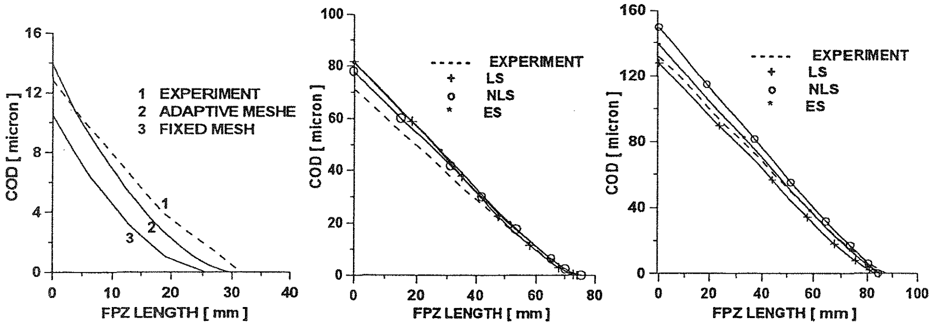


2293 (31%)

Fig. 7. Failure of specimen

The error can be further reduced by decreasing the minimum element size and with more number of refinements during progressive fracture.

Fig. 8 shows a comparison of predictions of process zone development with the test observations. A load of 1.695 kN was reached only with LS. The deviation in the local behaviour is quite significant for LS. Predictions with AFEA are more close to the observed deformations in the loading regime. The process zone details at a reduced load of 1.5 kN are given in fig. 9 for the three softening laws. During unloading (fig. 8 b and 8 c) analysis with fixed mesh also predicted close behaviour.



a) Load 1.695 kN b) Load 0.845 kN c) Load 0.396 kN

Fig. 8. COD in FPZ compared with finite element predictions

Fig. 10 shows a comparison of the global response of specimen with AFEA predictions. Analysis with fixed mesh with a reasonable number of nodes on the crack path predicted similar behaviour. But AFEA does not require apriori a good mesh and evolves it based on the posteriori error estimates. For better understanding, the energy expended in FPZ during loading phase of specimen is plotted in fig. 11. The cohesive model is capable of representing global force-equilibrium relation (fig. 12) with

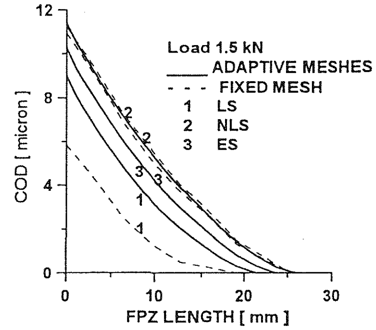


Fig. 9 Modelling of FPZ in Fixed & adaptive meshes

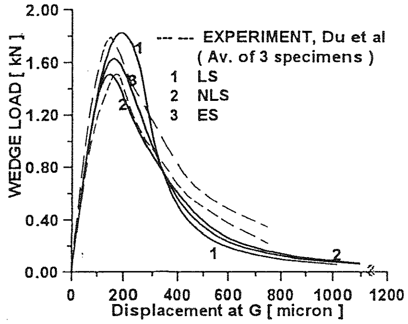


Fig. 10 Global response of specimen (AFEA)

reasonable FE meshes. But the variations in the energy expended in FPZ in pre-peak load range, will influence the member behaviour in the vicinity of the crack. Analysis with exponential approximation for material softening predicted better results (fig. 11).

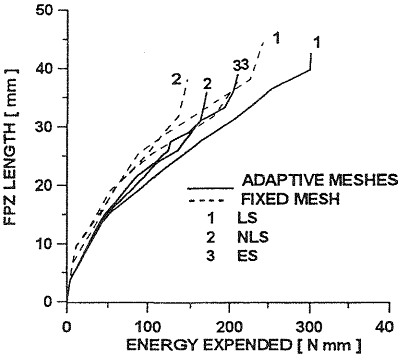


Fig. 11 Energy expended in FPZ

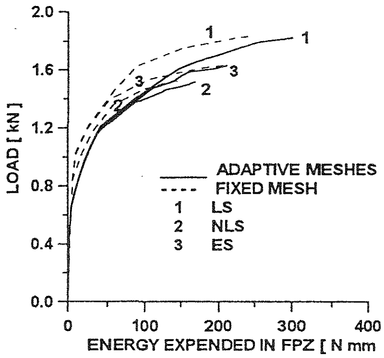


Fig. 12 Response upto peak load

6 Discussion and conclusions

Comparison of process zone development indicates the necessity of AFEA procedures when the local behavior of crack is of interest in the fracture analysis.

Results of FE simulations demonstrate the capability of the proposed computational frame work for AFEA to model fracture behaviour of concrete while improving the accuracy of FE computations.

Since the analysis procedure is adaptive and automated, it provides better FE framework for study and validation of fracture models. Work is in progress for extension of the procedure to non-planar crack conditions.

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