Fracture Mechanics of Concrete Structures
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FRACTURE BEHAVIOR OF REINFORCED HIGH STRENGTH CONCRETE TENSILE MEMBERS

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Abstract
A study was conducted to analyse the fracture behavior of reinforced concrete tensile members. The main objective was to improve the current understanding of the response of cracked high strength concrete elements based on fracture mechanics considerations. The average contribution of concrete in reinforced elements was experimentally analysed from reinforced panels subjected to uniaxial tension. The predictivness of a previously proposed fracture energy approach based on non-linear fracture mechanics was examined. A new method is derived to approximate the effective compliance for normal and high strength concrete members with multiple cracks. In addition, a simplified *R*-curve method is proposed to approximate the fracture behavior of cementitious materials with a pronounced brittle fracture response.
Key words: Cracking, fracture energy, high strength concrete, *R*-curve, reinforced concrete panels, uniaxial tensile strength

1 Introduction

Tensile failure in plain concrete with a single crack is approximately a brittle process. For the prediction of such brittle failure processes and to model certain non-linear failure aspects in plain concrete (e.g. influence of specimen geometry and size), several fracture mechanics based models have been previously proposed.

Fracture in reinforced concrete tensile members, on the other hand, is a more ductile process due to the development of multiple cracks and because steel reinforcement is transferring imposed loads predominantly across the cracks. A review of the literature reveals that models, proposed by the American Concrete Institute (ACI "Cracking" 1992) and CEB/FIP (CEB/FIP 1991), predict the average stress-strain behavior of reinforced concrete tensile members in an empirical manner. Such design concepts neglect fracture processes and failure mechanisms in the concrete matrix of reinforced concrete structures. In recent years, however, tests have demonstrated that a reliabale failure analysis of reinforced concrete membrane elements subjected to multi-axial stress states must include the tensile behavior of plain concrete (Vecchio and Collins (1986), Hsu (1993)).

According to these findings, some fracture mechanics concepts have been proposed for a rational description of cracking in reinforced concrete (e.g. Bazant and Oh (1983), Ouyang and Shah (1994)). So far, the predictivness of those models has been almost solely verified from test results on normal strength concrete specimen. To propose reliable fracture mechanic models, however, systematic studies are required to recognize all important parameters that govern the cracking response and to quantify their influences. Therefore, an extensive experimental and analytical investigation was conducted to study the cracking behavior of both normal and high strength reinforced concrete tensile members.

This paper provides a summary of the major findings of this study and presents a simplified R-curve method to predict fracture for cementitious materials with a pronounced brittle fracture response.

2 Experimental investigation

Reinforced panels were subjected to uniaxial tension to record the composite average stress - strain behavior. Uniaxial compression, uniaxial tension and three-point bend tests were conducted on cylindrical and prismatic plain concrete samples to, respectively, determine the corresponding mechanical properties and fracture parameters (e.g. f_c, f_t, K_{Ic}^s, $CTOD_c$ and E_c) of the concrete matrix. The overall objective was to evaluate the influence of reinforcing bar spacing, reinforcement ratio and strength on the cracking behavior of reinforced panels from a fracture mechanics point of view. The experimental work was presented in detail by Wollrab et al. (1996).

2.1 Test program

Twenty-three reinforced normal (NSC) and high strength concrete (HSC) panels with a cross-sectional area of 127 x 50.8 mm were loaded in uniaxial tension. The panels were tested in three series. The first and second test series were conducted to study the effect of reinforcing bar distribution and concrete strength on the average stress-strain behavior of concrete tensile members. In these two series, the reinforcement ratio, ρ, was approximately constant at 3%. The third test series investigated the influence of ρ on the cracking behavior in high strength concrete panels. In this series, reinforcement ratios of 2.2 and 0.8 percent were tested. The total length of the double-edge notched panels was 686 mm. Experimental data were continously recorded from the load cell, four LVDT's and two extensometers. Average stress-strain curves were determined for all panels based on the total load and the average displacement of the individual readings of the four LVDT's. The two extensometers were used to record the crack mouth opening displacement (CMOD) across the notches.

2.2 Experimental results

Average composite stress-strain curves and the matrix stress at first cracking were obtained for all panels based on the rule of mixtures. Average matrix stress-strain curves were obtained by subtracting the reinforcing bar contribution from the composite response. Interesting insight on the cracking behavior was obtained from the measurements of the CMOD at the notches. The following conclusions can be summarized:

- The matrix strength at first cracking for panels with a reinforcement ratio of 3% was found to decrease with increased spacing between the rebars. The authors believe that suppression of microcrack localization by the rebars can explain this phenomenon. The closer the rebar spacing, the better the suppresive action.
- The matrix strength at first cracking for high strength concrete panels with reinforcement ratios of 0.8%, 2.2% or 3% does not change.
- The post cracking behavior of concrete does not depend on the reinforcing bar spacing. For HSC panels, the average contribution of concrete increases with decreasing reinforcement ratios.
- Immediately after the peak load, the average concrete contribution is greater in HSC members. However, as average strains increase the concrete contribution declines considerably faster in HSC members.
- The slip between concrete and steel at the time of formation of the first crack was compared for normal and high strength concrete. It was found that slippage is higher in HSC. This may be explained by the fact that the cracking process in HSC is significantly more brittle. Thus, the separation of the crack faces is more sudden and larger.

Based on the experimental results a previously proposed fracture energy method was modified and a simplified R-curve method is proposed.

3 Theoretical investigation

3.1 Part I: Fracture energy model

Ouyang and Shah (1994) proposed a fracture energy concept using non-linear fracture mechanics. The model can predict the cracking behavior of reinforced concrete tensile members by balancing all dissipated energies (e.g. strain, sliding and debonding energies) during cracking. The concept is based on energy criteria that have been already successfully used to describe crack propagation in plain concrete plates of unit thickness with an initial crack length a_0. Accordingly, the energy equilibrium for a reinforced plate during cracking can be written as:

$$-\frac{1}{b \cdot t} \cdot \frac{\partial \varphi_c}{\partial N} = R_{Icf} + \frac{1}{b \cdot t} \cdot \frac{\partial(\varphi_d + \varphi_s)}{\partial N} \tag{1}$$

where b and t = specimen width and thickness, respectively; φ_c = strain energy of concrete containing N cracks; φ_d and φ_s = total debonding and sliding energies on all debonded interfaces associated with N cracks; R_{Icf} = energy consumed in crack propagation. The compliance of the concrete matrix with multiple cracks was determined with a self consistent method that has been successfully used to predict the effective elastic modulii of solids containing a large number of cracks (Kemeny and Cook (1986)).

The existing model was modified to better predict the tensile response of high strength concrete members. A new approach was proposed to evaluate the effective compliance of reinforced concrete members with multiple cracks. This was due to the fact that in reinforced concrete members, differently to solids, the number of cracks are not sufficiently large enough during the cracking phase. Therefore, the self-consistent method can only be used in an approximate manner. However, by using the new method for the evaluation of the effective compliance of cracked concrete, the model is suitable to predict the response of normal and high strength reinforced concrete members. The computational algorithm of the modified approach has been reported by Ouyang et al. (1997).

3.2 Part II: A simplified *R*-curve approach

In order to use the fracture energy model described in Part I, the fracture energy consumed in plain concrete during crack propagation, R_{Icf}, is needed to be determined (see Eq. 1). So far, various R-curve approaches have been applied to a wide range of materials to predict the onset of unstable crack propagation. Especially for quasi-brittle materials, such as cementitious materials with relatively large fracture process zones, R-curves are recognized as a useful method to simulate the fracture response. An R-curve method has been proposed by Ouyang and Shah (1991) to predict the fracture response of concretes. Since many experimental observations have indicated that high strength concrete is more brittle than normal strength concrete, the R-curve developed by Ouyang and Shah (1991) was further simplified and used for HSC in this paper.

3.2.1 Fracture response predicted with R-curves

As shown in Fig. 1, the R-curve proposed by Ouyang and Shah (1991) is a rising function of the crack extension. Points along the R-curve describe failure of specimens of different widths, but the same initial crack length. For a specimen of infinite size, a curve reach its maximum value of $R_{Ic_c}{}^{\infty} = (K_{Ic}{}^{s})^2/E_c$ at a critical crack length of $a_c{}^{\infty}$.

R-curves computed by using the method proposed by Ouyang and Shah for normal and high strength concrete, as shown in Fig. 1, illustrate that $R_{Ic}{}^{\infty}$ is greater for high strength concrete than for normal strength concrete. Accordingly, the crack extension in high strength concrete is smaller than in normal strength concrete, since the former exhibits a more brittle fracture behavior than the latter. As a result, the R-curve behavior of high strength concrete may be approximated with two straight lines connecting points AB and BC (see Fig. 2). Based on this observation, a simplified R-curve method can be proposed for concretes with a pronounced brittle fracture response.

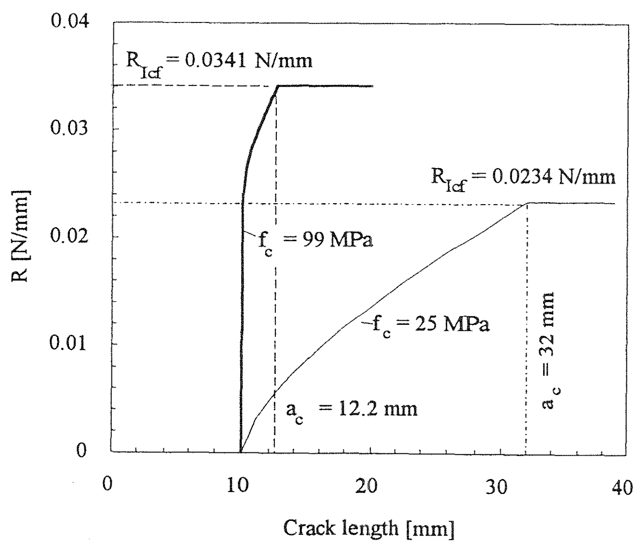

Fig. 1. Characteristic R-curves for normal and high strength concrete

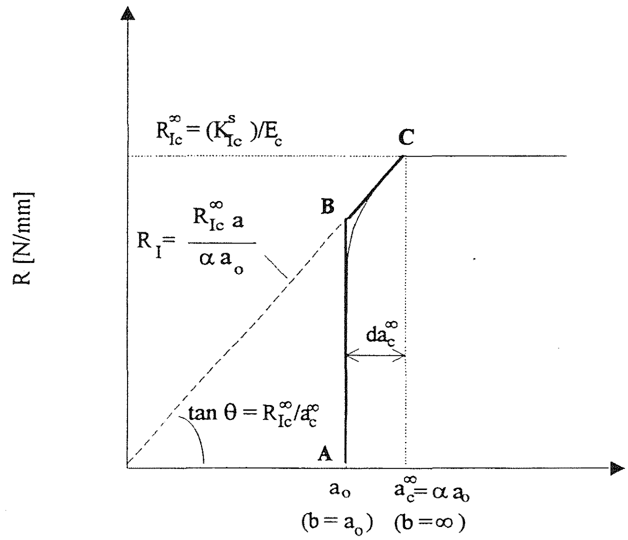

Fig. 2. A simplified *R*-curve approach for brittle cement-based matrices

3.2.2 Computational algorithm

The bi-linear *R*-curve in Fig. 2 illustrates, that an infinitely large specimen fractures at point C when $R_I = R_{Ic}^{\infty}$ and $a = a_c^{\infty} = a_0 + da_c^{\infty} = \alpha\, a_0$, where da_c^{∞} is the critical crack extension for a specimen of infinite size. Since the slope of line BC equals $R_{Ic}^{\infty}/(\alpha\, a_0)$ for $a_0 < a \leq a_c^{\infty}$, the fracture resistance of concrete with a given crack extension can be determined as:

$$R_{Ic} = \frac{R_{Ic}^{\infty}}{\alpha \cdot a_0} \cdot a \tag{2}$$

where $R_{Ic}^{\infty} = (K_{Ic}^{s})^2 / E_{c_c}$ for the plane stress condition and $\alpha = a_c^{\infty}/a_0$. The brittleness index α measures the crack extension and was introduced by Ouyang and Shah (1991). For a perfectly brittle material α equals 1 and increasing values indicate a more ductile material behavior. Using the material parameters K_{Ic} and $CTOD_c$, α is defined as:

$$\alpha = \frac{a_c^{\infty}}{a_0} = \frac{\pi E_c^2 f_1^2 CTOD_c^2}{32 a_0 K_{Ic}^2 f_2^2} + \sqrt{\left(\frac{\pi E_c^2 f_1^2 CTOD_c^2}{32 a_0 K_{Ic}^2 f_2^2}\right)^2 + 1} \tag{3}$$

where a_c^{∞} is the critical crack length for infinitely large structures and f_1 and f_2 are geometrical factors (Ouyang and Shah (1991)).

The critical crack extension is related to the specimen width since each R-curve actually represents the loci of critical G values for different specimen sizes. Therefore, point B in Fig. 2 corresponds to a specimen width of $b = a_0$ with a critical crack extension of 0, while point C corresponds to an infinitely large specimen with a critical crack extension of da_c^{∞}. Between these two points, a_c varies such that when b increases from a_0 to infinity, the corresponding critical crack length changes from 0 to $\alpha\, a_0$. For high strength concretes the variation of a_c is relatively small (Fig. 1). Therefore, a_c for finite sized specimens ($a_0 < b < \infty$) may be approximated with the following logarithmic interpolation function:

$$\frac{a_c}{a_0} = V_1 + V_2 \cdot \ln\left(\frac{b}{a_0}\right) \tag{4}$$

where the two constants, V_1 and V_2, can be determined from the two boundary conditions, $a_c = a_0$ when $b \to a_0$ and $a_c = a_c^{\infty} = \alpha\, a_0$ when $b \to \infty$. Since the specimen size cannot actually reach infinity, the second condition may be approximated by $a_c = \alpha\, a_0$ at $b = \Lambda\, a_0$, where Λ is a value large enough to obtain a reasonable prediction for a_c. The boundary conditions determine V_1 and V_2 such that Eq. 4 can be rewritten as:

$$\frac{a_c}{a_0} = 1 + \frac{\alpha - 1}{\ln(\Lambda)} \cdot \ln\left(\frac{b}{a_0}\right) \tag{5}$$

The relationship between the applied tensile stress and the fracture resistance of concrete is given as:

$$G_I = R_I = \frac{\sigma^2 \cdot \pi \cdot a \cdot g_1^2\left(\frac{a}{b}\right)}{E_c} \tag{6}$$

Substituting Eq. 2 and $a = a_c$ into Eq. 6 leads to an expression for the maximum tensile stress of concrete for a given specimen size as:

$$\max \sigma = f_t^R = \frac{1}{g_1\left(\frac{a_c}{b}\right)} \sqrt{\frac{E_c \cdot R_{Ic}}{\pi \cdot \alpha \cdot a_0}} \tag{7}$$

where $g_1(a_c/b)$ is a geometry function.

3.2.3 Evaluation of the simplified R-curve method

The applicability of the simplified R-curve method is evaluated by comparing the fracture parameter a_c, R_{Ic}, and f_t^R for specimens of different widths with predicted values by original R-curve method proposed by Ouyang and Shah (1991). A normal and high strength concrete is used for the comparison. For the computations $A = 20$ was selected. The values for the three fracture parameters predicted with the simplified and the original R-curve method are shown in Figs. 3 - 5.

The predicted values of the critical crack length for different specimen widths in Fig. 3, exhibit for normal strength concrete a rather large discrepancy. However, good agreement is obtained for the predicted values of high strength concrete. The same observations can be made for the parameter R_{Ic} (see Fig. 4). The results for the maximum tensile strength are shown in Fig. 5. Here, the curves show good agreement with the predicted tensile strengths for normal and high strength concretes.

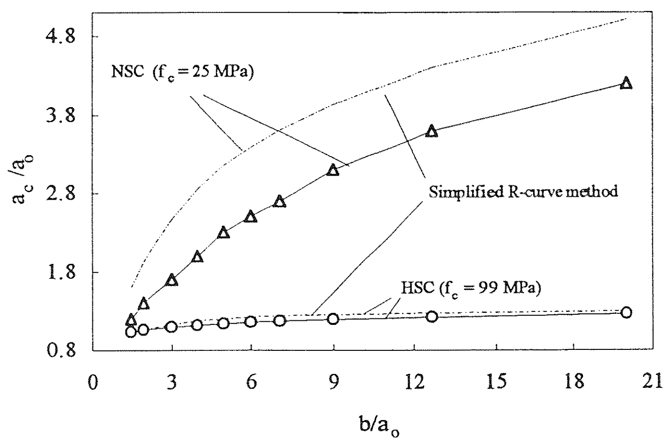

Fig. 3. a_c for specimens of different width and strength

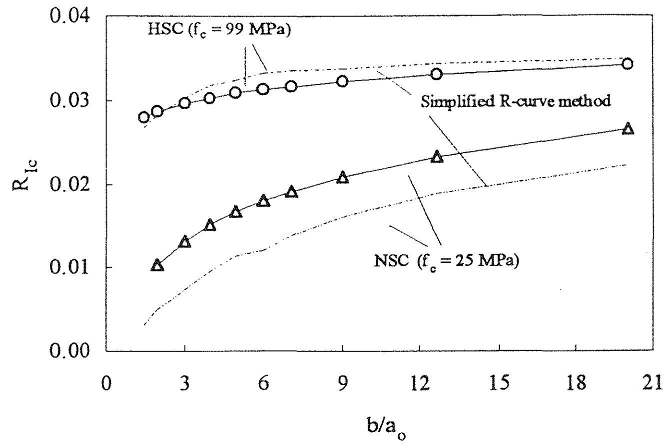

Fig. 4. R_{Ic} for specimens of different width and strength

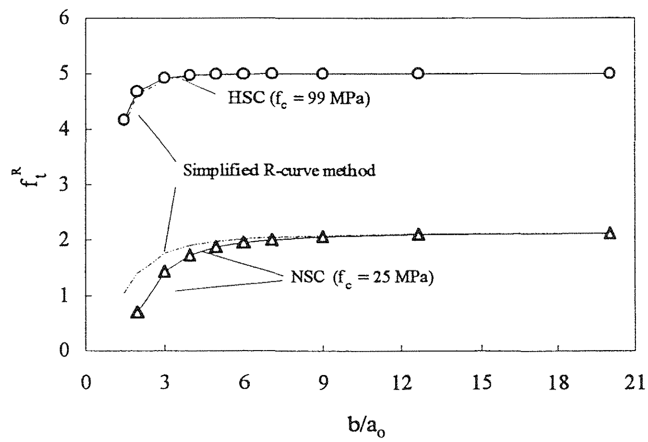

Fig. 5. f_t^R for specimens of different width and strength

The above comparison demonstrated that the simplified *R*-curve is suitable to predict the fracture behavior of high strength concrete specimens of different widths. However, the simplified method should only be used for concretes with a brittleness index of $\alpha \leq 1.5$. For concretes with $\alpha > 1.5$, the original *R*-curve should be used to determine a_c and R_{Ic}. For the determination of f_t^R, however, Eq. 7 can be used for both, normal and high strength concrete.

4 Concluding remarks

Interesting results were obtained from an experimental and theoretical study carried out to investigate the response of cracked normal and high strength reinforced concrete members. Experimentally, it was found that the cracking response in reinforced high strength concrete members is more brittle than in normal strength concrete members. A previously proposed fracture energy model was examined and modified to increase the predictivness of the cracking behavior especially for reinforced high strength concrete members. A new method was proposed to determine the effective modulus of elasticity of a plain concrete member with multiple cracks. A simplified *R*-curve method was proposed to approximate the fracture behavior of high strength concretes.

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