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BEHAVIOUR OF LIGHTLY REINFORCED CONCRETE BEAMS BY MEANS OF FRACTURE MECHANICS AND BOND-SLIP

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Abstract
A numerical model is proposed to study the behaviour of RC beams during the transition from the pre-cracked stage to the post-cracked one. During this transition, the softening branch of load-defection diagram, which is remarkable in lightly reinforced beams, is modeled by means of bond-slip and fracture mechanics and compared with experimental results.
Key words: bond-slip, cohesive crack, fracture mechanics, lightly-RC beams

1 Introduction

In the frame of beam theory, deformability is usually defined by means of the moment-curvature relationship M-$1/r$. Due to cracking phenomenon, in reinforced concrete (RC) beams it is difficult to define univocally and precisely a cross-sectional moment curvature relationship. In fact, for the same bending moment M it is possible to obtain different curvatures $1/r$ for cracked or uncracked cross sections. For this reason, for a representative portion of the beam, an average curvature comprised within stage I and stage II curvatures (Fig. 1) is usually considered.

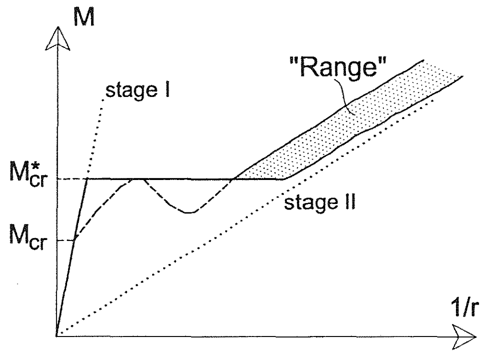


Fig. 1 Moment-curvature (M-$1/r$) relationships

Therefore, the behaviour of a beam could be modeled by means of particular moment-curvature relationships able to smear or average out all the different cross sectional curvatures of the considered portion ("cross section models"). This approach is adopted, for example, in CEB (1993), where the curvature $1/r$ is obtained for a given M, by a suitable reduction ("tension stiffening") of stage II curvature. However, the definition of an average curvature referred to a long beam portion with many cracks, is representative only for beams subjected to a virtually constant bending moment (Ferretti 1996). Conversely, some problems raise for beams characterized by peaks or high gradients of the bending moment diagram (Giuriani and Rosati 1984a). In these situations "local" type M-$1/r$ relationships, obtained by using representative portions delimited by two consecutive cracks, work better. Local type M-$1/r$ relationships are usually numerically obtained by means of the so-called "block models", which remove the hypothesis of perfect bond between steel and concrete and allow a slip between the two materials.

However, due to the randomness of the crack pattern, the distance between two consecutive cracks is uncertain and a univocal definition of the "local" M-$1/r$ relationship is still impossible. For this problem, in Avalle et al. (1994) a block model is proposed, able to encompass, in a region called "range", all the possible M-$1/r$ curves (Fig. 1). Both in cross sectional and block models, the first cracked stage, which corresponds to the transition from the uncracked (stage I) to the stabilized crack pattern, claims for particular attention. In this situation, a simplified change of deformability as the linear joint between the pre-cracked stage and the post cracked one proposed in Fig. 1, could be oversimplified for a reliable structural analysis (Giuriani and Rosati 1984a). In fact, when the first crack forms, both the tests carried on controlling the displacement (Bosco et al. 1990) or the crack mouth opening w (Giuriani and Rosati 1984b; Planas et al. 1995) show a remarkable softening behaviour (e.g. dashed line in Fig. 1).

As pointed out by Giuriani and Rosati (1984b), the study of the softening branch, which is particularly important in lightly-reinforced concrete beams, requires the knowledge of the local behaviour near the crack rather than classical sectional theories of RC beams. In effect, in the early cracked stage, crack width is very small and it seems necessary to take into account the bond-slip behaviour between reinforcing steel and concrete, as well as non-linear fracture mechanics of tensile concrete. Since the first studies by means of fracture mechanics, the local deformability near a crack is usually considered by assuming perfect bond between steel and concrete. Therefore, the deformability of lightly reinforced concrete beams is attributed only to fracture mechanisms in the process zone which can be defined (Gerstle et al. 1992) by using a cohesive model (e.g. CEB 1993; Liaw et al. 1990), by means of LEFM (Bosco and Carpinteri 1992) or by considering both a cohesive model and LEFM (Balunc et al. 1992).

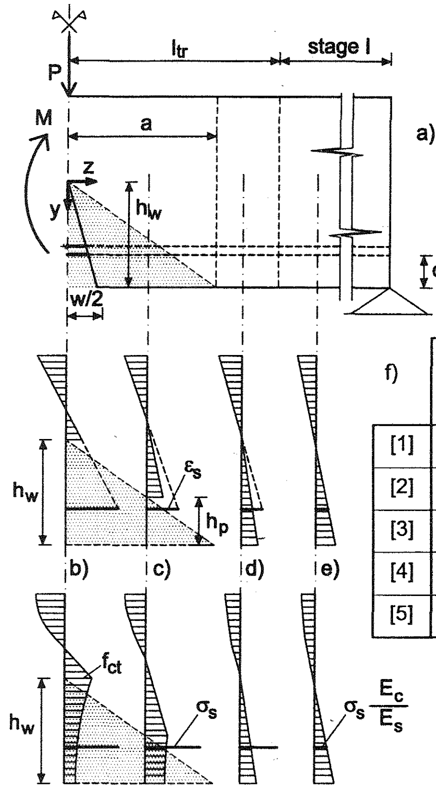
The models based on the aforementioned hypothesis provide consistent results only when perfect bond occurs. This is true, for example, when the crack affects only the concrete cover. On the contrary, when the crack goes through the reinforcing zone, slip between concrete and bars occurs and the perfect bond hypothesis does not hold at least in a certain zone of the beam. This zone, comprised between the crack and the point where slip extinguishes, is named "transmission length l_{tr}". Within the transmission zone, the variation of stresses and strains in steel and concrete due to bond stresses should be considered. In the literature, the bond-slip behaviour is included (Bazant and Cedolin 1980; Planas et al. 1995) by using an experimental pull-out relationship coupled with a cohesive crack model. As the pull-out relationship is only global, no information is provided within the transmission length.

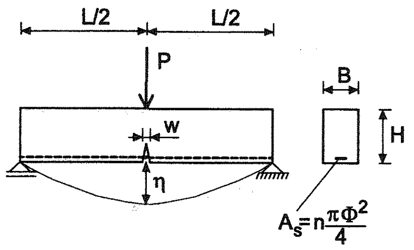
Aim of this paper is to propose a numerical model which is able to study the stable or unstable behaviour of RC beams during the formation and growth of the first crack, by computing the stress and strain fields within the transmission length l_{tr}.

2 Numerical model

The flexural deformability of RC beams where one crack is present (Fig. 2) should be studied by means of a two dimensional analysis, by using, for instance, the finite element method (e.g. Riva and Plizzari 1992). Nevertheless, by introducing a suitable simplified strain profile in every cross-section of the beam, the problem can be solved by means of a one-dimensional analysis, as already proposed for tensile members (Fantilli et al. 1998).

In this way, the beam of Fig. 2 is divided in two different portions. Within the first portion, of length l_{tr}, the tensile stresses of the reinforcing bar are transmitted to the surrounding concrete by means of bond stresses τ. Conversely, within the second portion, perfect bond occurs and stage I condition





f)	B [mm]	H [mm]	n	Φ [mm]	f_c $\left[\frac{N}{mm^2}\right]$	f_y $\left[\frac{N}{mm^2}\right]$	E_c $\left[\frac{kN}{mm^2}\right]$	G_f $\left[\frac{N}{m}\right]$	N_P
[1]	50	250	1	6	38.1	481	29.2	-	-
[2]	150	200	1	5	77.5	569	34.3	4.84	0.13
[3]	150	100	2	8	77.5	441	34.3	4.84	0.73
[4]	50	150	2	2.5	39.5	538	30.3	6.25	0.20
[5]	50	150	4	2.5	39.5	538	30.3	6.25	0.40

[1] Giurani and Rosati (1984b)

[2], [3] Bosco et al. (1990)

[4], [5] Planas et al. (1995)

Fig. 2 Three point bending beam: a) two analyzed portions; b), c), d), e) stress and strain hypotheses; f) geometrical and mechanical properties of the beams tested by various authors

applies (Fig. 2e). In the midspan cracked cross section (Fig. 2b), the strain profile for the steel and uncracked concrete is assumed to be plane. Moreover in the same cross-section, the crack width w varies linearly from the crack mouth ($y = h_w$) to the crack tip ($y = 0$). In the fracture process zone, cohesive stresses are introduced by means of a fictitious crack model. According to Giuriani and Rosati (1987), whose model was valid for plain concrete beams, the cohesive stresses propagate in the diffusion zone depicted with dotted hatch in Fig. 2a. In a generic cross section within the diffusion zone (Fig. 2c), plane strain profile is assumed for steel and concrete in compression, whereas the strain of tensile concrete above the diffusion zone lays on a different plane. Considering the same cross-section, only the equilibrium condition is required in the diffusion part of height h_p. Finally, for a cross-section comprised between diffusion and stage I zones, the strain profile is represented by considering two different planes (Fig. 2d). For a generic section of the block, together with these compatibility assumptions (Fig. 2), the equilibrium equations are considered, namely:

$$\int_{A_c} \sigma_c dA_c + \sigma_s \cdot A_s = 0 \qquad (1a)$$

$$\int_{A_c} \sigma_c \cdot y_c dA_c + \sigma_s \cdot y_s \cdot A_s = M(z) \qquad (1b)$$

where σ_c and σ_s are the stresses in concrete and steel respectively, A_c and A_s are the areas of concrete and steel and $M(z)$ is the applied bending moment.

For a portion of the beam of infinitesimal length dz, the equilibrium equation for the reinforcing bar alone can be written:

$$\frac{d\sigma_s}{dz} = \frac{p_s}{A_s} \cdot \tau(s(z)) \qquad (2)$$

where p_s is the perimeter of the reinforcement in tension, while $\tau(s(z))$ is the bond stress. Referring to the same portion of the beam, it is also possible to define the slip $s(z)=s_s(z)-s_c(z)$ as the difference in the displacements between two initially overlapping points belonging to steel and concrete. Hence, by derivation with respect to z:

$$\frac{ds}{dz} = -[\varepsilon_s(z) - \varepsilon_c(z)] \qquad (3)$$

where ε_s is the strain of the reinforcing bar and ε_c is the tensile strain in the surrounding concrete. To solve the problem, the equilibrium and compatibility equations must be associated with the constitutive laws of the materials and with the bond-slip relationship τ-s. In particular, the ascending

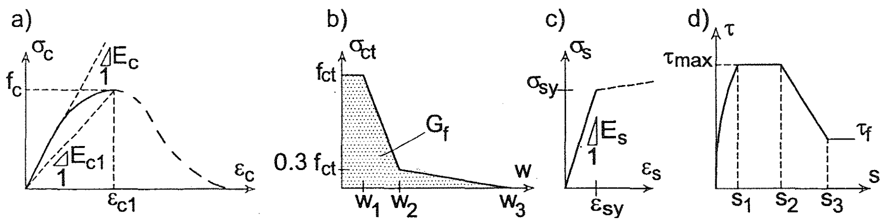


Fig. 3 Adopted constitutive laws: a) stress-strain relationship for concrete in compression (CEB 1993); b) cohesive relationship for tensile cracked concrete (Liaw et al. 1990); c) stress-strain relationship for reinforcing steel (CEB 1993); d) bond-slip relationship (CEB 1993)

branch (Fig. 3a) of the relationship proposed by CEB (1993) is considered as reference for concrete in compression. Concrete in tension is assumed to have a linear elastic stress-strain relationship, with modulus of elasticity E_c, up to the ultimate tensile strength f_{ct}. Initial tension crack starts to form when the tensile strength of concrete f_{ct} (Fig. 3b) is reached in the midspan section of the beam. The crack is modeled by means of the fictitious crack model represented by the relationships of Liaw et al. (1990) shown in Figure 3b :

$$
\begin{aligned}
&\sigma_c = f_{ct} && if \quad w \le w_1 \\
&\sigma_s = f_{ct} - 0.7 \cdot f_{ct} \cdot \frac{w - w_1}{w_2 - w_1} && if \quad w_1 < w \le w_2 \\
&\sigma_s = 0.3 \cdot f_{ct} \cdot \frac{w_3 - w}{w_3 - w_2} && if \quad w_2 < w \le w_3
\end{aligned}
\tag{4}
$$

For the stress-strain relationship of reinforcing steel in tension or compression, the bilinear law depicted in Fig. 3c is assumed.

In accordance with CEB (1993), the following bond-slip τ-s relationships are adopted:

$$
\begin{aligned}
&\tau = \tau_{max} \cdot \left(\frac{s}{s_1}\right)^{\alpha} && if \quad 0 \le s < s_1 \\
&\tau = \tau_{max} && if \quad s_1 \le s < s_2 \\
&\tau = \tau_{max} - (\tau_{max} - \tau_f) \cdot \frac{(s - s_2)}{(s_3 - s_2)} && if \quad s_2 \le s < s_3 \\
&\tau = \tau_f && if \quad s \ge s_3
\end{aligned}
\tag{5}
$$

with the proposed CEB (1993) reduction of bond stresses near cracks. For the solution of the problem, in addition to the foregoing equations, suitable boundary conditions are also adopted, that is to say, cohesive stress condition for the cracked section and stage I condition for the sections with perfect bond. From a mathematical viewpoint, Eqs. 1 - 5 (which generally cannot be uncoupled), together with the boundary conditions, constitute a differential "two point boundary value problem". In principle, this problem can be solved with the aid of various numerical analysis techniques, such as relaxation through finite differences, finite element methods and shooting techniques. The shooting method was adopted herein for the numerical solution (Ferretti 1996). In particular, to avoid double solutions due to the sof-

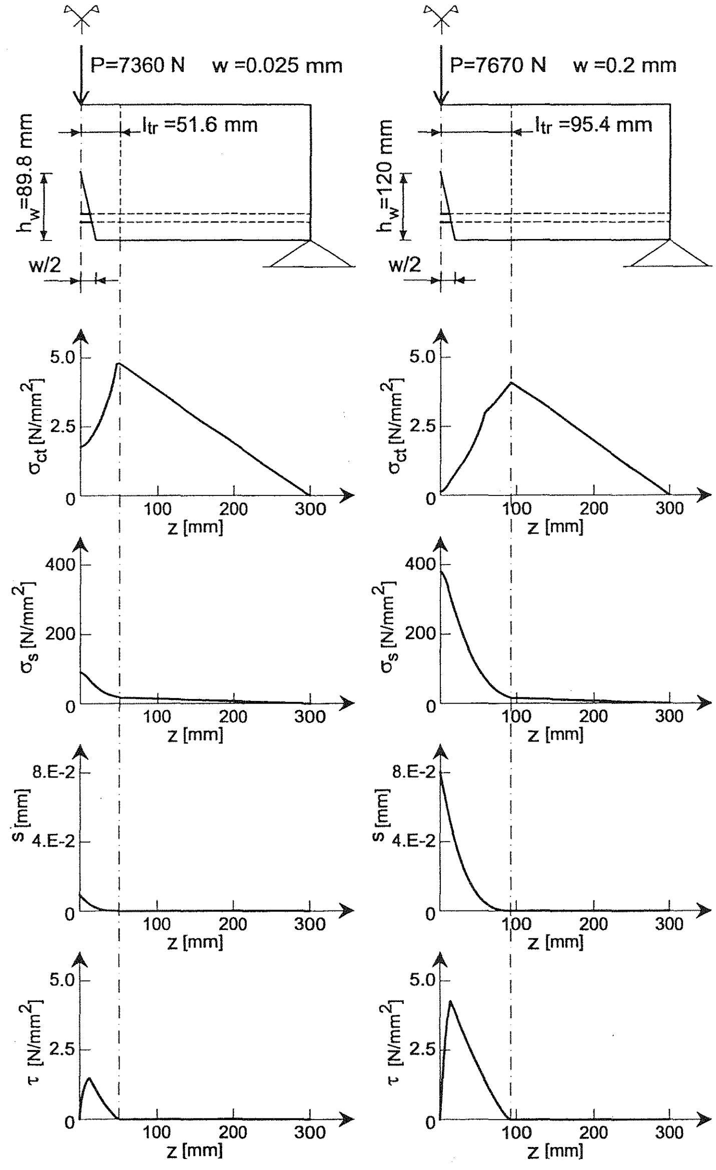


Fig. 4 Some numerical outcoming for the beam [4]

tening phenomenon, the crack mouth opening (CMOD) is assumed as the independent variable (Fig. 2a). In this way, Eqs. 1 - 5 can be integrated starting from the cracked cross section up to the opposite state I cross section, where it is necessary to control if all the boundary conditions are verified. If these requirements are not satisfied, the initial value of h_w must be adjusted by means of a trial and error procedure.

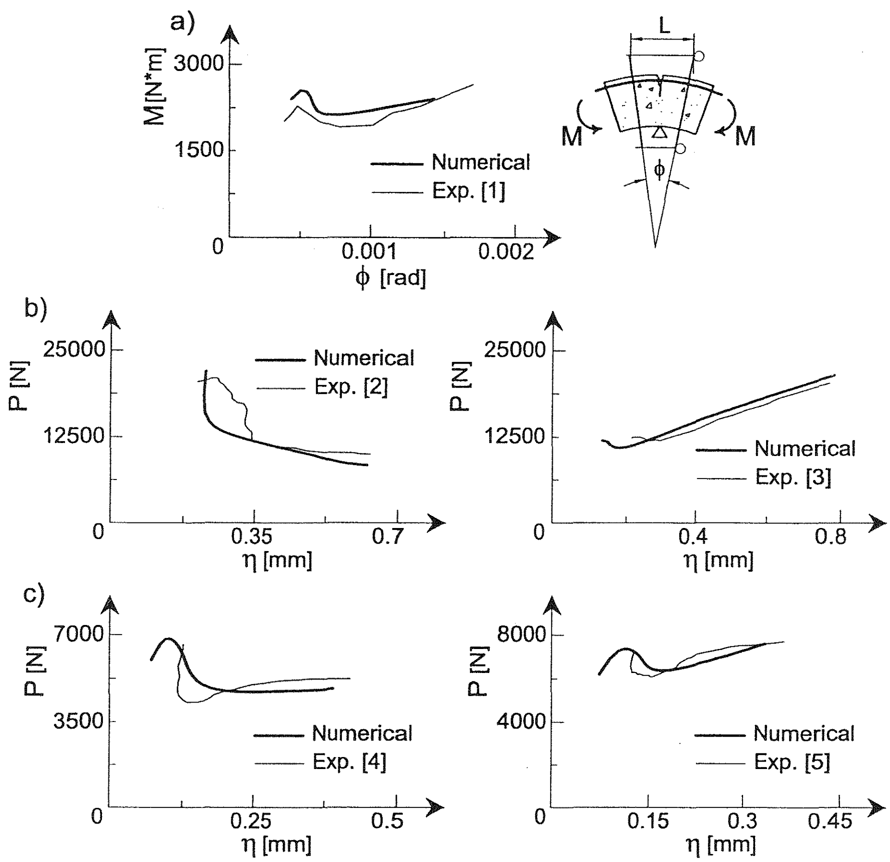


Fig. 5 Comparison, for the beams of Fig.2f, between the proposed model and the experimental data

3 Numerical results

For an imposed value of *w,* the solution of the problem yields the distribution along the beam of all the static and kinematic unknowns (Fig. 4): the state of tensile stress in concrete an steel (σ_{ct}, σ_s), the slip (s) and the bond stress (τ). When the state of stress and the consequent state of strain are known, it is possible to plot the moment-rotation relationship M-Φ by increasing crack width w, as shown in Fig. 5a for the beam [1] whose mechanical and geometrical properties are collected in Fig. 2f. In Fig. 5a, the remarkable softening branch of the M-Φ diagram, measured in the test results of Giuriani and Rosati (1984b) by means of the overlapping moirè, is correctly simulated. Therefore, the proposed model seems also capable to compute correctly the load-displacement P-η response of RC beams during crack growth. In particular, in Fig. 5b, the numerical and experimental P-η diagrams of the beams [2] and [3] of Bosco et al. (1990) (whose mechanical and geometrical properties are still represented in Fig. 2f) are compared. Moreover Fig. 5 shows a substantial agreement between numerical and experimental outcomings, both for the beam [2] having a low value of the brittleness number $N_p=(f_y H^{0.5} A_s)/(K_{Ic} A_c)$ (Carpinteri 1984), and for the beam [3] having an higher value of N_p (Fig. 2f). The same agreement still remain by reducing the dimensions of the beams, as shown in Fig. 5b for the members [4] and [5] tested by Planas et al. (1995). If the first crack appears when the applied bending moment M reaches the value M_{cr}, the subsequent growth of the crack is sometimes accompanied by a remarkable increase of M, as pointed out by the numerical and experimental analyses of Fig. 5. When the maximum value of M is reached (effective crack bending moment M_{cr}* of Fig. 1), the tensile stress of concrete σ_{ct} in the stage I zone could exceed the tensile strength f_{ct} without the possible growth of a new crack. This paradox can be eliminated, for example, by removing the stage I boundary condition and by assuming only $\sigma_{ct} = f_{ct}$ at the bottom of the section where $s = 0$. In this way, as shown in Fantilli (1998), the paradox disappears without significant effects on the results. Hence, the choice of the boundary condition does not appear to be a crucial aspect regarding the softening simulation.

4 Conclusions

The comparison between numerical outcomings and test data allows to state out the following conclusions:

- since the proposed model is able to simulate correctly the remarkable softening branches due to the first crack formation, it seems useful for a correct structural analysis between the pre-cracked stage and the post-cracked one.
- Moreover, as the model can define correctly the structural behaviour of RC beams with different brittleness number and scales (Fig. 2f), it could be employed to compute the minimum reinforcement ratio and to put its size-dependence into evidence (Bosco and Carpinteri 1992).

5 References

Avalle, M., Ferretti, D., Iori, I. and Vallini, P. (1994) On the deformability of reinforced concrete members in tension and bending, in **Proceedings of Euro-C 1994** (eds. M. Mang, H. Bicaninc and R. de Borst), Pineridge Press, Innsbruck, 723-734.

Baluch, M.H., Azad, A.K. and Ashmawi, W. (1992) Fracture mechanics application to reinforced concrete members in flexure, in **Applications of**

Fracture Mechanics to Reinforced Concrete (ed. A. Carpinteri), Elsevier Applied Science, London, 413-436.
Bazant, Z.P. and Cedolin, L. (1980) Fracture mechanics of reinforced concrete. **J. of the Eng. Mech. Div.** (ASCE), 106, 1257-1306.
Bosco, C. and Carpinteri, A. (1992) Fracture mechanics evaluation of minimum reinforcement in concrete structures, in **Applications of Fracture Mechanics to Reinforced Concrete** (ed. A. Carpinteri), Elsevier Applied Science, London, 347-377.
Bosco, C., Carpinteri, A. and Debernardi, P.G.(1990) Fracture of reinforced concrete: scale effects and snap-back instability. **Engineering Fracture Mechanics**, 35 (4/5), 665-677.
Carpinteri A. (1984) Stability of the fracture process of RC beams. **J. of the Structural Engineering** (ASCE), 110, 544-558.
CEB -Comitè Euro-International du Bèton- (1993) **CEB-FIP Model Code 1990. CEB Bulletin d'Information 213-214**. Thomas Telford Service Ltd, London.
Fantilli A. P. (1998) **Deformabilità e duttilità delle strutture in c.a.** PhD thesis, Politecnico di Torino (to be published).
Fantilli, A.P., Ferretti, D., Iori, I. and Vallini, P. (1998) Analysis of RC tensile members considering fracture mechanics and bond between steel and concrete, **in Proceedings of Euro-C 1998**, (eds. M. Mang, H. Bicaninc and R. de Borst), Balkema, Badgastein, 807-815.
Ferretti D. (1996) **Sul comportamento a breve termine di elementi in conglomerato cementizio armati**. PhD thesis,11, Politecnico di Torino.
Gerstle, W.H., Dey, P.P., Prasad, N.N.V., Rahulkumar, P. and Xie, M. (1992) Crack Growth in flexural members - A fracture mechanics approach. **ACI Structural Journal**, 89(6), 617-625.
Giuriani, E. and Rosati G. (1984a) Effect of a single crack in continuous beams. **Studi & Ricerche**, 6, 119-149.
Giuriani, E. and Rosati G. (1984b) Experimental investigation on RC elements subjected to a single crack. Analysis of the instable branches of the moment-rotation relationship. **Studi & Ricerche**, 6, 151-199.
Giuriani, E. and Rosati G. (1987) An analytical model for the study of the crack propagation in plain concrete elements under bending. **Studi & Ricerche**, 9, 107-127.
Liaw, B.M., Jeang, F.L., Du, J.J., Hawkins, N.M. and Kobayashi A.S. (1990) Improved nonlinear model for concrete fracture. **J. of Eng. Mech.** (ASCE), 116, 429-445.
Planas, J., Riuz, G. and Elices, M. (1995) Fracture of lightly reinforced concrete beams: theory and experiments, in **FRAMCOS-2** (ed. F. H. Wittmann), AEDIFICATO Publishers, Freiburg, 1179-1188.
Riva, P. and Plizzari, G.(1992) Modelling steel to concrete interaction using rebar elements in nonlinear finite elements analysis, in **Proceedings of Int. Conference Bond in Concrete** (ed. CEB), Riga, 2.44-2.53.