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PROBABILISTIC STUDY ON THE TENSILE STRENGTH OF MULTIPLE-FRP TENDONS

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Abstract

This study analyzed the failure load of FRP multiple-tendon systems having a design strength of 130 tf, by using the weakest link model and the Monte Carlo method to examine the tensile strength characteristics of the systems. It was found that the mean value and the standard deviation of a tension test could be simulated accurately by the Monte Carlo method with variations in the tensile stiffness. Accordingly, based on numerical analysis using this method, the authors examined the appropriate number of cables for tendon systems with a design strength of 130 tf, and how to evaluate the design strength of systems using different numbers of cables.

Key words: multiple-FRP tendon, chain failure, failure probability, reliability analysis

1 Introduction

Multiple-tendon systems using brittle materials such as FRP cables exhibit chain failure in most cases. It is presumed that their failure strength is likely to fit the weakest link model, and their average tensile strength is lower than the total obtained by multiplying the average tensile strength of an individual component cable by the number of cables used in a multiple-tendon system. Accordingly, for multiple-tendon systems to be put to practical use, their tensile strength characteristics must be understood. However, it is often difficult to experimentally determine the tensile strength of systems using a number of specimens.

This study analyzed the failure loads of multiple-tendon systems having a design strength of 130 tf made of carbon and aramid fiber reinforced plastic cables (CFRP and AFRP), by using the weakest link model and the Monte Carlo method. The authors simulated the results of tension testing of five specimens and also examined the appropriate number of cables to be used in systems with a design strength of 130 tf, and how to evaluate the design strength of systems using different numbers of cables.

2 Summary of tensile tests

Table 1 shows the details of the multiple-tendon systems used in the experiment. Both CFRP and AFRP used friction bonding anchors. The loading method, the number of specimens and the size of the specimens followed the standards set in the "Proposed tension test method for FRP cables" and the JSCE standard "Proposed performance test method for anchors and couplers used in prestressed concrete construction." Table 2 shows the results of the tension test. For both systems, each specimen lost its proof stress when the cables broke, with no failure occurring at the anchoring sections. Typical chain failure occurred in three of the five specimens in both systems.

3 Analysis of failure strength

Table 3 shows the failure strength and distribution of failure strain of CFRP and AFRP cables. Since it was confirmed that the data fitted the normal distribution, the failure loads of systems were computed based on this.

Table 1. Details of multiple-tendon systems

		CFRP	AFRP
Cable specifications	Diameter (mm)	12.5	14.7
	Specified load (tf)	14.5	24.0
	Type of fiber	PAN-type	Para-type
	Matrix	Epoxy resin	Epoxy resin
	Configuration	Twisted	Braided
No. of cables used		11	7
Specimens	Anchorage length	500 mm	600 mm
	Anchorage diameter	100 mm	120 mm
	Standard strand length	2,000 mm	3,000 mm
Filler		Epoxy resin	Cement expander

Table 2. Results of the tensile test of the multiple-tendon systems (experimental)

	CFRP	AFRP
No. of specimens	5	5
Minimum failure load (tf)	165.0	160.0
Maximum failure load (tf)	180.0	178.0
Mean failure load (tf)	172.6	169.5
Standard deviation (tf)	6.580	7.071
No. of specimens subjected to chain failure	3	3

3.1 Analysis by the weakest link model

If it is assumed that a system fails at the load at which the weakest link model fails, its failure load can be determined by multiplying the lowest failure load (f_s) of n cables by the number of cables. On the other hand, the failure probability density function of the system is determined by multiplying the probability that the failure load of at least one of n cables is f_s by the probability that the failure loads of the remaining cables all exceed f_s. The probability distribution function is determined by integrating this probability density function as given by the following expression:

$$P_m = 1 - (1 - P_s)^n \quad (1)$$

P_m: failure probability of the system
n: the number of cables
P_s: failure probability of cables with the lowest failure load (f_s)

The authors specified the failure probability P_m in expression (1) to reverse-calculate P_s, and multiplied the failure load f_s of a cable corresponding to P_s by n to determine the failure load f_m of the multiple-tendon system corresponding to that specific failure probability P_m.

Table 3. Cable statistics

		CFRP	AFRP
Failure load (tf)	Modulus	100	98
	Mean value	17.120	27.206
	Standard deviation	0.805	0.887
Failure strain (%)	Modulus	30	30
	Mean value	1.561	2.171
	Standard deviation	0.112	0.147
Coefficient of Correlation (ρ)	Modulus	30	30
	ρ	0.055	0.247

3.2 Analysis by the Monte Carlo method

In the Monte Carlo method, failure loads of the systems were calculated by modeling the multiple-tendon systems as follows:

1. Cables do not pull free of the anchors. The system fails as the individual cables fail.
2. Statistically, cable failures occur independently.
3. The correlation between the failure load and the failure strain of cables, expressed as the correlation coefficient ρ, is taken into consideration.
4. The correlation between the load and the strain of the cables forms a straight line connecting the origin and the crossing points of the failure load and the failure strain. The failure loads for the two cases shown in Fig. 1 are computed. Case 1 is a model in which the stiffness is fixed, while the tensile stiffness changes according to the correlation of the load and the strain in Case 2.
5. When variance in the strand lengths is taken into consideration, the mean cable length is assumed to be the standard cable length and the range of lengths fits a normal distribution within ±2σ (95.4%; see Fig. 2). The length of each cable is calculated with a maximum variance of 4σ.

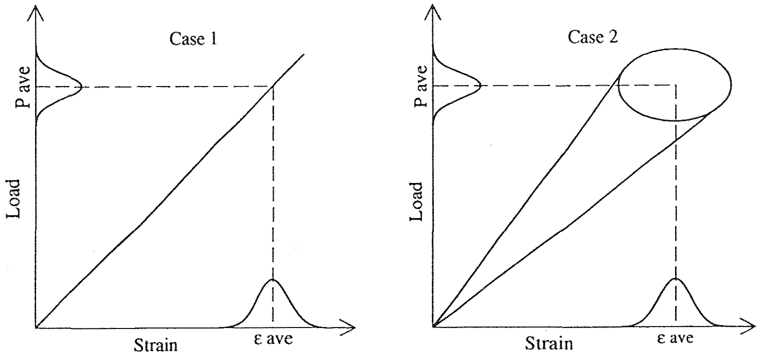


Fig. 1. Correlation model between the load and strain of cables

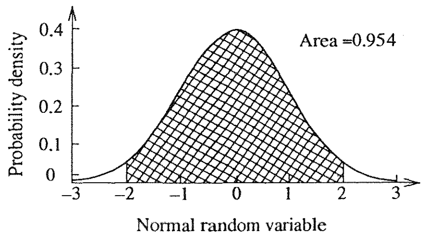


Fig. 2. Distribution model of cable length

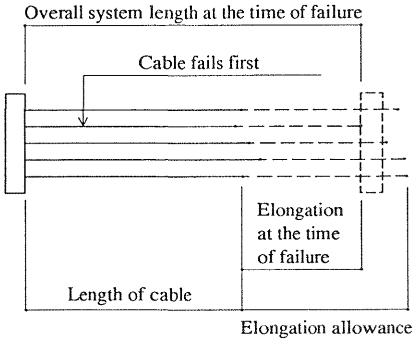


Fig. 3. System failure model with variation in length

In the analysis, each probability distribution function value is defined first by generating random numbers in the range [0,1], to calculate the elongation of the cable by picking up failure loads, failure strains and the length of cables, based on the above assumptions. Then, based on the relationship between the different cable lengths and the elongation, the cable which will fail first is identified (Fig. 3). The overall system length at the time of failure is defined and the elongation of each cable is calculated. The system failure load can be calculated as the sum of the loads applied to each cable. If the system failure load defined by failure of a single cable is lower than that of the failure of (n-1) cables, the system failure load is defined by the failure of (n-1) cables, indicating that chain failure does not occur in the system.

4 Results of analysis

4.1 Comparison of experimental and analytical results

Five specimens were selected randomly out of 50,000, and 10,000 pairs of mean failure loads and standard deviations were determined from them. Table 4 shows the mean values of these results. Analysis was performed for Cases 1 and 2 in the Monte Carlo method.

Table 4. Mean failure load and mean value of standard deviations (experiment & analysis)

		Mean failure load		Standard deviation	
		Computation results (tf)	Error (%)	Computation results (tf)	Error (%)
CFRP	Test result	172.6		6.580	
	Case 1	174.3	-1.0	4.721	28.3
	Case 2	167.9	2.8	6.121	7.0
AFRP	Test result	169.5		7.071	
	Case 1	182.1	-7.4	3.616	48.9
	Case 2	173.5	-2.4	6.213	12.1

Comparing the mean values of failure loads, the variance for Case 1 of AFRP was 7.4%, while the variances for Case 2 were less than 3% for both CFRP and AFRP, indicating that Case 2 estimated the mean value of failure loads more accurately. In addition, the mean value estimated by Case 2 had a greater safety margin than Case 1. Comparing the standard deviations, more accurate analyses were obtained in Case 2, although the variance for each case was relatively large at around 10%. From these comparisons, it is shown that Case 2 reproduces the failure load characteristics of the multiple-tendon systems better than Case 1 and with a larger safety margin.

4.2 Examination of the number of cables

When the lengths of cables composing the system are not uniform, the failure load of the system is reduced. To verify how this variance in cable length affects a system's reliability and how many cables are appropriate for the design strength, this study calculated the failure load of systems using the maximum variance of the cable length and the number of cables as parameters. The analysis was made using Case 2 in the Monte Carlo method, setting the standard cable length of CFRP and AFRP to 3000 mm. It was assumed that the variance in cable length would fit a normal distribution as shown in Fig. 2, and six levels of maximum variance were set from 0 to 30 mm (±15 mm). The failure loads of 10,000 specimens were analyzed for each level of maximum variance.

Figure 4 shows the relationship between the maximum variance of the cable length and the failure probability. The failure probability was determined by dividing the number of specimens having a failure load lower than the design strength of 130 tf by the total number of specimens. When the standard failure probability of the design-strength system is set to P = 0.135%, the number of cables of AFRP cannot be reduced from seven to six. If the number of CFRP cables is reduced to ten, the failure probability is higher than the standard probability of 0.135% with a maximum variance of 0.3% (10 mm) or more. Considering the fact that some variance will occur inevitably during the production of systems, the number of cables used in the systems examined in this study are generally appropriate relative to the design strengths for both the systems.

4.3 Evaluation of design strength introducing the multiple-tendon coefficient

The design strength of PC cables, which have ductility, is calculated by expression (2) based on the design strength of individual component cables that comprise the multiple system. This expression cannot be applied, however, to FRP cables, which are brittle, because their failure strength is mostly governed by the weakest link model. The authors

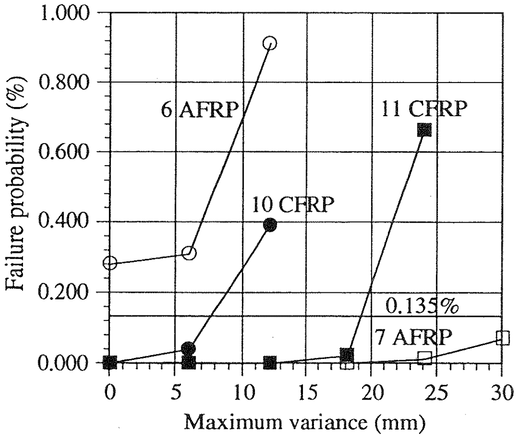


Fig. 4. Relationship between failure probability and maximum variance of cable length (analysis)

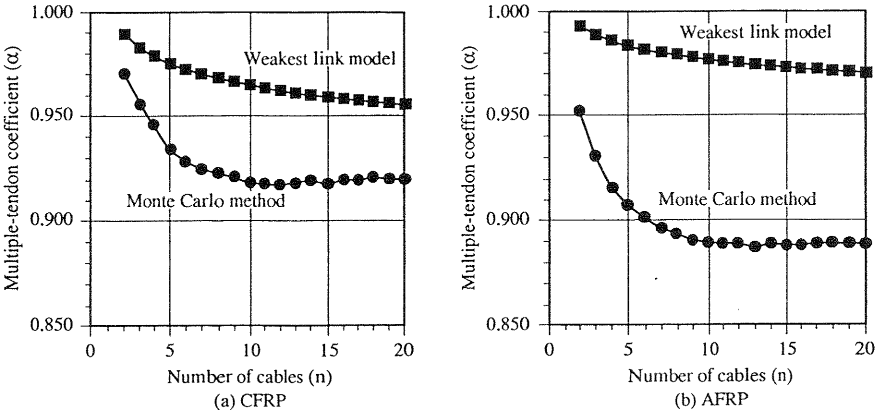
propose predicting the design strength with expression (3), which introduces the multiple-tendon coefficient that takes into consideration the reduction in strength due to multiplication. The multiple coefficient was estimated by the Monte Carlo method using Case 2.

$$f_{mpc} = f_{spc} \times n \times \eta \tag{2}$$
$$f_{mfrp} = f_{sfrp} \times n \times \alpha \times \eta \tag{3}$$

f_{mpc}, f_{mfrp}: design strength of the multiple-tendon system using PC or FRP cables.
f_{spc}, f_{sfrp}: design strength of a PC or FRP cable.
n: the number of cables.
η: anchor coefficient, the reduction coefficient of the tensile strength due to anchoring.
α: multiple-tendon coefficient, the reduction coefficient of the tensile strength due to multiplexing.

To obtain α, first the failure load was calculated 10,000 times with the number of cables n ranging from 2 to 20, and the design strength fmfrp of the multiple-tendon system was determined for individual cases. The obtained values of f_{mfrp}, f_{sfrp} and $\eta = 1$ were entered in expression (3) to calculate α as a numeric experimental value. The failure probability of the system was $p = 0.135\%$.

Figure 5 shows the relationship between the multiple-tendon coefficient (α) and the number of cables. The figure also shows the multiple-tendon coefficient obtained based on the weakest link model for comparison. Figure 6 shows the number of specimens which exhibited chain failure and the number of cables. For both FRPs, the multiple-tendon coefficient by the Monte Carlo method decreased linearly with the increase in the number of cables up to around 7. However, the coefficient leveled off as more cables were used, averaging around 0.91 for CFRP and 0.88 for AFRP. Since most specimens caused chain failure with 6 cables or fewer, the failure load of the multiple-tendon system was governed by the minimum failure load of individual cables comprising the system. The minimum failure load of cables decreased with the increase in the number of cables, so reducing the multiple-tendon coefficient. The minimum failure load of cables continued to decrease as the number of cables exceeded 7, but the rate was not significant. The probability of chain failure also decreased with the increase in the number of cables. As a result, the failure load of the multiple-tendon system was sometimes larger than the minimum failure load multiplied by the number of cables. This maintained the multiple-tendon coefficient at a certain level while the number of cables increased. To determine the load specifications with a suitable



(a) CFRP

(b) AFRP

Fig. 5. Relationship between the multiple-tendon coefficient and the number of cables

safety margin, it is suggested that a multiple-tendon coefficient of 0.90 be used for CFRP and 0.85 be used for AFRP.

At the end of this study, the anchor coefficient was examined with expression (3) using the design strength calculated by the 3σ method based on the test results and the multiple-tendon coefficient calculated from the analysis. Table 5 shows the results. The anchor coefficient was almost η = 1.0 for CFRP but slightly below that for AFRP. This suggests that the anchorage performance of CFRP is slightly better than AFRP.

Table 5. Examination of the anchor coefficient

	f_{sfrp} (tf)	n	α	f_{mfrp} (tf)	η
CFRP	14.705	11	0.918	152.860	1.029
AFRP	24.545	7	0.896	148.287	0.963

5 Conclusions

The following conclusions were obtained from this experimental and analytical study:

1. The failure load of multiple-tendon systems can be calculated accurately and safely by correctly modeling their tensile stiffness and by Monte Carlo simulation.
2. The authors proposed an expression to obtain the design strength of a multiple-tendon system from the design strength of the cables

comprising the system, introducing the multiple-tendon coefficient. Test calculations using the Monte Carlo method suggested that the design strength of the multiple-tendon system can be calculated in a reasonably simple manner when the number of cables in the system is 20 or below.

3. It was demonstrated that the anchor coefficient of an FRP multiple-tendon system which has been tested for tensile strength can be evaluated by using the proposed expression and the test calculation of the multiple-tendon coefficient.

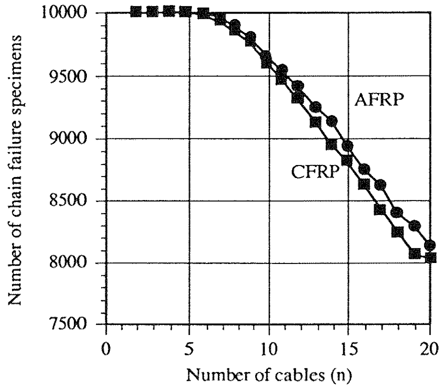


Fig. 6. Relationship between the number of chain failure specimens and the number of cables

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