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PLATE END SHEAR DESIGN FOR EXTERNAL CFRP LAMINATES

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Abstract
In the design of externally bonded reinforcement, the anchoring design is extremely important, because failure is very often initiated in the end zones. Expressions for the anchoring capacity in both the serviceability limit state and the ultimate limit state are derived from the classical differential equation of Volkersen. Therefore non-linear concrete properties, describing the pre- and post-cracking behaviour are considered. A relation with the mode II-fracture energy has been set up. The model parameters are simply determined from the concrete compressive strength and the concrete tensile strength.
Key words: Concrete strengthening, CFRP, Anchoring stresses

1 Introduction

Since many years externally bonded steel plates have been used to strengthen deteriorated, damaged or inadequate concrete structures. Appropriate design rules to determine the bending capacity of concrete beams were set up, Van Gemert et al. (1990). At the end zones, where the forces are transferred from the concrete to the external reinforcement, the anchorage was usually done by mechanical devices, such as dowels or external stirrups.

Recently new high-grade materials, such as CFRP laminates are used instead of the classical steel plates, Brosens and Van Gemert (1996). However, when using CFRP laminates the traditional mechanical anchorage systems can no longer be used. By drilling a hole in the longitudinally oriented fibres, all fibres would be cut and would no longer be able to transfer any forces. Because premature failure often is initiated in the anchoring zones and because of to the high forces in the CFRP laminates, it has become very important to know the behaviour in the force transfer zone and to determine the anchorage capacity of a bonded concrete-CFRP connection. Moreover, both the serviceability limit state and the ultimate limit state have to be considered. The determination of the load situation can be done by conventional design codes.

2 Differential equation

2.1 Theory of Volkersen

Many authors have used the classical theory of Volkersen (1938) for the description of the stress situation in bonded connections subjected to shear (Bresson (1971), Ranisch (1982)). The following assumptions were made:

- All the materials are linear elastic materials
- Bending effects are neglected
- The normal stresses are uniformly distributed over the cross-section

With these assumptions the following differential equation for bonded concrete-CFRP laminates can be derived.

$$\frac{\partial^2 s_l}{\partial x^2} - \frac{1 + m_l \gamma_l}{E_l h_l} f(s_l) = 0 \qquad (1)$$

With s_l The relative displacement between concrete and laminate (= the slip between the two materials)

m_l The stiffness ratio ($m_l = E_l / E_c$)

γ_l The cross sectional area ratio ($\gamma_l = A_l / A_c$)

$f(s_l)$ The relation between the shear stress τ_l and the slip s_l ($\tau_l = f(s_l)$)

All the properties of the connection are expressed by the relationship $\tau_l = f(s_l)$. The essential step in reliably predicting the bearing capacity of the connection is the choice of this relationship.

2.2 Bilinear relationship

Many researchers investigated the influence of the function $\tau_l = f(s_l)$. Bresson (1971) proposed a simple linear relationship whereas Wicke and Pichler (1991) used a non-linear relationship. Both models did not take into account the post-cracking behaviour. However, better results were obtained considering this post-cracking behaviour. Ranisch (1982) used a bilinear relationship. The advantage of this approach was that an analytical solution could be found. Other possibilities were suggested: an elasto-plastic relationship as for steel and a special relationship for steel fibre reinforced concrete. In fact, with a modern mathematical integration program every function can be introduced and the differential equation solved.

Holzenkämpfer (1994) found out that for bonded steel plates sufficiently accurate results were obtained by Ranisch' bilinear model, figure 1. In the first section (I), the shear stress grows linearly with increasing slip until a maximum value τ_{lm} is reached. At this point the first crack is initiated but stress transfer is still possible by aggregate interlock. This post-cracking

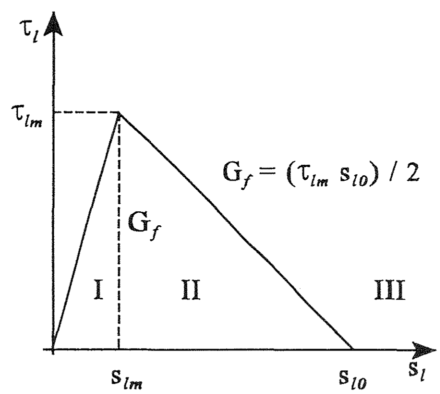

Fig. 1. Bilinear relationship $\tau_l = f(s_l)$

behaviour is described by the linearly descending branch (section II). When the slip attains the value s_{l0} the shear stress τ_l has become zero (section III). The area under the τ_l - s_l curve is defined as the fracture energy G_f.

When executing a pure shear test, following stages are found, figure 2:

- figure 2a At small forces, the whole concrete-CFRP connection is located in section I. This is called the elastic state. The specimen is still uncracked. The shear stress distribution along the connection is a hyperbolic function with its maximum value at the end where the force is introduced. When the maximum value τ_{lm} has been reached, a crack is initiated.
- figure 2b After attaining τ_{lm}, the connection is in a combined elasto-plastic stage. When the crack length increases, the shear stress peak τ_{lm} moves to the other end of the laminate. In the elastic region (section I), the concrete is still uncracked whereas in the plastic

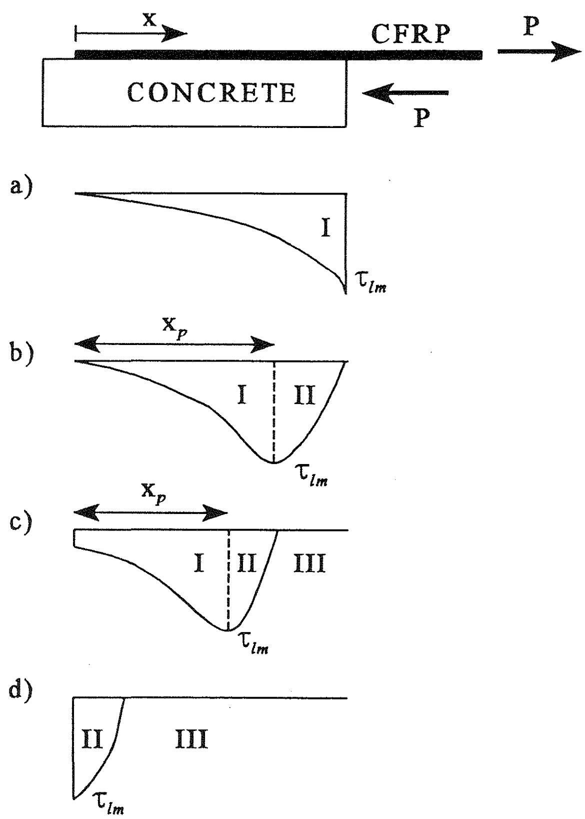

Fig. 2. Different stages at a simple shear test

region (section II) the concrete is cracked (micro-cracks) but still able to transfer forces. The length of the elastic region is indicated by x_p.

- figure 2c At a certain moment, the slip will have attained the value s_{l0} and the shear stress τ_l has become zero (section III). In this region no stresses can be transferred over the crack. The crack has become a macro-crack.
- figure 2d When the shear stress peak τ_{lm} reaches the other end of the laminate, the whole connection is in the plastic state. In section II, forces can still be transferred whereas in section III a macro-crack without stress-transfer is present. Section II decreases until the connection completely fails.

2.3 Serviceability limit state

In the serviceability limit state, no cracks are allowed. The whole connection must be in the elastic state. The maximum transferable force appears when the shear stress reaches the value τ_{lm} at the end where the force is introduced, figure 2a. When solving the differential equation (1) the following expression for P can be obtained.

$$P(l) = \frac{b_l\sqrt{2\,G_f E_l h_l}}{\sqrt{1 + m_l\gamma_l}}\tanh\sqrt{\frac{\tau_{lm}^2\,l^2}{2\,G_f E_l h_l}} \tag{2}$$

with

$$G_f = \frac{\tau_{lm}\,s_{lm}}{2} \tag{3}$$

G_f is the area under the elastic part of the τ_l-s_l relationship and l is the bonding length. Since the CFRP laminates are very thin ($A_l << A_c$), a simplification can be done by putting $m_l\gamma_l \approx 0$. Then the limit for P for large values of l is

$$P_{max} = b_l\sqrt{2\,G_f E_l h_l} \tag{4}$$

The anchorage length l_a is defined as the length needed to attain 97 % of P_{max}. Taking into account that tanh(2) = 0.97, following equation can be derived.

$$l_a = \frac{2 P_{max}}{b_l \tau_{lm}} \tag{5}$$

This is the formula for a simple triangular stress distribution which was earlier proposed by Brosens and Van Gemert (1997). Taking a variation coefficient of 12 %, Holzenkämpfer (1994), the characteristic value for the maximum force can be found.

$$P_k = P_m(1 - 1.64 \text{ x } 0.12) \approx 0.8 P_m \tag{6}$$

This means that the characteristic force which can be transferred by the anchoring length l_a is equal to

$$R_k = 0.97 \text{ x } 0.8 \text{ x } P_{max} \approx \frac{3}{4} P_{max} \tag{7}$$

2.4 Ultimate limit state

In the ultimate limit state, the failure load is determined. The maximum load occurs when the connection is in a combined elastic-plastic state. Starting from Volkersen's equation (1) with the bilinear τ_l-s_l relationship and after a few calculations, the equations for determining the maximum load in function of the bonded length can be obtained.

$$P(l) = \frac{E_l A_l \lambda \omega s_{l0} \sin(\lambda \omega (l - x_p))}{1 + m_l \gamma_l} \tag{8}$$

with

$$\lambda = \sqrt{\frac{s_{lm}}{s_{l0} - s_{lm}}} \tag{9}$$

$$\omega = \sqrt{\frac{\tau_{lm} (1 + m_l \gamma_l)}{s_{lm} E_l h_l}} \tag{10}$$

x_p can be found from the relation

$$\tanh(\omega x_p) = \lambda \tan(\lambda \omega (l - x_p)) \tag{11}$$

The problem in this set of equations is that x_p is defined as an implicit function and can only be found by iteration. However it can be shown that for large values for l and with the simpification that $m_l \gamma_l \approx 0$, equation (8) converges to

$$P_{max} = b_l \sqrt{2\, G_f E_l h_l} \tag{12}$$

with

$$G_f = \frac{\tau_{lm}\, s_{l0}}{2} \tag{13}$$

Here G_f is the area under the complete τ_l-s_l curve (elastic and plastic state). Note that the same expression for the maximum force has been found as in the serviceability limit state. The only difference lies in the definition of the fracture energy G_f.

As before, the anchoring length l_a is defined as the length needed to attain 97 % of P_{max}. Following equation can be derived.

$$l_a = \frac{2\,\lambda + Atan\left(\dfrac{tanh\,(2)}{\lambda}\right)}{\lambda\,\omega} \tag{14}$$

With this anchoring length l_a, it can be shown that

$$\frac{P(l_a)}{P_{max}} \geq tanh\,(2) \approx 0.97 \tag{15}$$

Again, with a variation of 12%, the maximum allowable force for the connection is

$$R_k = 0.97\ x\ 0.8\ x\ P_{max} \approx \frac{3}{4} P_{max} \tag{16}$$

In figure 3, the equations (2) and (14) are drawn for a practical case. Both the anchoring lengths for the serviceability limit state and for the ultimate limit state are shown. The reinforcement consists of 3 layers of CFRP sheets. In the serviceability limit state the anchorage length l_a equals 79 mm. In the ultimate limit state l_a becomes 225 mm.

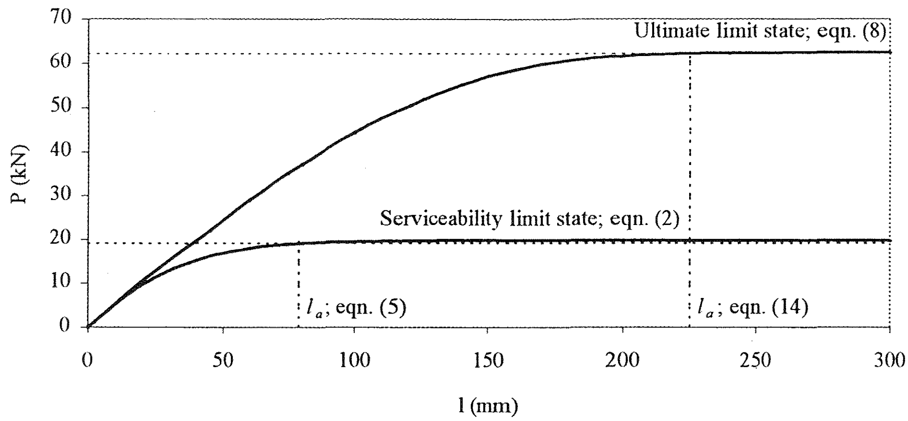

Fig. 3. Force as a function of the bonding length l
with $E_l = 200000$ N/mm^2, $h_l = 0.501$ mm, $b_l = 100$ mm, $\tau_{lm} = 5$ N/mm^2, $E_c = 30000$ N/mm^2, $h_c = 75$ mm, $b_c = 150$ mm, $s_{lm} = 0.08$ mm, $s_{l0} = 0.8$ mm

2.5 Model parameters

When using the above-mentioned formulae, the τ_l-s_l relationship has to be known. Therefore it is necessary to set up empirical formulae or experimental test methods to determine the value of τ_{lm}, s_{lm} and s_{l0}. At practical applications, the only material parameters which can easily be found, are the concrete compressive strength and the concrete pull-off strength. Hence a relation between these parameters was searched.

For the maximum shear stress τ_{lm}, the fracture criterium of Mohr-Coulomb can be used, figure 4. The Mohr-Coulomb line is described as

$$\tau = \frac{1}{2}\sqrt{f_{ck} f_{ctm}}\left[1 + \left(\frac{f_{ck}}{f_{ctm}} - 1\right)\frac{\sigma_y}{f_{ck}}\right] \qquad (17)$$

When $\sigma_y = 0$, τ_{lm} is found.

$$\tau_{lm} = \sqrt{f_{ck} f_{ctm}} \qquad (18)$$

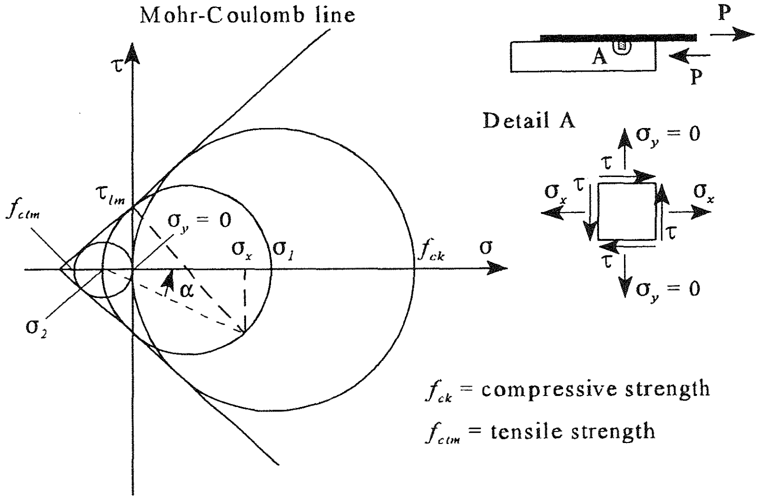

Fig. 4. Fracture criterium of Mohr-Coulomb

According to the new European Standard, Eurocode 2 (1995), the relation between the tensile strength and the compressive strength is

$$f_{ctm} = 0.3 f_{ck}^{2/3} \tag{19}$$

Using (19) in (18) and after a linear interpolation between τ_{lm} and f_{ctm} or f_{ck}, following expressions can be found.

$$\tau_{lm} = 0.137 f_{ck} \quad or \quad \tau_{lm} = 1.8 f_{ctm} \tag{20}$$

For the slip however it is more complicated to find adequate expressions. Assuming that the slip is the sum of the slip in the glue and the slip in the outer part of the concrete, following expression can be derived (Poisson's ratio ν is taken 0.25).

$$s_{lm} = 2.5\,\tau_{lm} \left(\frac{h_g}{E_g} + \frac{d_{ref}}{E_c} \right) \tag{21}$$

with h_g Thickness of the glue layer
E_g Young's modulus of the glue layer
d_{ref} Reference distance, here taken 50 mm

In the theory of Volkersen, the assumption is made that the stresses are uniformly distributed over the cross section. In reality, the stress distribution near the contact zone between concrete and CFRP is not uniform. Over a certain distance, where the stresses are influenced by the external bonded CFRP laminate, a stress gradient occurs. This distance is defined as d_{ref}. The influence zone is taken 2.5 - 3 times the maximum aggregate size, usually about 50 mm.

The slip s_{l0} is determined by using the fracture energy G_f. From a large number of experiments done by Holzenkämpfer (1994), a linear relationship between G_f and f_{ctm} was found.

$$G_f = C_f f_{ctm} \tag{22}$$

For C_f the value 0.092 was found, but this value must be handled with care. The experiments were done with bonded steel plates. CFRP laminates are much thinner and unidirectional. Therefore, further research is needed to check the validity of equation (22) with respect to CFRP laminates. When equation (22) can be accepted, the value of s_{l0} is very easily found.

$$s_{l0} = \frac{2 G_f}{\tau_{lm}} \tag{23}$$

At the Laboratory Reyntjens of K.U.Leuven, Belgium, a research program is running to determine the τ_l-s_l relationship of bonded concrete-CFRP connections.

3 Non linear fracture mechanics

The force transfer mechanism can also be solved using non linear fracture mechanics (NLFM), Täljsten (1994). A crack will be instable when the energy needed to extend the crack with an infinitesimal length becomes larger than the elastic energy stored in the body. Here the fracture energy G_f is defined as the energy needed to bring an unit area into complete fracture. A distinction is made between the kind of fracture (Mode I, II or III). For the simple shear test, the fracture energy for mode II, G_{fII}, is used. In Täljsten (1994) is explained how to determine this parameter experimentally. Finally the following equation is obtained.

$$P_{max} = b_l \sqrt{\frac{2\,G_{fII}\,E_l\,h_l}{1 + m_l\,\gamma_l}} \tag{24}$$

By taking $m_l\gamma_l \approx 0$ this equation is similar to (4) and (12). The only difference is the definition of the fracture energy. Here the fracture energy for mode II, G_{fII}, has been taken. Again, for practical applications a simple determination of the parameters is necessary.

4 Conclusions

The anchoring capacity of bonded connections is very important in the design of externally bonded reinforcement for structural renovation. A proper anchoring design assures the safety of the system, because possible failure often is initiated by delamination in the end zones. Therefore an idea of magnitude of the maximum transferable load under service conditions is indispensable. In the serviceability limit state no cracks, not even micro-cracks are allowed. However, there is still an additional reserve because the global bearing capacity of the bonded connection is given by the ultimate limit state. The failure load of the connection is as a rule higher than the maximum service load. Of course all the common safety factors used in the ultimate limit state and the serviceability limit state have to be taken into account. A good understanding of the anchoring phenomena is the basis for a proper design of externally bonded reinforcement.

5 Acknowledgement

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