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SIZE EFFECTS IN TOUGHNESS INDUCED BY CRACK CLOSE TO FREE EDGE

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Abstract

This paper explains, using fracture mechanics principles, that the common size effects observed in the fracture toughness of concrete and similar materials can be clarified by failures of test samples with crack tips close to either front or back surfaces. From the strength and toughness criteria a simple asymptotic function, akin to the original Bazant's size effect law, is acquired for edge cracks at the front surfaces. The key parameter used in the asymptotic function, the reference crack length a*, is similar to the original Hillerborg's characteristic length l_{th} introduced for concrete. The size effect due to crack tips close to the back surfaces is further considered by a local energy distribution function g_f. It shows that the size effects in toughness are inevitable when the distance from a crack tip to either front or back surface is comparable to the crack-tip fracture process zone.

Key words: Size effects, fracture toughness, crack, free edge

1 Introduction

The significance of size effects in concrete fracture has been well-documented since the pioneer work of Bazant (1984) more than ten years ago. Is concrete really different to metals and ceramics as "size effect" is virtually never mentioned for these materials? This paper addresses this problem by firstly examining typical fracture characteristics of metals with cracks close to free surfaces, and then provides a fresh explanation to the size effect by linking a simple asymptotic function describing the ductile/brittle failure transition for metals to the more familiar size effect law commonly used for concrete fracture.

The objective of the present study is therefore to show that there are many common features among concrete, metal, ceramics and various composites, and applications of fracture mechanics to concrete are no difference to other materials i.e. they all should obey the fundamental principles.

2 Small Edge Crack at Free Front Surface

2.1 "Ductile/brittle" failure transition and size effect

To start with, let us consider the tensile failure of a metal plate shown in Fig. 1. For simplicity, a perfect elastic/plastic stress and strain relation is assumed so that the plastic failure is controlled by the yield strength S_y even if the plate may contain a very small edge crack. However, if the edge crack is sufficiently long, the linear elastic fracture mechanics (LEFM) criterion, the fracture toughness K_{IC}, can be applied. Both criteria have been represented by the pale straight lines in the diagram of failure strength σ_f versus crack length a in Fig. 1.

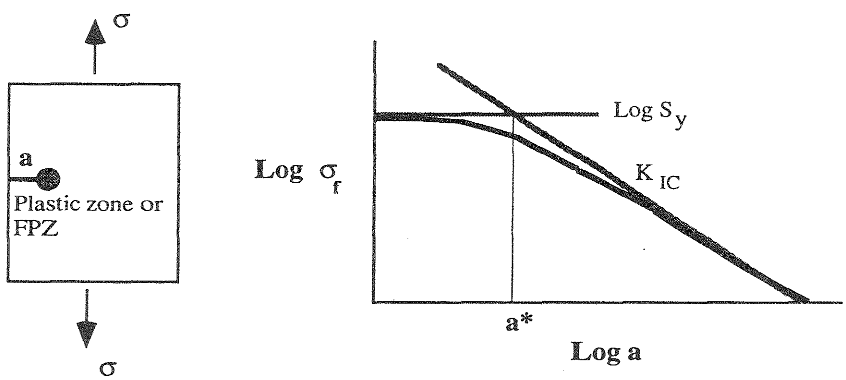

Fig. 1. Tensile failure of a large metal plate with a small edge crack

An interesting point in Fig. 1 is the reference crack length a* given by the intersection of S_y and K_{IC} criteria. From the basic LEFM equation for stress intensity factors, it is obtained that:

$$a^* = \frac{1}{\pi Y^2} \cdot \left(\frac{K_{IC}}{S_y} \right)^2 \qquad (1)$$

where the geometry factor Y is a constant (= 1.12) for the case shown in Fig. 1, and hence the reference crack a* is a material constant determined solely by the fracture toughness K_{IC} and yield strength S_y.

Obviously, an abrupt transition from ductile to brittle failure can never occur. Elastic/plastic failure needs to be considered as indicated by the solid curve in Fig. 1 with asymptotic behaviours at two extremes: a << a* and a >> a*. If a << a*, the very short crack can be completely shielded by the crack-tip plastic zone or fracture process zone (FPZ) commonly used for concrete-like materials so that the S_y criterion prevails. On the other hand, if a >> a*, the crack-tip plastic zone is very small in comparison with the crack size and other material dimension so that the K_{IC} criterion prevails. When a is around a*, elastic/plastic fracture occurs.

A simple asymptotic function has been assumed based on such considerations of the a/a* ratio and elastic/plastic fracture (Hu, 1998):

$$\sigma_f = \frac{S_y}{\sqrt{1+a/a^*}} \tag{2}$$

It can be easily proven from Eqns (1) and (2) that Eqn (2) also satisfies both the S_y and K_{IC} criteria when a << a* and a >> a*.

Now, let us adopt the common notations used for concrete to replace those parameters in Eqns (1) and (2). Eqn (1) becomes:

$$a^* \propto \left(\frac{K_{IC}}{S_y}\right)^2 \leftrightarrow \frac{E \cdot G_f}{f_t^2} = l_{ch} \tag{3}$$

As usual, f_t is the tensile strength, G_f the specific fracture energy and E the Young's modulus. Eqn (3) shows that the reference crack a* introduced for the ductile to brittle failure transition of metals is the same as the characteristic length of concrete defined by Hillerborg et al (1976). Thus for concrete, Eqn (2) can be rewritten as:

$$\sigma_N = \frac{\alpha \cdot f_t}{\sqrt{1+\beta \cdot \frac{a}{l_{ch}}}} \tag{4}$$

where α and β are dimensionless constants that depend on the geometry of test samples. Here, the nominal fracture strength of concrete σ_N is used to replace the tensile failure strength of metal σ_f. The parameter α

is introduced because σ_N is not necessarily measured under pure tensile conditions. It can be seen that Eqn (4) is almost identical to the following size effect law for geometrically similar specimens originally obtained from dimensional arguments by Bazant (1984):

$$\sigma_N = \frac{A \cdot f_t}{\sqrt{1 + B \cdot W}} \tag{5}$$

where A and B are constants depending on the geometry and material properties, and W is the specimen width or a characteristic specimen dimension. Obviously, α is the same as A. However, B in Eqn (5) has the length dimension, and it appears that the a/l_{ch} ratio in Eqn (4) has a similar role of W in Eqn (5).

2.2 Results from experiment and other model

Now, let us see how Eqn (4) originated from Eqn (2) can be applied to concrete-like materials. Experimental results on the critical stress intensity factor K_C of various cement paste specimens from Higgins and Bailey (1976) have been studied by Cotterell and Mai (1987, 1996). For convenience, their results are shown in Fig. 2.

To show the influence of small edge cracks and avoid the influence of the back surface, results from small a/d ratios are considered only. Since nearly all the experimental K_C results for various d values are available for a/d around 0.2 and it is sufficiently less than 1, the average experimental results at a/d = 0.2 are chosen to test the validity of Eqn (4). As usual, the standard stress intensity factor expression should be used (as α has been introduced in Eqn (4) to account for different stress conditions):

$$K_C = \sigma_N \cdot Y \cdot \sqrt{\pi a} \tag{6}$$

The size effect in the toughness K_C is determined by the nominal strength σ_N given by Eqn (4). For a given a/d ratio, $a \propto d$, and it can be derived from Eqns (4) and (6) that:

$$K_C = \sqrt{\frac{\alpha_1 \cdot d}{1 + \beta_1 d}} \tag{7}$$

where α_1 and β_1 are geometrical and material constants since the material constants f_t and l_{ch} have been combined into them. As expected, Eqn (7) shows that the size effect disappears when d is very large. And the true size-independent fracture toughness K_{IC} is given by:

$$K_{IC} = \sqrt{\alpha_1/\beta_1} = \sqrt{E \cdot G_f} \quad (8)$$

The significance of Eqn (7) is that it can be used to determine K_{IC} even if only small specimens with limited d values are tested (because the geometrical and material constants α_1 and β_1 are independent of the specimen size). Therefore, Eqn (7) not only indicates the size effect in K_C, but also gives the true size-independent fracture toughness K_{IC}. Relevant results from Fig. 2 and Eqn (7) are listed in Table 1.

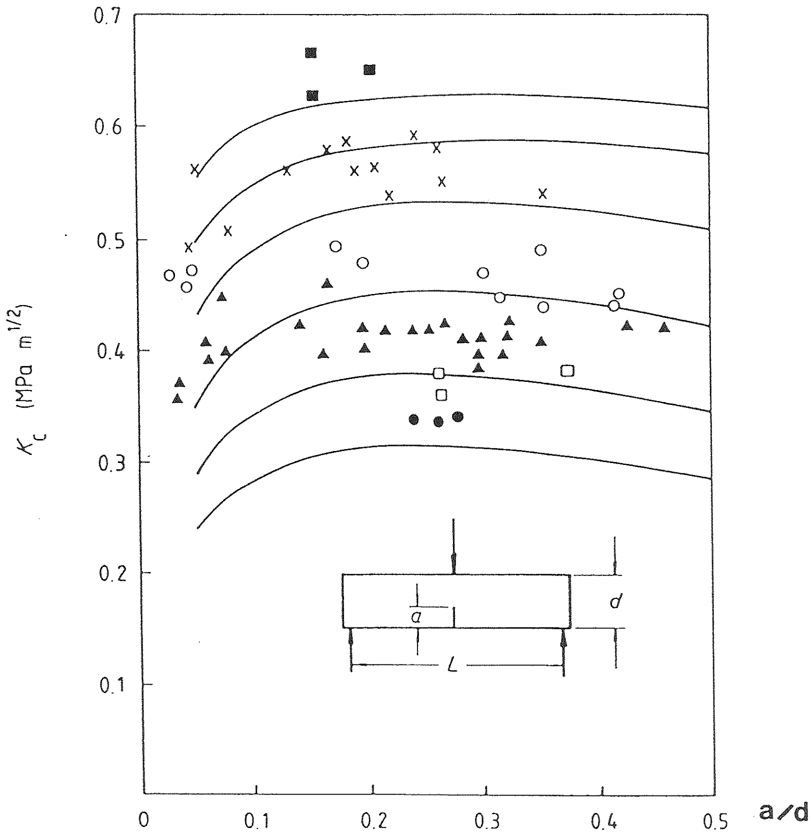

Fig. 2. Critical stress intensity factor as function of a/d ratio. d (mm) = 5 ●, 8 □, 14 ▲, 28 O, 56 X, 110 ■, after Cotterell and Mai (1987, 1996) and Higgins and Bailey (1976)

It is clear that with constant α_1 and β_1 for a/d = 0.2, Eqn (7) fits the experimental results very well, and it is even closer to the crack growth resistance K_R-curve model developed independently by Cotterell and

Mai (1987, 1996). It is even more interesting to see that Eqn (7) can give a perfect fit to their analytical results for a/d = 0.05 with constant α_1 and β_1. Substituting the α_1 and β_1 results in Table 1 into Eqn (8), it is obtained that K_{IC} = 0.65 and 0.62 MPa√m for a/d = 0.2 and 0.05. K_{IC} at a/d = 0.1 is √(0.019/0.044) = 0.66 MPa√m . These results are indeed very close, and almost identical to the plateau toughness value of the K_R-curve for very large specimens obtained by Cotterell and Mai (1987, 1996).

Table 1. Fitting of toughness results in Fig. 2 (all in MPa√m)

	a/d = 0.2			a/d = 0.05	
d (mm)	K_C (exp.)[1]	K_C (theo.)[2]	K_C (theo.)[3]	K_C (theo.)[4]	K_C (theo.)[3]
5	0.34	0.32	0.31	0.24	0.24
8	0.37	0.38	0.38	0.29	0.29
14	0.41	0.45	0.45	0.35	0.35
28	0.49	0.52	0.52	0.43	0.43
56	0.57	0.57	0.58	0.50	0.50
110	0.65	0.61	0.62	0.55	0.55

[1]Experimental results from Higgins and Bailey (1976)
[2]Results from Eqn (7) with α_1 = 0.027 and β_1 = 0.064
[3]Analytical results from Cotterell and Mai (1987)
[4]Results from Eqn (7) with α_1 = 0.013 and β_1 = 0.034

3 Crack Tip Close To Free Back Surface

3.1 Typical failure of metals

As before, let us first consider metal failures. Obviously, a plastic zone has a profound influence when a crack tip is very close to the free back surface of a test sample, as in Fig. 3 (a).

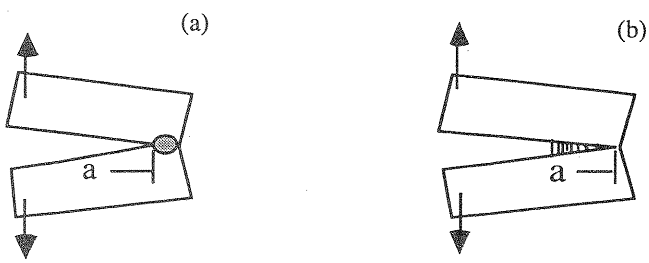

Fig. 3. (a) Large scale plastic yielding in a metal specimen
(b) large scale bridging in brittle-matrix-composite

Everyone knows in this case fracture is controlled by large scale yielding even if the bulk of the specimen is still elastic. The K_{IC} or K_C criterion simply does not apply. A plastic energy criterion such as the "specific work of fracture" model (Mai and Cotterell (1980), Atkins and Mai (1988)) should be used.

The questions are: in similar situations can the K-concept still be used for concrete-like materials, and is the K_R-curve approach based on the integral of bridging stresses with a constant G_f applicable? It appears this has seldom been questioned until recently (Hu (1997)).

3.2 Fibre-reinforced brittle matrix composites

For these materials, the crack-tip plastic zone is very limited, and can be ignored in comparison with the large scale crack bridging behind the crack-tip. Adopting the common resistance R-curve definition and applying the K-superposition principle to include the bridging stress contribution K_b, one routinely obtains that:

$$K_R = K_a(P,a) = K_{IC} + K_b(\sigma_b,a) \qquad (9)$$

where K_{IC} is the toughness for the matrix material. LEFM equations for K are derived for stress-free cracks, i.e. the applied load P approaches to 0 when the crack a approaches the free back surface so that $K_a(P,a) \equiv K_{IC}$, or the toughness is independent of the initial crack length. However, LEFM may not be valid for bridged cracks. For instance, even if the fictitious crack tip in Fig. 3(b) is right at the back surface, P is still bigger than zero because the bridging stress $\sigma_b \neq 0$, which makes $K_b(\sigma_b,a) \rightarrow \infty$, or $K_a(P,a) = K_R \rightarrow \infty$. In this case, a high K_R does not mean more energy is dissipated, and any associated size effects (e.g. a limited specimen width in comparison of the crack length, or when a crack approaches the back surface) are more likely artefacts. It is rational to argue that the situations in Fig. 3(a) & (b) should be treated exactly the same, i.e. LEFM equations should not be used if either a plastic zone or a crack-bridging zone is near the specimen back surface as the problem is no longer elastic. If Eqn (9) were used for cases similar to that in Fig. 3(b), severe size effects in toughness would be observed (e.g. Cotterell and Mai (1996)).

3.2 Concrete-like materials

It has been observed by Hu and Wittmann (1990, 1992) that when a crack is close to the back surface of a mortar specimen, the length of a FPZ or bridging zone is reduced. Obviously, the limited remaining specimen ligament imposes a constraint on further development of the bridging zone, which shows unmistakably the effect of a free edge (the specimen back surface).

The fictitious crack model proposed by Hillerborg et al (1976) has been widely adopted for concrete fracture. The mechanism behind the idealised bridged crack model is that the FPZ in concrete-like materials has a finite width within which bridging stresses are generated through crack-surface interlocking and friction (e.g. Mai and Lawn (1987), Hu and Mai (1992)). It is possible that when the FPZ length is reduced, so is its width as illustrated in Fig. 4(b). The later influences the specific energy G_f and then the crack bridging stresses. As a result, smaller G_f values are obtained if cracks are too close to the back surfaces (Hu and Wittmann (1992)).

Since the results from long cracks close to the back surfaces show the influence of a free edge (back surface), they should be similar to those obtained from short edge cracks showing the influence of the other free edge (front surface). The results in Fig. 2 show just that!

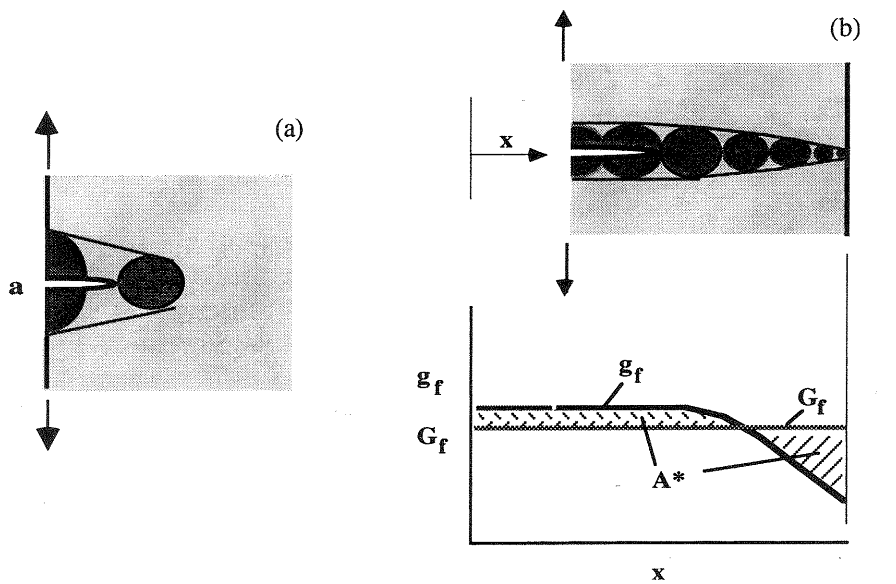

Fig. 4. (a) Contour of FPZ for short edge cracks and (b) contour of FPZ for cracks close to the free back surface.

In order to derive the size independent G_f values for concrete-like materials, Hu and Wittmann (1992) have introduced a local energy g_f function to account for the variation in the FPZ width. Only the normal G_f tests are required to determine $G_f = G_f(a)$ as a function of the initial notch length a. The local energy distribution function $g_f(a)$, as a function of the position parameter a, can then be determined as follows:

$$g_f(a) = G_f(a) - (1 - a/W) \cdot \frac{dG_f(a)}{d(a/W)} \qquad (10)$$

The above equation can be better understood with the sketch in Fig. 4(b). For a given specimen with an initial notch length a, one measurement of G_f is obtained, which is the averaged value of the local energy function g_f. G_f will vary with the notch length a because of the "size effects". By testing specimens with different notch lengths, the function $G_f(a)$ is determined. Then $g_f(a)$, as a function of the location parameter a, can be evaluated from Eqn (10). The maximum g_f can be taken as the size-independent G_f value for large structures.

4 Discussion and Concluding Remarks

It is well-known that the thickness B of a metal specimen has to satisfy the condition, $B \geq B^* = 2.5(K_{IC}/S_y)^2$, to ensure a thickness, or size, independent toughness measurement. From Eqn (1), it can be obtained that $B^* = 10a^*$ for $Y = 1.12$. In other words, to restrict the influence of two free side surfaces and ensure plane strain fracture, the thickness has to be ten times of the ductile/brittle transition crack size a*. Similarly, if $a/a^* \geq 10$, the influence of the free front surface can be neglected, as shown by Eqn (2). It is clear that the size effect in toughness is not unique to concrete. The difference is it is interpreted in terms of elastic/plastic fracture for metals. The key parameter for the size effects or fracture transition is the reference crack size a* or characteristic length l_{ch}. For steels, typical $K_{IC} = 60$ MPa$\sqrt{m}$ and $S_y = 700$ MPa. For advanced structural ceramics, e.g. alumina and zirconia, $K_{IC} = 4$ MPa$\sqrt{m}$ and $\sigma_f > 1000$ MPa. Using these parameters, a* has been calculated from Eqn (1) and listed in Table 2 together the typical l_{ch} for various cementitious materials.

Table. 2 Approximate a* and l_{ch} for various materials

ceramic	steel (← a*)	cement paste (l_{ch} →)	mortar[1]	concrete[1]	fibre reinforced mortar[1]
< 4 μm	2 mm	10 mm	90 mm	200 mm	> 2000 mm

[1]After Cotterell and Mai (1996)

It is self-evident that because of the extremely small a*, surface polishing is important for ceramics to avoid cracking due to surface scratches. Since the inherent processing defects in ceramics are normally bigger than 4 μm, fracture of ceramics is always controlled by the K_{IC} criterion. However, due to the size variation in their inherent defects, the strengths of ceramics become size-dependent. Steels seldom contain any processing defects bigger than 2 mm so that normal tensile failure is always controlled by the S_y criterion. For an artificial crack shorter than 20 mm, elastic/plastic fracture, or free edge or size effects, is observed. For steels, specimens with the width of 50 mm or bigger are often used to determine K_{IC}. That would transfer to concrete specimens of 5 m! Therefore, size effects will always be observed in laboratory, and Eqn (7) derived from Eqn (2) or (4) will always have a special role in applications of fracture mechanics to concrete.

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