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DIFFERENT ASPECTS OF THE SIZE EFFECT IN CONCRETE AND RC STRUCTURES

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Abstract
There are many experimental evidence on the existence of the size effects in concrete and reinforced concrete (RC) structures. It is difficult to derive a general size effect formula that would cover complete size range and all structural geometries. However, it may be useful to classify structures and related size effect formulas into two typical classes: (1) for structures which exhibit unstable crack growth and (2) for structures which before failure show a stable cracking. In the present paper the size effect on the nominal strength for beams and columns is discussed. Furthermore, some additional important aspects of the size effect are also considered: minimum reinforcement requirement, influence and importance of the distributed reinforcement, ductility and contribution of the concrete tensile resistance to the overall resistance of RC beams.
Key words: Beams, columns, concrete, ductility, fracture, reinforcement.

1 Introduction

Relative change (decrease) of the structural properties (peak resistance, ductility, etc.) when the structure size increases is known as the size effect. In quasibrittle materials such as concrete it is a well known phenomenon and there are a number of experimental and theoretical studies (Kani, 1967; Bazant, 1984; Bazant, Ozbolt and Eligehausen, 1994) which confirm existence of it. Unfortunately, in most experiments the size effect was studied for rather small structures for which it exists independent of the structure type. Therefore, more experimental and theoretical studies for larger structures of different geometries are needed.

The size effect in concrete and reinforced concrete structures is controlled by balance between the energy released from the structure as a consequence of concrete cracking and the energy consumed by the material. Assuming validity of the linear elastic fracture mechanics (LEFM) for brittle materials, two typical structural configurations (geometries) exist: (1) Structures which after the crack initiation exhibit an unstable crack growth i.e. at constant nominal stress the stress intensity factor increases when the crack length increases and (2) structures for which the crack after initiation grows in a stable manner until the critical length is reached i.e. at constant nominal stress the stress intensity factor decreases when the crack length increases.

Two above mentioned types of the geometries are limit cases where it is assumed that the size of the fracture process zone (FPZ) is negligible compared to the structure size (brittle materials). However, for concrete and reinforced concrete structures one has to account for some nonlinear effects such as: relatively large FPZ, influence of the reinforcement and bond between concrete and reinforcement, enviromental conditions etc.

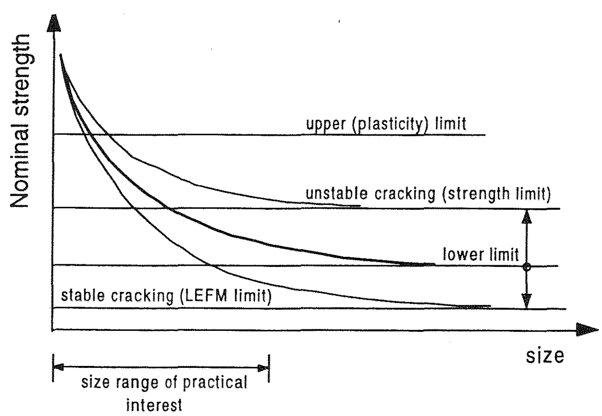

Fig. 1. Size effect formula for structures which exhibit: a) stable crack growth and b) unstable crack growth

When accounting for the nonlinear effects mentioned above, the nominal strength of any initially unnotched structure has upper (plasticity) and lower (strength) limit (Ozbolt, 1995; see Fig. 1). The size effect on the nominal strength for unnotched geometries which exhibit stable crack growth as well as for all geometries with proportionally scaled flaw is well defined by Bazant's size effect formula (Bazant, 1984). For large structures of these type the nominal strength yields to the lower limit which is relatively to the upper (plasticity) limit small. For these structures the size effect is strong in a broad size range. On the contrary, larger structures which exhibit unstable crack growth have a lower (strength) limit that is compared to the plasticity limit relatively large. Consequently, LEFM type of the size effect formula can not be used (Ozbolt, 1995). For these structures the size effect on the nominal strength exists only for limited size range (Ozbolt, 1995; see Fig. 1).

Besides the nominal strength the size effect has also some other aspects which are important for the structural safety. For instance, influence of the structure size on the required minimum reinforcement ratio, the role of the distributed reinforcement in large structures, ductility of RC structures and other still open questions. To demonstrate some of these effects an extensive numerical study for concrete and RC structures of different sizes was carried out (Ozbolt, 1995). In the following sections some of the results for RC beams and columns are shown and discussed.

2 Bending of slender RC beams

To investigate fracture behavior of RC beams of different sizes the numerical fracture finite element analysis on geometrically similar RC beams with different reinforcement ratios was carried out (Ozbolt, 1995). The analysis was performed by the finite element code *MASA* which is based on the nonlocal microplane model (Ozbolt and Bazant, 1996; Ozbolt et al., 1997). In the analysis were used four-node plane finite elements.

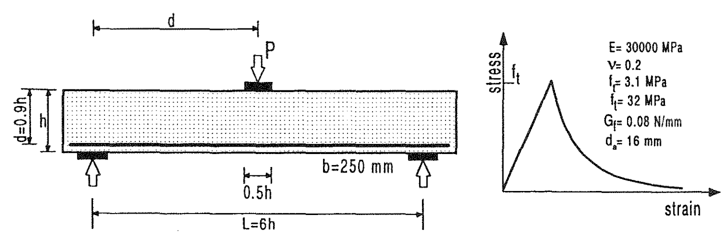

Fig. 2. Geometry of the beam and uniaxial tensile stress-strain curve used in the analysis

The numerical study was carried out for reinforced concrete beams loaded in three-point bending. Geometrically similar beams of five different sizes (h= 100, 200, 400, 800 and 1600 mm; see Fig. 2) were analyzed. The beam width (b= 100 mm) as well as the span-beam depth ratio (L/h= 6) were kept constant. The reinforcement ratio (μ $=100 A_s/(bh)$) was varied from 0 to 5% for each beam size. The bending reinforcement was assumed to be placed at the beam bottom with an concrete cover of $0.1h$ (see Fig. 2). Furthermore, the amount of distributed reinforcement was varied as well.

Concrete properties employed in the analysis were constant for all beam sizes and taken as follows: uniaxial tensile strength f_t= 3.1 MPa, uniaxial compressive strength f_c= 32 MPa, fracture energy G_F= 0.08 N/mm and maximum aggregate size d_a= 16 mm. An ideally elasto-plastic stress-strain relationship for steel was adopted with Young's modulus E_s= 210000 MPa and yield limit σ_y= 420 MPa. The reinforcement was introduced in a smeared sense i.e. as a layer(s) of elements in the bottom zone of the beam. Although a bond-slip relationship between the steel and concrete was not explicitly specified, it was taken into account in an integral form through the rows of finite elements below and above the reinforcement.

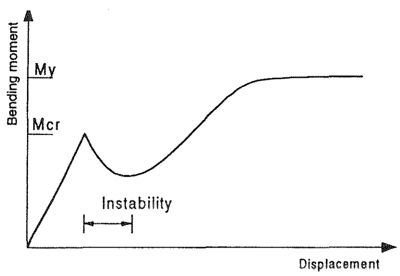

Fig. 3. Unstable beam response after M_{cr} is reached

2.1 Minimum reinforcement

Under minimum reinforcement one understands concentrated bending reinforced area placed at the beam bottom which provides: (1) Stable beam response after M_{cr} is reached i.e. the load-displacement curve after approaching M_{cr} should not exhibit significant load drop (energy equlibrium; see Fig. 3) and (2) the ultimate bending moment due to the yielding of reinforcement should be approximately equal or larger than the bending moment at initiation of the first bending crack ($M_Y \geq M_{cr}$, stress equlibrium; see Fig. 3).

Fig. 6 shows required minimum reinforcement ratio as a function of the beam size obtained in the numerical analysis. The distributed reinforcement was designed according to the CEB-FIP (1990). For comparison the minimum reinforcemnet requirement according to CEB-FIP (1990) and ACI-318 (1989) design codes are plotted as well. Smaller beams without distributed reinforcement exhibit an decrease of the minimum reinforcement ratio (μ_{min}) with increase of the beam size. The reason is the size effect on the cracking moment M_{cr}. However, for beams larger than approximately 300 mm μ_{min} starts to increase with the increase of the beam depth. The increase is approximately proportional to the square root of the beam depth. There are two reasons for this: (1) Unstable (explosive) crack growth in large beams after the tensile strength at the beam bottom is reached and (2) relative decrease of the bond resistance when the beam size increases. These results are an extreme case since in the analysis the concrete was assumed to be rather brittle and the bond resistance volume was assumed to be constant and size independent. For comparison in Fig. 4 is also plotted the curve proposed by Carpinteri and coworkers (Bosco and Carpinteri, 1992). This curve is based on the stress equilibrium criteria and therefore predicts decrease of required μ_{min} when the beam size increases. Obviously, further tests on larger beams are needed to clarify the above aspects of the minimum reinforcement requirement.

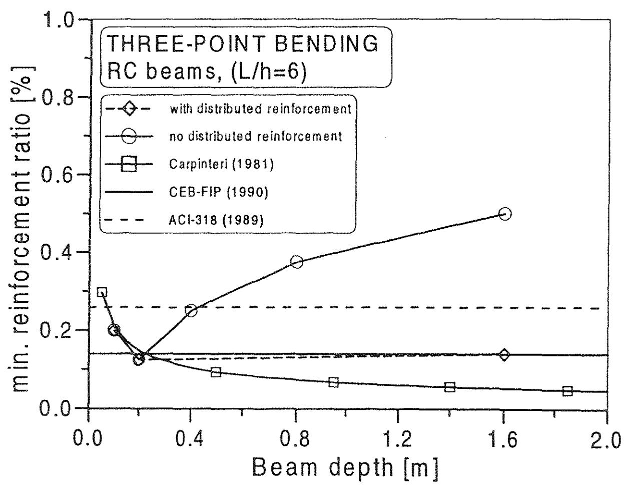

Fig. 4. Relation between depth of the beam and required minimum reinforcement ratio

From Fig. 4 it can be seen that μ_{min} is independent of the beam size if distributed reinforcement is present. In large beams distributed reinforcement stabilize the crack growth and improve the bond capacity. This is demonstrated in Fig. 5 which shows calculated load-displacement curves for large beam ($h=$ 1.6 m, $\mu =$ 0.14%) with and without distributed reinforcement. In contrast to the beam without distributed reinforcement, the beam with it exhibits an ductile response after the cracking moment is reached. The reason is the distribution of damage caused by concrete cracking over a larger volume what makes the consumption of structural energy release possible and assures an stable crack growth. In contrast to large beams in small beams one needs practically no distributed reinforcement since small beams are already without it relatively ductile.

Further aspect related to the minimum reinforcement requirement is the fact that because of the environmental conditions tensile strength of concrete in the practice can be relatively low. Consequently, the concrete is less brittle and the minimum reinforcement ratio should be independent of the size. To prove this further experiments in different environmental conditions are needed.

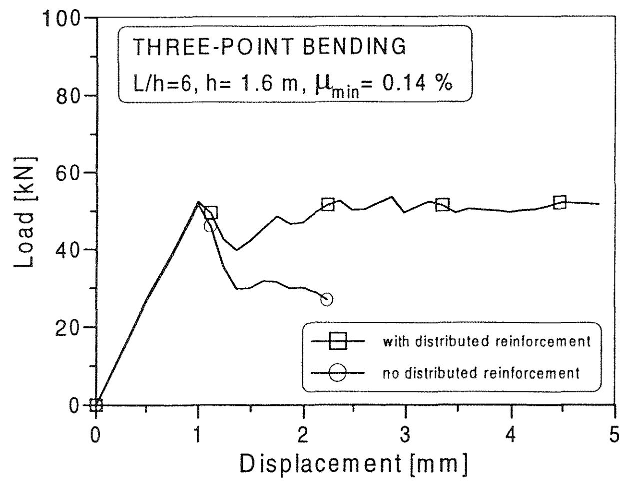

Fig. 5. Calculated load-displacement curve for large beam with and without distributed reinforcement

2.2 Contribution of concrete to the ultimate bending moment at ductile failure

To demonstrate the contribution of concrete to the peak resistance for beams with relatively small reinforcement ratio (μ= 0.25 -- 0.375 %) which fail in a ductile manner, the nominal bending moment which corresponds to the peak load is in Fig. 6a plotted as a function of the beam depth. The bending moment obtained from the numerical analysis is normalized to the lowest possible bending (yielding) moment calculated as:

$$M_Y = \sigma_y A_s (0.9d) \tag{1}$$

with d= effective beam depth. Eq. (1) is used in design practice and it neglects the contribution of concrete to the peak load. Fig. 6a shows that small RC beams with low reinforcement ratio have always higher peak resistance than the ultimate resistance predicted by (1).

The reason for the relatively high nominal bending moment in small beams is due to the contribution of concrete tensile resistance (see Fig. 7). This contribution, compared with the contribution of reinforcement (small reinforcement ratio) is relatively large and it is a consequence of the stable crack growth in small beams. When the beam size increases the contribution of concrete decreases and for very large beam the maximal bending moment is controlled only by the reinforcement. Note, however, that the numerical results probably show too high values for the contribution of concrete to the peak resistance. The reason is due to the high assumed tensile strength of concrete and relatively low yield stress of reinforcement (420 Mpa). Furthermore, in the analysis the hardening of the reinforcement was neglected. Beams with a low reinforcement ratio fail by rapture of steel. Therefore, depending on the steel hardening ratio the failure moment will increase and the relative contribution of concrete at peak load should decrease. Experimental studies are needed to clarify the contribution of the concrete tensile strength on the ultimate load for beams with low reinforcement ratio.

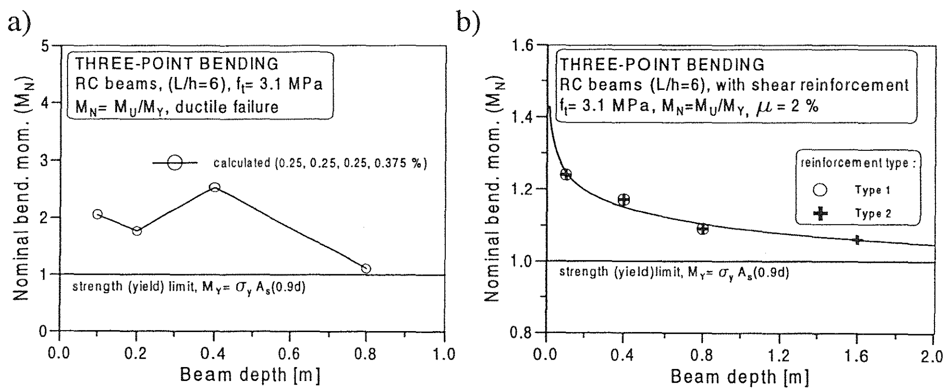

Fig. 6. The nominal bending moment at peak load as a function of the beam depth (ductile failure): a) low and b) high reinforcement ratio

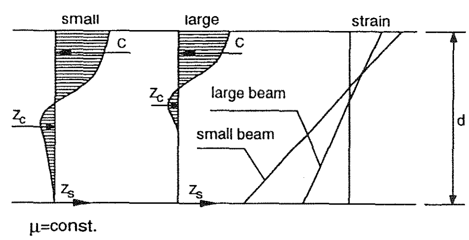

Fig. 7. Qualitative distribution of stresses and strains

Ductile failure of RC beams with higher reinforcement ratio (RC beams with $\mu = 2\ \%$) was investigated for the size range $h = 100$ to 1600 mm. To prevent diagonal shear failure the beams were in the left and the right third of the span reinforced by stirrups (ϕ 10/100 mm). In the mid third the beams were optionally provided by distributed reinforcement (type 2 with distributed reinforcement -- 8% of the main bending reinforcement, type 1 without it). This reinforcement is assumed to be ideally elasto-plastic with the same yield limit as for the bending reinforcement (420 MPa). In Fig. 6b is the nominal bending moment at peak load plotted as a function of the beam size. As can be seen, in contrast to the RC beams with relatively small reinforcement ratio, the size effect on the nominal bending moment is not significant. Namely, for doubling the beam depth (from 400 to 800 mm) the nominal strength decreases by about 5%. The reason for relatively small size effect in beams with higher reinforcement ratio is due to small contribution of the concrete tensile resistance.

3 Eccentric compression of concrete and RC columns

Behaviour of eccentrically loaded plain and RC columns of different sizes was recently experimentally and numerically investigated (Meyer, 1997; Ozbolt and Li, 1998). The experimental study was carried out for different concrete qualities and for different reinforcement ratios. For the constant column thickness of $b =$ 160 mm and slenderness $h/d = 2.5$ the width of the columns was varied from $d = 160$ to 480 mm. Columns were loaded by deformation control of the end cross sections. For RC columns the concrete cover was kept constant (for more detailes see Meyer, 1997). The numerical study of the same columns was performed by the use of the three-dimensional finite element code (*MASA*).

In Fig. 8a are plotted and compared the experimental and numerical results for nominal strength as a function of the column size. The figure shows that the size effect on the nominal strength exists. However, it is relatively weak and limited to smaller columns. This is valid not only for plain concrete but also for RC columns. The size effect for RC columns seems to be slightly stronger (Ozbolt and Li, 1998). The reason for relatively week size effect on the nominal strength is due to the unstable crack growth after the concrete compressive strength is reached.

Fig. 8b shows the relative nominal stress as a function of the maximal compressive and tensile strains for different plain concrete column sizes, measured in the experiment and in the analysis. It can be seen that with increase of the size ductility decreases. Similar as the nominal strength, decrease of ductility when the size increases is not significant, at least not for the size range investigated. The same tendency is also valid for RC columns especially if the stirrups are not spaced close enough (Ozbolt and Li, 1998).

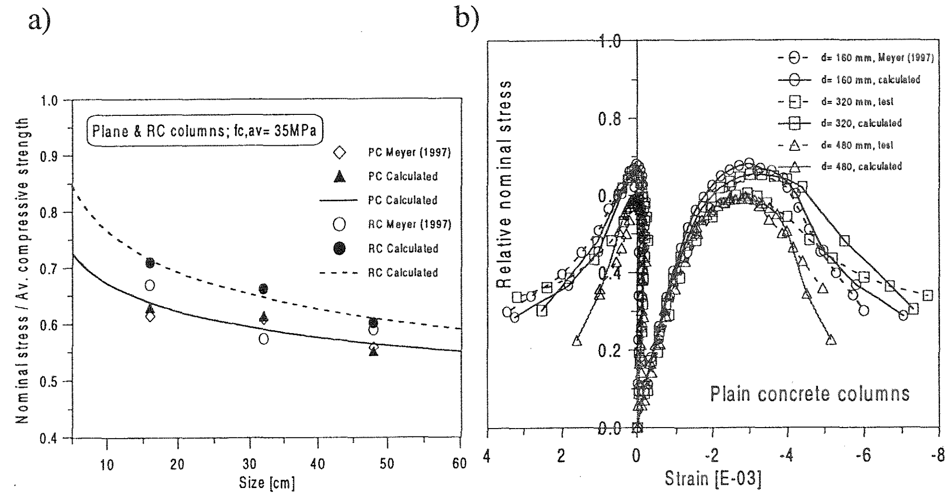

Fig. 8. Eccentric compression failure of plain concrete columns (average f_c = 35 Mpa): a) the relative nominal strength as a function of size and b) the relative nominal stress as a function of tensile and compressive strains measured over the total column length

4 Conclusions

- Each concrete or RC structure exhibits the size effect on the nominal strength. The size range in which it is strong depends on the structure type. If the stable crack growth in a broad size range is possible, for $d \rightarrow \infty$ the nominal strength yields approximately to zero. On the contrary, the nominal strength for large structures has a strength limit.
- Apart from the nominal strength, which for most concrete and RC structures does not yield to zero, the quantities which control structural ductility decrease when the structure size increases. Therefore, to describe behavior of concrete and RC structures in a general sense the size effect type formula of Bazant seems to be appropriate.
- Numerical analysis indicates that in large RC beams the required minimum bending reinforcement ratio increases when the beam size increases. However, for beams with distributed reinforcement, what is normal case in the practice, the required minimum bending reinforcement ratio seams to be independent of the beam size.

- There is a clear indication that the importance of the distributed reinforcement becomes more pronounced in large structures.
- In small beams with low reinforcement ratio that fails in a ductile manner contribution of concrete to the ultimate load is relatively high.
- The size effect on the nominal strength and on the deformationl capacity of concrete and RC columns is relatively small.

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