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SIZE EFFECTS OF REINFORCED CONCRETE BEAMS IN SHEAR

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Abstract
The theory of fracture mechanics enables to deal with the size effect problem more rationally. Some studies on the shear behavior of concrete beam exist. However, it is very difficult to find comprehensive experimental studies on the size effect of reinforced concrete beams.

The purpose of the present study is to explore more realistically the size effect of reinforced concrete beam under shear loading. To this end, a series of tests have been conducted, and the major test variables are the relative depth of beam, shear reinforcement ratio, and longitudinal reinforcement ratio. The test results indicate that the size effect is never small and exhibit rather large effect. A size effect formula is derived to predict the shear resistance of reinforced concrete beam. The proposed formula shows good correlation with experimental data. The present study indicates that less size effect is observed for moderately reinforced concrete beam.
Key words : size effect, shear, reinforced concrete

1 Introduction

The size effect of reinforced concrete beam in shear has been studied by several researchers(Bazant and Sun).

In regard to the size effect of shear, several formulas have been proposed and some of which are derived by dimensional analysis and similitude arguments from simplifying assumption, namely that energy loss due to cracking is a function of both the fracture length and the area of the cracking zone which is assumed to have a constant width at its front, proportional to the maximum aggregate size(Bazant and Oh).

The present study focuses on the size effect of reinforced concrete beam in shear. To this end, a comprehensive experimental study has been conducted. The test result is analyzed and a prediction formula derived.

2 Test procedure

2.1 Major parameters of experiments

Major experimental variables are the effective depth(size) of the beam(d), longitudinal reinforcement ratio(ρ), and shear reinforcement ratio(ρ_v).

The test specimens have compressive strength of 280 kg/cm^2(28 MPa), maximum aggregate size(d_a) of 25mm, and W/C ratio of 43%, respectively. The specimen is three-point loaded by Material Testing System(MTS).

2.2 Design and fabrication of test specimens

The dimensions and details of test specimens are shown in Tables 1 and 2. The longitudinal reinforcement ratio ρ is varied from zero to ρ_{max} by zero, ρ_{min}(200/f_y, where f_y in psi), 0.5ρ_{max}, ρ_{max}(0.75ρ_b, where ρ_b is balanced reinforcement ratio).

The shear reinforcement ratio ρ_v defined by A_v/bs, where A_v is area of one stirrup, b is width of the member, and s is spacing of stirrups, is appropriately increased according to the increase of ρ.

Table 1. Dimensions of test specimens

Type of Specimen	Length(L) (cm)	Span(S) (cm)	Width(b) (cm)	Effective Depth(d) (cm)	Height(h) (cm)
A	90	80	20	10	13
B	180	160	20	20	25
C	260	240	20	30	35
D	340	320	20	40	45
Shear span(a/d) is 4 for all specimens					

Table 2. Details of test specimens

Specimen Index	Longitudinal Reinforcement Ratio(ρ)	Shear Reinforcement Ratio(ρ_v)	Effective Depth(d) (cm)
R0D10	0	0	10
R0D20			20
R0D30			30
R0D40			40
R1D10	ρ_{min}	0.00150	10
R1D20			20
R1D30			30
R1D40			40
R2D10	$0.5\rho_{max}$	0.00194	10
R2D20			20
R2D30			30
R2D40			40
R3D10	ρ_{max}	0.00301	10
R3D20			20
R3D30			30
R3D40			40

3 Test results

To observe the size effect experimentally in shear behavior of reinforced concrete beam, the logarithmic value of relative shear, $\log(V_{max}/V_n)$, and relative beam depth, $\log(d/d_a)$, are plotted according to each steel ratio as shown in Fig. 1. The V_{max} is the measured maximum shear force and V_n is the nominal shear strength based on design code described below.

Fig. 1 indicates that the size effect is greatly observed. Fig. 2 shows that the normally reinforced beam($\rho=0.5\rho_{max}$) exhibits less size effect than other members.

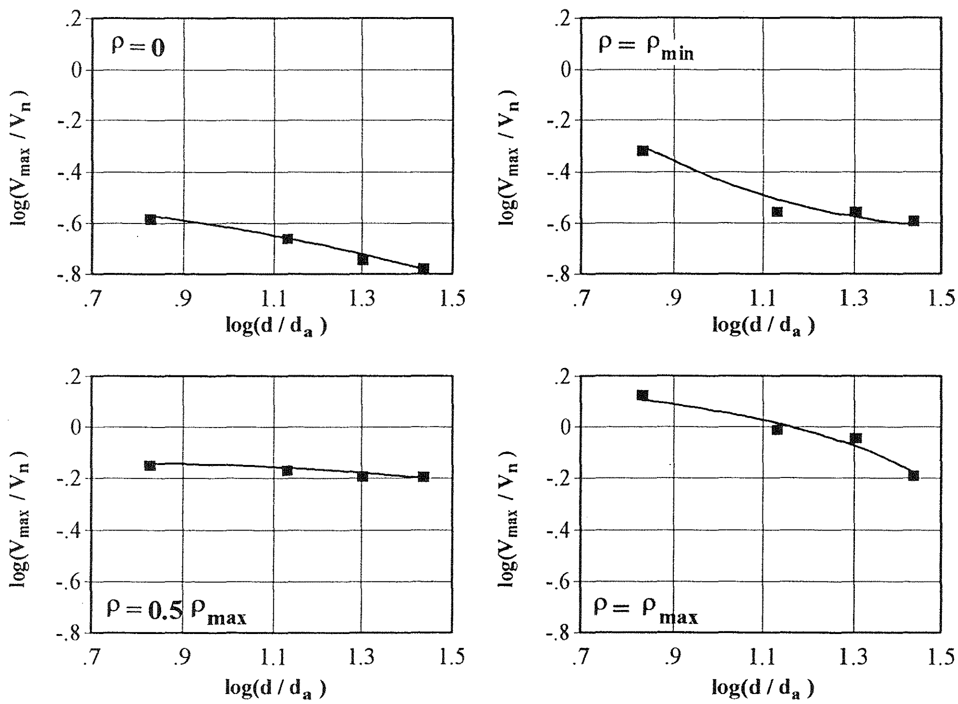

Fig. 1. Test results on the size effect of RC beams in shear for various steel ratios

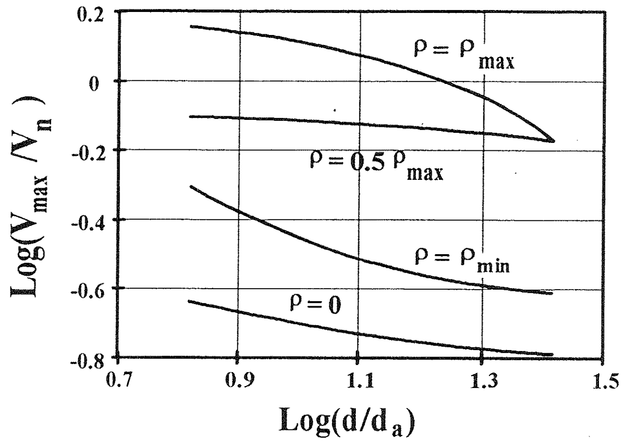

Fig. 2. Comparison of test results in shear

4 Theoretical prediction formula for size effect in shear

The size effect formula in shear strength of reinforced concrete beams can be written as follows.

$$V_{max} = V_n\left(\frac{\alpha_0}{\sqrt{1+\frac{d}{\lambda_0 d_a}}}\right) \tag{1}$$

where,

$$V_n = V_c + V_s \tag{2}$$

$$V_s = \frac{A_{s_v} f_y d}{s} \tag{3}$$

$$V_c = (0.50\sqrt{f_c'} + 176\frac{\rho V_u d}{M_u})bd \quad (kg) \tag{4}$$

In Egn (1), V_{max} is ultimate shear strength at failure, and V_c is shear strength of concrete given by ACI and KSCE codes. The coefficients α_0 and λ_0 are functions of the longitudinal reinforcement ratio ρ, shear reinforcement ratio ρ_v, and can be assumed as follows.

$$\alpha_0 = a_1 + a_2(\frac{\rho}{\rho_v}) \quad (5)$$

$$\lambda_0 = b_1 + b_2(\frac{\rho}{\rho_v}) + b_3(\frac{\rho}{\rho_v})^2 \quad (6)$$

From the measured values of test members, the values of α_0 and λ_0 in each longitudinal and shear reinforcement ratio can be calculated as the points in Fig. 3 and 4. Then, the unknowns in Eqns (5) and (6) can be determinded by the regression of ρ/ρ_v, α_0 and λ_0.

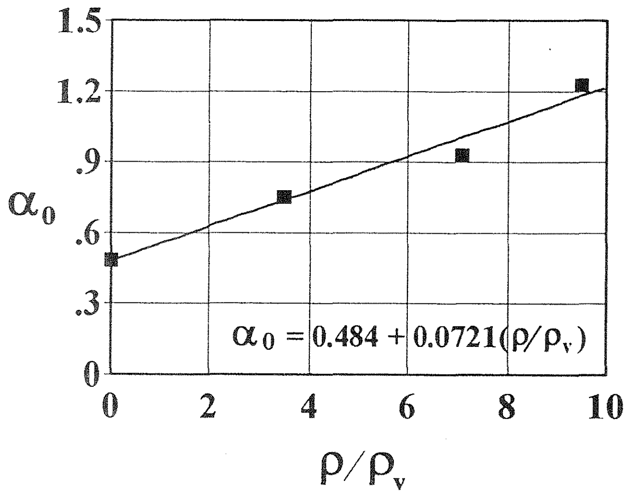

Fig. 3. α_0-ρ/ρ_v relation in size effect formula of shear

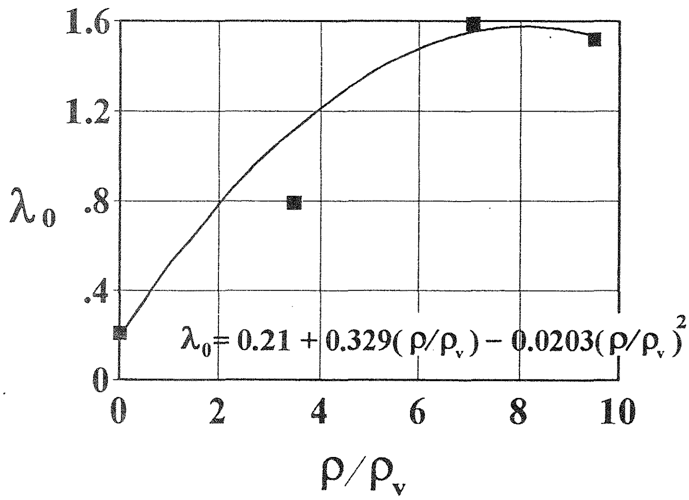

Fig. 4. λ_0-ρ/ρ_v relation in size effect formula of shear

To compare the test results with other studies, we define nominal shear strength as conventional form in which the same V_s, the steel contribution, based on the 45° Truss Model is used, but different V_c, the concrete contribution, is used depending on each study.

The shear strength equation proposed by Bazant, Kim, and Pfeiffer is,

$$V_c = [6.5\rho^{\frac{1}{3}}(\sqrt{f_c'} + 3000\sqrt{\frac{\rho}{(a/d)^5}})\frac{1 + \sqrt{0.2/d_a}}{\sqrt{1 + d/25d_a}}]bd \quad (lb) \qquad (7)$$

The shear strength equation proposed by Bazant and Sun is,

$$v_c = v_c^0\frac{1 + \sqrt{0.2/d_a}}{\sqrt{1 + 25(1+\rho_v/\rho_0)d_a}} \quad (lb/in^2)$$

$$V_c = v_c bd \quad (lb) \qquad (8)$$

where,

$$v_c^0 = 6.5\rho^{\frac{1}{3}}(\sqrt{f_c'} + 3000\sqrt{\frac{\rho}{(a/d)^5}})$$

$$\frac{1}{\rho_0} = a_0[1 + \tanh(2\frac{a}{d} - 5.6)]$$

The proposed formula in this study shows good correlation with test data and especially matches well in the case of ρ equal to $\rho_{max}(0.75\rho_b)$. However, other formulas show some deviation from test data.

5 Conclusion

The shear tests have been conducted in three point loading condition to study the size effect of reinforced concrete beam in shear. The major test variables are the effective depth of the beam(d), longitudinal reinforcement ratio(ρ), and shear reinforcement ratio(ρ_v). The size effect formula for the shear resistance of reinforced concrete beams is derived and compared with test data as well as other studies. The proposed formulas show relatively good agreement with test data. The present study indicates that the size effect is generally pronounced in shear of reinforced concrete beam and the effect is little bit less for moderately reinforced concrete beams.

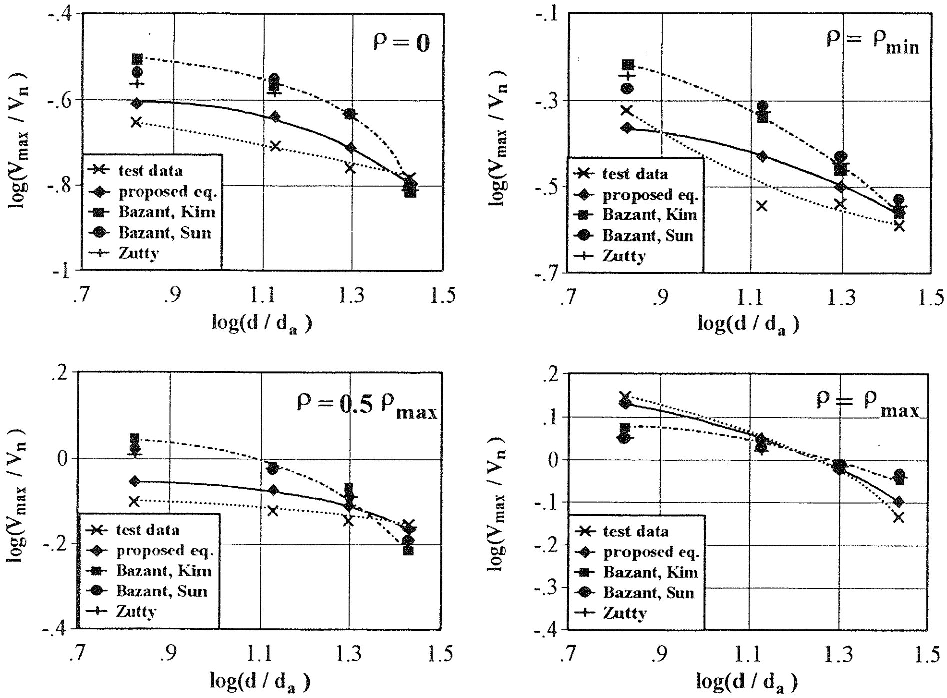

Fig. 5. Comparison of proposed formula with other studies

6 References

Bazant, Z.P., Kim, J.K. and Pfeiffer, P.A. (1986) Determination of properties from size effect test, **Journal of Structural Engineering**, ASCE, 112, 2, 289-307.

Bazant, Z.P. and Oh, B.H. (1983) Crack band theory in fracture of concrete, **Materials and Structures**, 16, 93, 155-177.

Bazant, Z.P. and Sun, H.H. (1987) Size effect in diagonal shear failure : influence of aggregate size and stirrups, **ACI Material Journal**, 84, 4, 259-272.

Oh, B.H. (1984) Nonlinear fracture analysis of concrete structures, **KSCE Journal**, 4, 4, 1-11.