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NONLINEAR SIZE EFFECT ANALYSIS OF HEADED ANCHORS EMBEDED IN CONCRETE BLOCKS UNDER VARIOUS SUPPORTING CONDITIONS

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Abstract

In this paper the size effect on both ultimate strength and failure cone surface formation in pull-out tests under various boundary conditions was studied through precise analysis. This analysis shows that the fictitious crack approach can be extended with two orthogonal rod elements as a new technique, which can successfully predict the size effect on the pull-out strength. The numerical investigation has been carried out by a computer simulation using our original program ANACS. Using Arc-length technique the post peak behavior was captured well even when the snap back instability occures. To judge the validity of the results, the numerical results was compared with the empirical equation of Eligehausen and Sawade (1989), and test results of RILEM report (1991).
Key words: concrete fracture, size effect, finite element method, discrete model, fictitious crack, rod elements, pull-out tests, headed anchors

1 Introduction

This paper represents an energy approach of the fracture behavior of concrete where the failure cone is simulated by a discrete crack sewed by two orthogonal rod elements. The experimental results are available only for embedded depths up to 150mm. Therefore, the comparison with experimental results is presented only for shallow embedded depths up to 150mm.

In this investigation the arc-length method was employed to capture the post peak behavior, whether the failure type is ductile or brittle with or without snap-back behavior. The pull-out problem is physically three dimensional, however, it can be considered as axially symmetric if its geometry and material properties are independent of the circumferential coordinate θ.

2 Finite element modeling and fictitious crack simulation

The concrete elements around the crack path are assumed to follow an elastic stress-strain relation in tension. Then, the failure cone surface can be modeled based on the fictitious crack approach.

It is noted that, there is no scope of compression failure. The studied cases in this paper are concerned with the tensile cone failure of concrete, therefore compressive stresses are confirmed to remain within the elastic range.

In the present size effect analysis, the concrete element size is taken proportional to the specimen size. Since the fracture energy model is implemented in the program and the fracture energy G_F is kept constant for all concrete blocks of different sizes, the mesh sensitivity is considered to be insignificant, Morgan, Niwa and Tanabe (1997).

The fracture zone is modeled by two orthogonal rod elements (Fig. 1). Each rod element can be considered as a virtual element with unit length (L=1). The rod element perpendicular to failure cone surface exhibits nonlinear stress-strain behavior of concrete by using the 1/4th softening curve, Morgan, Niwa, and Tanabe (1997).

Although many investigations are currently in progress, as that by Karihaloo (1995), unfortunately the mode II fracture properties of concrete are not yet well established. Therefore, a very simple model has been assigned to the rod element parallel to failure cone surface by Morgan, Niwa, and Tanabe (1997).

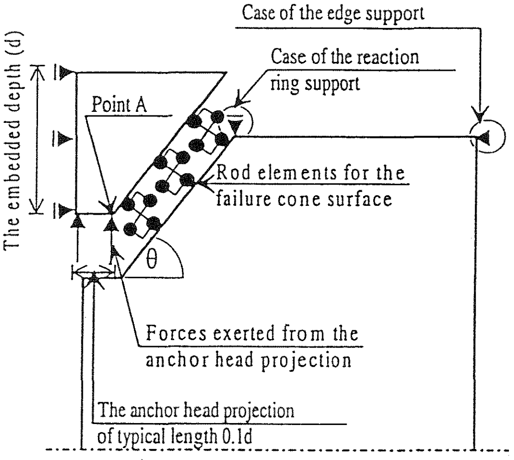

Fig. 1 The failure surface, and the supporting conditions

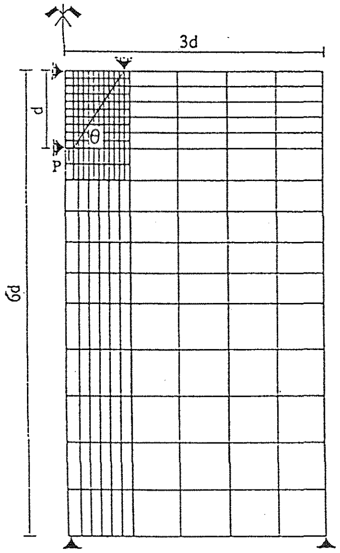

Fig. 2 Typical mesh for the concrete block

The value of the ultimate stress chosen for the rod element which is parallel to failure cone surface is equal to 30.0MPa. This large value reflects the fact that the crack propagation and the ultimate strength mainly depend on the tensile fracture energy stored in the rod element perpendicular to failure cone surface.

3 The problem stated

A typical geometry of the cylindrical concrete block and the mesh division are illustrated in Fig. 2. The finite element model used for the pull-out analysis consists of 268 elements including quadrilateral elements with 6,7 and 8 nodes. The number of nodes in this model is 858. The geometrical dimensions of the analyzed model are determined according to the RILEM requirements of Anchor Bolts (1991).

In this investigation, three kinds of supporting conditions are considered. The first case is analyzed by considering the reaction ring as an inverted roller support on the top surface of the specimen (Fig. 1), to simulate the standard practice of pull-out tests. The second case is analyzed without a ring support, but by considering a hinged support at the top edge surface of the specimen (Fig. 1) to study the behavior of failure cone surface without the effect of the reaction ring support. The third case is analysed by considering neither the reaction ring support nor the top edge support, i.e. the top surface of the specimen is free from any restrictions. The third case is presumed to simulate the behavior of headed anchors in real practice.

4 The size effect on the inclination of failure cone surface

Nine embedded depths are considered, as d=50, 150, 450, 600, 1000, 2000, 5000, 10000 and 12500 mm. Concrete properties are identical for all nine concrete blocks; as compressive strength f_c'=30.0MPa, tensile strength f_t=3.0MPa, fracture energy G_F=100N/m and Young's modulus E_C=30.0GPa.

The failure cone surface is assumed to be oriented at angles between (26°-76°), thus 11 finite element models are rearranged, for every embedded depth in every supporting condition with respect to the chosen crack path to trace all crack inclinations. For the sake of studying the size effect on the crack inclination which gives the minimum pull-out strength for all boundary condition cases considered, 297 types of finite element models are prepared.

Results are shown in Figs. 3-11. It has been found that in the case of the reaction ring support the inclination of the failure cone surface to give the minimum pull-out strength is ranging between 53°-60° for embedded depths up to 5000mm as shown in Figs. 3-9.

On the other hand, for such large embedded depths as 10000mm and 12500mm, it has been found that the inclination of the crack surface for the minimum pull-out strength is ranging between 38°-51° as shown in Figs. 10 and 11. Moreover, in the case of the edge support, it is found that the inclination of the failure cone surface, for the minimum pull-out strength is about 60° for embedded depths up to 2000mm, and it is ranging between 51°-55° for large embedded depths such as 5000mm or more. Furthermore, in the case of the top surface free from restrictions, it is found that the inclination for the minimum pull-out strength is ranging between 60°-63.5° for the embedded depths up to 2000mm, and it is about 53° for deep embedded cases more than 5000mm.

From Figs. 3-11 it is noticed that the results of the second boundary condition and of the third are almost identical, and they give lower pull-out strengths than the case of the reaction ring support. Also, from all figures, it has been found that there is a significant change in the overall trend of the resulting pull-out strengths.

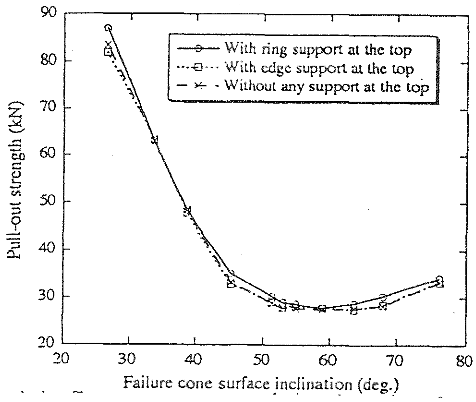

Fig. 3 Variation of pull-out strength with cone failure angle θ (d=50mm)

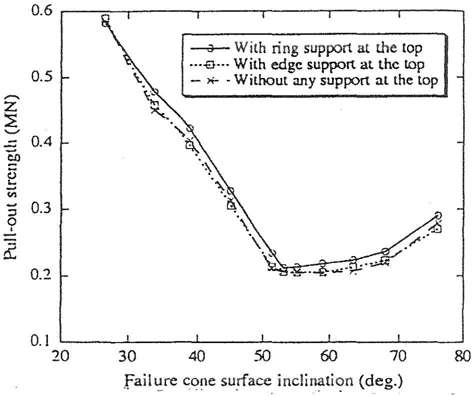

Fig. 4 Variation of pull-out strength with cone failure angle θ (d=150mm)

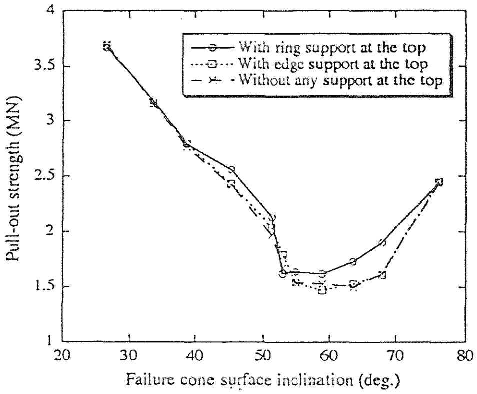

Fig. 5 Variation of pull-out strength with cone failure angle θ (d=450mm)

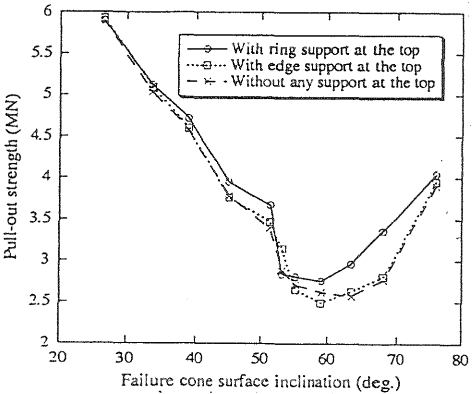

Fig. 6 Variation of pull-out strength with cone failure angle θ (d=600mm)

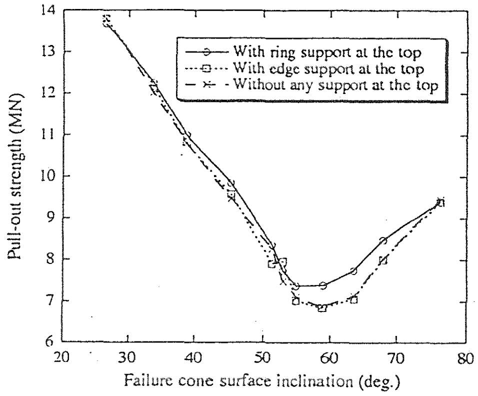

Fig. 7 Variation of pull-out strength with cone failure angle θ (d=1000mm)

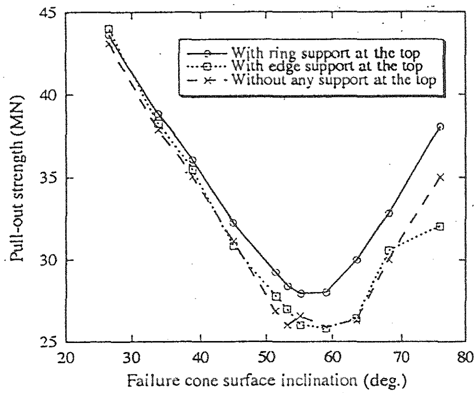

Fig. 8 Variation of pull-out strength with cone failure angle θ (d=2000mm)

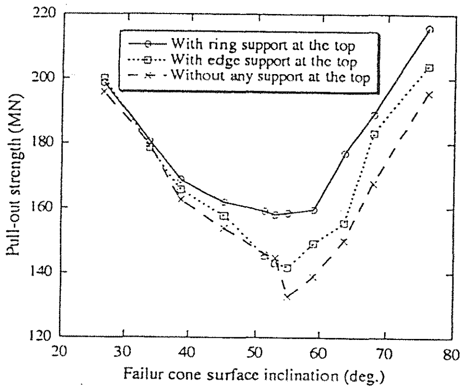

Fig. 9 Variation of pull-out strength with cone failure angle θ (d=5000mm)

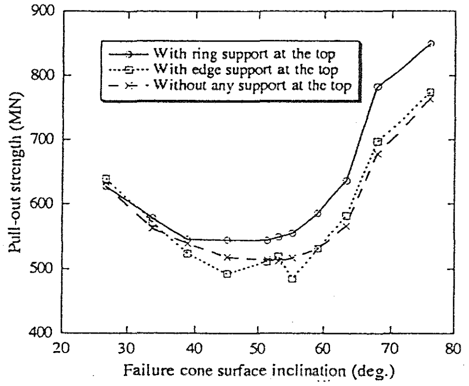

Fig. 10 Variation of pull-out strength with cone failure angle θ (d=10000mm)

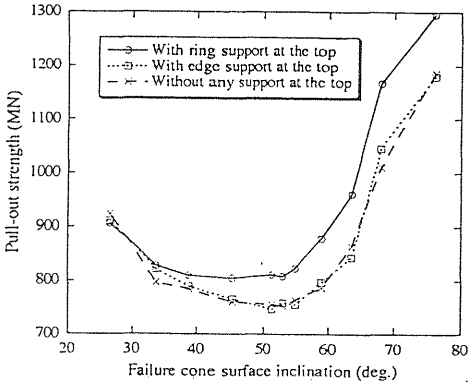

Fig. 11 Variation of pull-out strength with cone failure angle θ (d=12500mm)

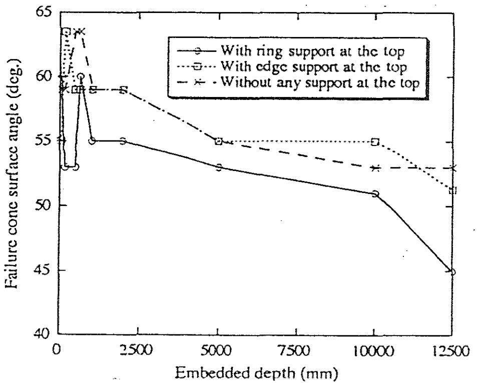

Fig. 12 The size effect on failure cone surface inclination angle θ

Fig. 12 shows that the size effect on the angle of failure cone surface. For such large embedded depths such as 10000mm and 12500mm, the cone failure surface inclination is getting small as compared with shallow embedded depths. Furthermore, from Fig.12 it can be noticed that the inclination of the failure cone surface, which gives the minimum pull-out strength in the second and third supporting condition is getting steeper than the case of the reaction ring support. Consequently, the smaller failure cone surface area becomes, the smaller ultimate pull-out strength becomes.

5 The effect of the anchor head on the ultimate pull-out strength

For further parametric study, the effect of the anchor head projection inside the concrete block on the resulting pull-out strength is studied.

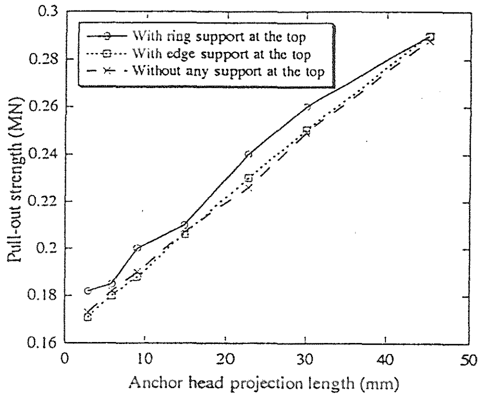

Fig. 13 Effect of anchor head projection

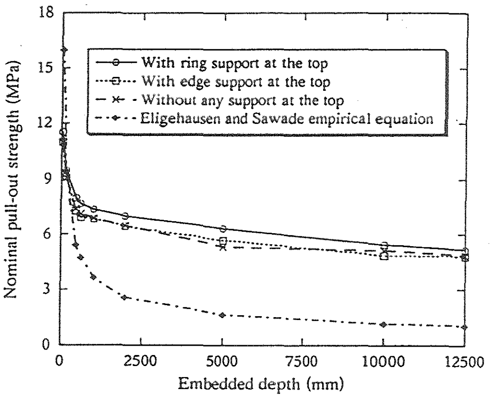

Fig. 14 Size effect analysis

To perform this study 7 finite element models are prepared for every supporting condition case. The embedded depth of this study is chosen 150mm, and the inclination of failure cone surface is taken as 53°. The anchor head projection has such values as 0.02d, 0.04d, 0.06d, 0.1d, 0.15d, 0.2d and 0.3d, where d is the embedded depth. The anchor head projection represented the distance from the left side of the specimen at the axis of symmetry to the crack position at point A in Fig. 1.

Fig. 13 shows the tendency of the resulting pull-out strength with the increase of the anchor head projection for all boundary conditions. From Fig. 13, it is apparent that no minimum pull-out strength can be detected from these analyses. Therefore, the anchor head projection of 0.1d, which is taken as a standard practice in many researches as Ozbolt and Eligehausen (1992) and RILEM report (1991), has been utilized for all preceding and following analyses.

6 Size effect analysis

The minimum pull-out strengths resulted from Figs. 3-11 are utilized for this study. Fig. 14 shows the tendency of the nominal pull-out strength with the increase of the embedded depth for all the boundary conditions. The decrease of the strength is known as the size effect. For the three boundary conditions in Fig. 14, the size effect moderates for large embedded depths and the nominal pull-out strength of the concrete blocks tends to reach a certain limit. Fig. 14 proves that the proposed analytical model can predict the size effect of the pull-out strength, due to the explicit difference between the detected pull-out strength for small and large embedded depths and moderating the size effect for large embedded depths.

The nominal pull-out strength in Fig. 14 is calculated by dividing the resulted minimum pull-out strength by the square of the embedded depth, as Ozbolt and Eligehausen (1992),

$$\sigma = N_u / d^2 \tag{1}$$

where σ is the nominal pull-out strength, N_u is the pull-out strength, and d is the embedded depth.

The results of the empirical equation, by Eligehausen and Sawade (1989) are given in Fig. 14. Their equation gives the relation between the induced pull-out strength N_u and the embedded length h_{ef}, as follows;

$$N_u = 2.1 (E\, G_F)^{0.5} h_{ef}^{\,1.5} \tag{2}$$

where E is the Young's modulus of concrete, G_F is the fracture energy of concrete and h_{ef} is the embedded depth of the anchor.

It is better to mention that Eligehausen and Sawades' empirical equation was only verified up to 450 mm. The presented results in Fig. 14 have reasonable agreement with their empirical equation in embedded depths up to 450mm for all supporting condition cases.

Furthermore, the results are compared with the analytical and experimental results appeared in the RILEM report (1991) as shown in Table 1. From Table 1, it can be noticed that the present analytical results have a rather good agreement with the previous analytical and experimental results by RILEM (1991).

Figs. 15 and 16 show the load-displacement diagrams of embedded depths 150mm and 5000mm, respectively, for the three boundary conditions. By using the arc-length method, the full pull-out force versus displacement diagram can be obtained. The convergence criterion is maintained at all load levels before and after the peak load. In the both figures the vertical displacements are determined at the point A (Fig. 1).

In the case of small embedded depths such as 150mm, Fig. 15 shows that the failure mode is ductile and the snap back phenomenon does not occur whatever the boundary condition is. On the other hand, for large embedded depth such as 5000mm, Fig. 16 shows a post peak snap back response, which reflects the brittle behavior of such deep embedded anchors.

Table 1 Comparison of the present analysis with the previous results

Analyzer/ Tester	Embedded depth=50mm The crack inclination θ=45°	Embedded depth=150mm The crack inclination θ=45°
1. Barr and Tokatly, tests 2. Ozbolt, analysis 3. Palm and Gylltoft, analysis	22.5	64-220 191 227
ELIGEHAUSEN and SAWADE empirical equation	40	211.3
The present analysis	27.6 at the crack inclination θ=60°	212.5 at the crack inclination θ=53°

The ultimate strength is in kN.

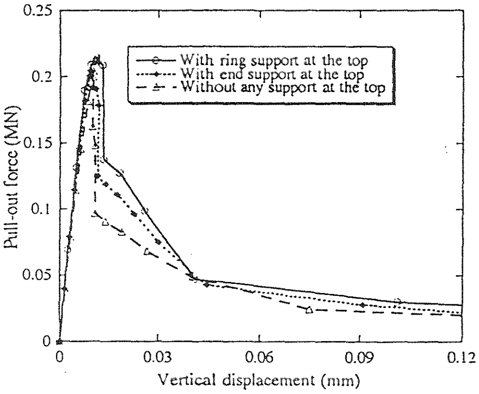

Fig. 15 Force-displacement diagram in the case of d=150mm

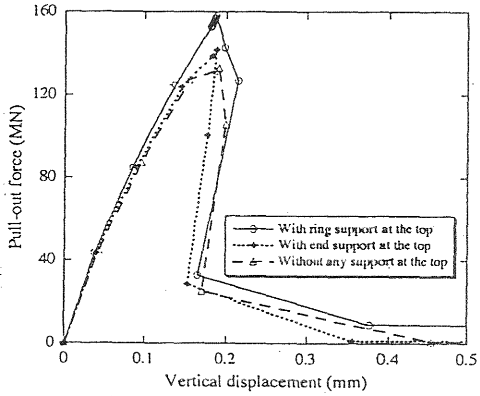

Fig. 16 Force-displacement diagram in the case of d=5000mm

The snap back occurs, as a result of a bifurcation process which leads to a sudden drop in both load and deflection. It can be concluded that the pull-out behavior of headed anchors is significantly affected by the embedded depth and the concrete block size. The failure mode changes from ductile to brittle as the embedded depth and the size of concrete block increases. In other words, fracture of concrete leads to brittle failures due to the size effect of decreasing strength with increasing structural size.

7 Conclusion

It is demonstrated that the influence of different variables on pull-out strength of headed anchors embedded in concrete blocks can be studied numerically by means of nonlinear fracture mechanics. It has been observed that for small embedded depths the pull-out capacity of headed anchors embedded in concrete blocks is profoundly affected by the size effect. On the other hand, for large embedded headed anchors, the numerical predictions showed that the size effect becomes insignificant. Moreover, the snap back occurs when both the embedded depth and the concrete block size increase. Thus the brittle behavior of concrete blocks becomes significant. Also, it is found that in the case of the reaction ring support, the inclination of failure cone surface with range 53°-60° gives the minimum pull-out strength for the embedded depths up to 5000mm. In contrast, for large embedded depths as 10000mm and 12500mm, the inclination 38°-51° of the failure cone gives the minimum pull-out strength. In the case of the edge support and in the case of the free top surface, the inclination of the failure cone becomes more steeper consequently the resulting ultimate pull-out strength becomes smaller, than the case of the reaction ring support. The conclusion indicates that the commonly adapted method assuming 45° failure surface yields exaggerated resisting load. Finally the pull-out strength of cone failure is mainly dependent on mode I fracture.

8 References

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