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SIZE EFFECTS OF REINFORCED CONCRETE BEAMS IN FLEXURE

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Abstract
The size effect is one of the most important and compelling aspects of fracture mechanics in concrete structures, and is distinguished from stress-based failure theories. In this study, the size effect in flexural behavior of reinforced concrete beams has been studied. The major variables of test are the size(relative depth) of the members as well as the longitudinal reinforcement ratio. According to the results of this study, the size effect of reinforced concrete members in flexural resistance is pronounced as that of shear resistance, and the use of steel ratio ρ equal to $0.5\rho_{max}$ exhibits less size effect than that of ρ_{min}, or ρ_{max} in flexure, in which ρ_{max} is the maximum longitudinal tensile reinforcement ratio prescribed in the ACI design code.

The prediction formula for the size effect of reinforced concrete beams in flexure is proposed and shows good correlation with other experimental data.
Key words : size effect, flexure, reinforced concrete

1 Introduction

Concrete is a brittle material, and thus its mechanical behavior is critically influenced by crack growth. The study of concrete fracture including size effect may provide the fundamental bases in understanding the behavior of concrete structures.

According to the classical theories which use some type of strength limit or failure criterion in terms of stresses, the nominal stress is constant, and independent of structure size. This may be illustrated by considering the elastic and plastic formulas for the strength of beams in bending, shear, and torsion. It is seen that these formulas are of the same form except some factors. Thus, plotting the $\log(\sigma_n)$ vs $\log(d)$, the failure stated according to the strength or yield criterion is always given by horizontal line, i.e., exhibits no size effect. In contrast with this, fracture governed by LEFM exhibits rather dominant size effect(Fig. 1.).

The fracture mechanics is ignored by designer until now, but it is very important in structural design.

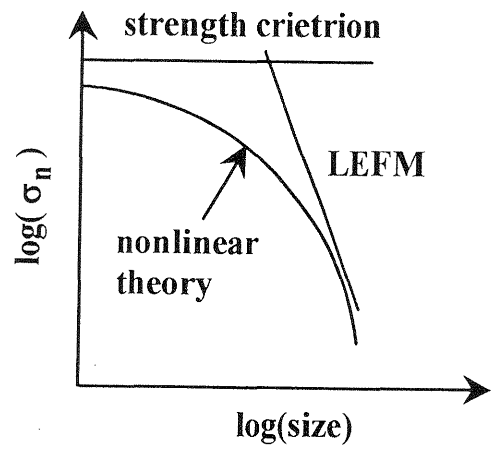

Fig. 1. Size effect in fracture mechanics compared with strength criterion

2 Test procedure

2.1 Major parameters of experiments

Major experimental variables are the effective depth(size) of the beam(d) and longitudinal reinforcement ratio(ρ).

The test specimens have compressive strength of 280 kg/cm^2(28 MPa), maximum aggregate size(d_a) of 25mm, and W/C ratio of 43%, respectively. The specimen is three-point loaded by Material Testing System(MTS).

2.2 Design and fabrication of test specimens

The dimensions and details of test specimens are shown in Tables 1 and 2. The longitudinal reinforcement ratio is varied from zero to ρ_{max} by zero, ρ_{min}(200/f_y, where f_y in psi), 0.5ρ_{max}, ρ_{max}(0.75ρ_b, where ρ_b is balanced reinforcement ratio).

Table 1. Dimensions of test specimens

Type of Specimen	Length(L) (cm)	Span(S) (cm)	Width(b) (cm)	Effective Depth(d) (cm)	Height(h) (cm)
A	90	80	20	10	13
B	180	160	20	20	25
C	260	240	20	30	35
D	340	320	20	40	45

Table 2. Details of test specimens

Specimen Index	Longitudinal Reinforcement Ratio(ρ)	Effective Depth(d) (cm)	Specimen Index	Longitudinal Reinforcement Ratio(ρ)	Effective Depth(d) (cm)
R0D10	0	10	R2D10	0.5ρ_{max}	10
R0D20		20	R2D20		20
R0D30		30	R2D30		30
R0D40		40	R2D40		40
R1D10	ρ_{min}	10	R3D10	ρ_{max}	10
R1D20		20	R3D20		20
R1D30		30	R3D30		30
R1D40		40	R3D40		40

3 Test results

To observe the size effect experimentally in flexural behavior of reinforced concrete beam, the logarithmic value of relative moment, $\log(M_{max}/M_n)$, and relative beam depth, $\log(d/d_a)$, are plotted for various steel ratios as shown in Fig. 2. The M_{max} is the measured maximum external moment and M_n is the nominal moment strength based on the basic concept of ultimate strength design.

Fig. 2 indicates that all the beams exhibit considerable size effect according to relative size(depth) of the beams. Fig. 3 shows that the normally reinforced beam($\rho=0.5\rho_{max}$) exhibits less size effect than other members.

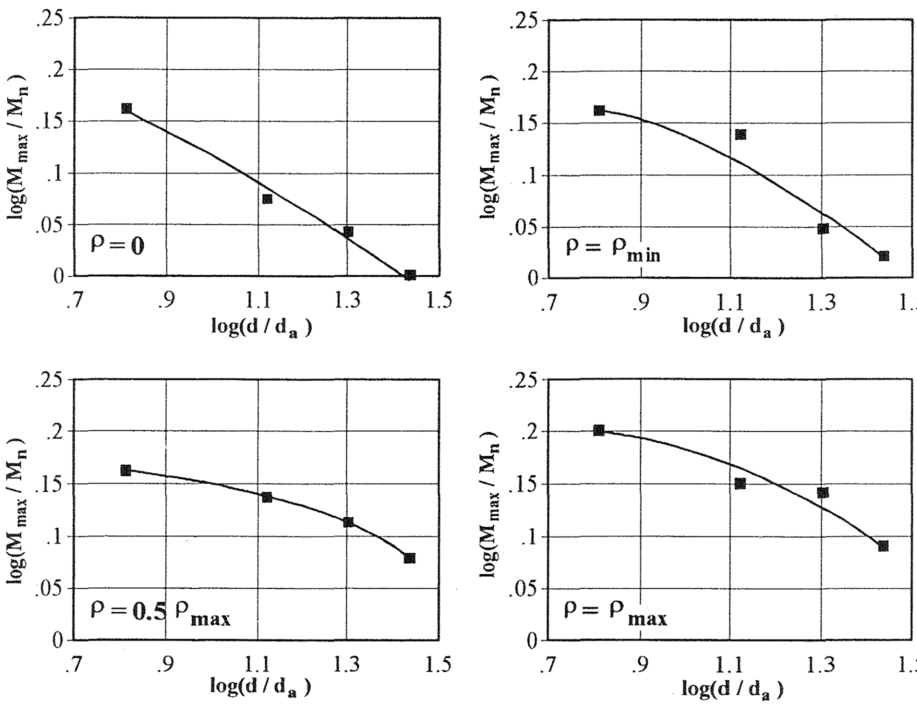

Fig. 2. Test results on the size effect of RC beams in flexure for various steel ratios

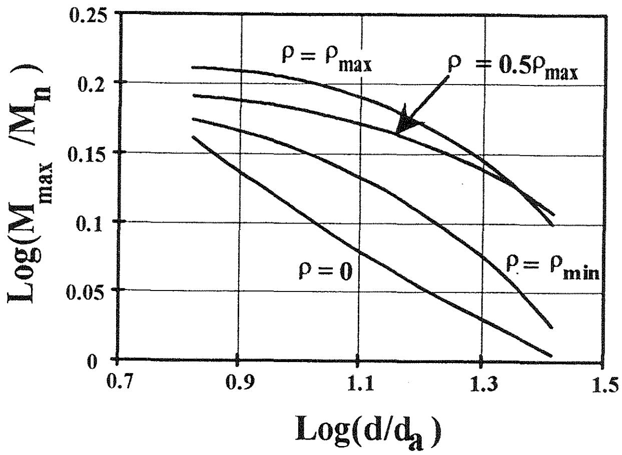

Fig. 3. Comparison of test results in flexure

4 Theoretical prediction formula for size effect in flexure

The size effect formula in moment strength of reinforced concrete beams can be written as follows, which has the similar form of Bazant's formula in shear.

$$M_{max} = M_n\left(\frac{\alpha_0}{\sqrt{1+\frac{d}{\lambda_0 d_a}}}\right) \tag{1}$$

Here, M_{max} is ultimate moment strength at failure and M_n is nominal moment strength of designed section as mentioned above. The coefficients α_0 and λ_0 are functions of the longitudinal reinforcement ratio ρ, and can be assumed as parabolic equations.

$$\begin{aligned} \alpha_0 &= a_1 + a_2\rho + a_3\rho^2 \\ \lambda_0 &= b_1 + b_2\rho + b_3\rho^2 \end{aligned} \tag{2}$$

From the measured values of test members, the values of α_0 and λ_0 in each longitudinal reinforcement ratio can be calculated as the points in Fig. 4 and 5. Then, the unknowns in Eqn (2) can be calculated by the regression of ρ, α_0 and λ_0. The determined coefficients are also shown in Fig. 4 and 5.

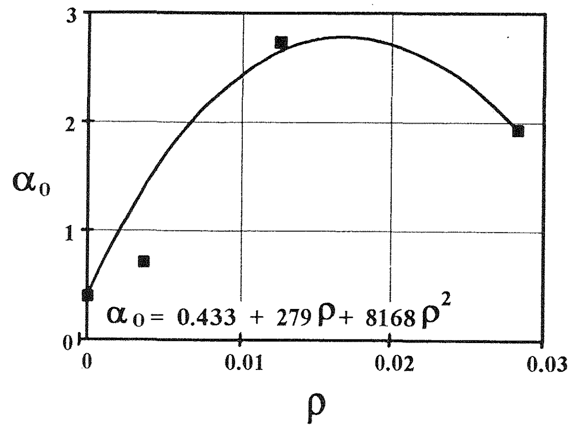

Fig. 4. α_0-ρ relation in size effect formula of flexure

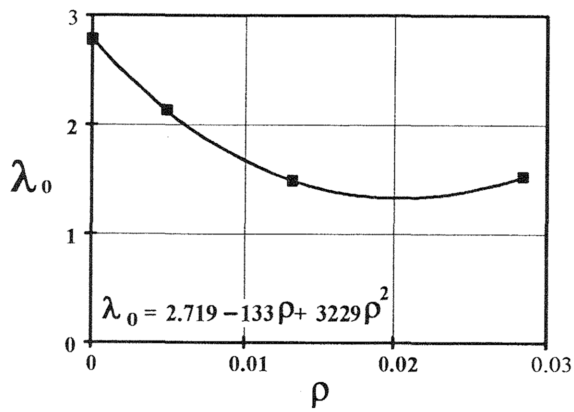

Fig. 5. λ_0-ρ relation in size effect formula of flexure

To compare the test results of flexural behavior with Dey's study, the normalized moment M, and normalized depth β of Dey's study on fictitious crack model of reinforced concrete should be converted into the nominal moment M_n and the relative depth d/d_a, respectively. Then these values are analyzed by regression and compared with test results.

The proposed formula shows good agreement with Dey's study especially in the ρ equal to $0.5\rho_{max}(0.375\rho_b)$ as shown in Fig. 6.

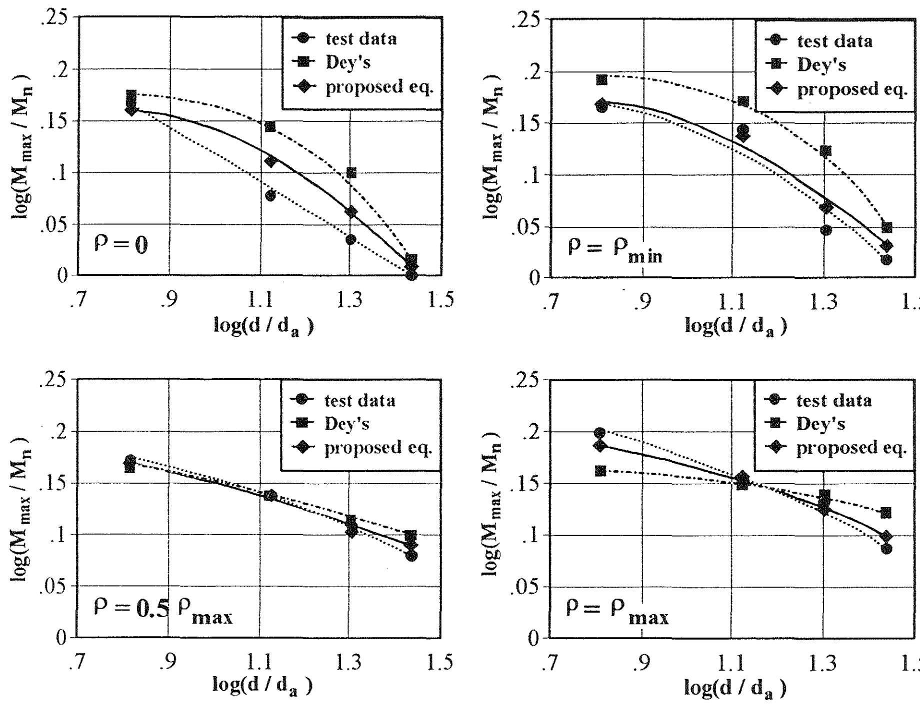

Fig. 6. Comparison of proposed formula with Dey's study

5 Conclusion

The flexural behavior of reinforced concrete beams has been studied to explore the size effect in flexure. The major experimental variables are the effective depth of the beam(d) and longitudinal reinforcement ratio(ρ).

It is generally known that the size effect of flexural behavior of reinforced concrete beams is not pronounced as much as that of shear behavior. However, according to the results of this study, the size effect of reinforced concrete members in flexure is pronounced as that of shear resistance.

The size effect formulas for flexural behavior of reinforced concrete beams are derived based on the test results and compared with other study. The proposed formulas show relatively good agreement with other study.

Generally, the size effect is known to decrease with the increase of longitudinal reinforcement ratio, but no apparent trend is observed in this study. Moreover, the size effect is also pronounced for heavily reinforced beams. However, it's proved that the use of steel ratio ρ equal to $0.5\rho_{max}(0.375\rho_b)$ exhibits less size effect than that of ρ_{min}, or $\rho_{max}(0.75\rho_b)$ in flexure. Thus, the advantage of practical use of steel ratio adjacent to $0.5\rho_{max}$ is verified in the light of size effect.

6 References

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