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SIZE EFFECT ON FLEXURAL RESISTANCE DUE TO BENDING SPAN OF CONCRETE BEAMS

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Abstract

Many researchers have studied the size effect on the flexural resistance of concrete beams under bending moment due to the size of cross section, but the size effect due to the length of beams has not been studied enough. The influence of beam length, therefore, has generally been modeled using a series linking model, which is a probability model.

In this study, the effect of beam length on flexural resistance was investigated by experiments and the series linking model was evaluated. In the experiments, two series of bending tests were performed on plain concrete beams. In Series A (B), a 4.5 x 4.5 cm (8.5 x 8.5 cm) cross section was used. In each series, all specimens had the same shear span of 8 cm (20 cm), and bending spans of 5, 7, and 9 cm (5, 10, and 20 cm) were used.

The experiments showed that the flexural resistance decreased as the bending span increased, and that the series linking model overestimates the flexural resistance. The series linking model, therefore, was modified, and the modified model showed good agreement with the test results.

Key words: Size effect, series linking model, flexural resistance, beam

1 Introduction

Many researchers have confirmed experimentally the size effect of concrete members, and theoretical studies and evaluations of the size effect have been dependent for their basis on such sciences and theories as fracture mechanics (Uchida et al.(1992), Hidaka et al.(1994), Rokugo et al. (1994)) and probability theory (Izumi et al.(1981), Kosaka et al.(1985)). The size effect on the flexural resistance of concrete beams due to beam length have been studied only to the extent that a generalized series linking model has been proposed. Applicability of that model to concrete beams, however, has not yet been investigated experimentally.

In this study, many four-point bending tests were carried out on concrete beams of varying lengths to investigate the size effect on flexural resistance due to beam length. Then, on the basis of the test results, applicability of the series linking model to the size effect on the flexural resistance of concrete beams due to beam length was investigated. A newly developed model designed to express this size effect is presented in this paper. In the tests, a fixed shear span length was used to make the influence of energy stored in the shear span uniform, and the relationship between the pure bending span and the flexural resistance was investigated by varying the pure bending span.

2 Application of the series linking model to size effect on flexural resistance due to beam length

Let us assume that positive bending moment M is acting on a beam of length L, as shown in Fig. 1. This beam is divided into a number of "standard-beams" of length λ which is equal to the spacing between cross sections at which each flexural resistance can be considered mutually independent. That is, the beam of length L is thought of as a chain of n standard-beams. This series linking model assumes that failure occurs independently in each of these n standard-beams, and that the beam of length L is considered to have failed when at least one out of the n standard-beams has failed.

If bending moment M=m acts on each standard-beam of length λ, and that the probability distribution function of the flexural resistance of each standard-beam is P(m), then the probability distribution function $P_n(m)$ of the flexural resistance for pure bending span L ($=n\cdot\lambda$) can be expressed by Eq. (1).

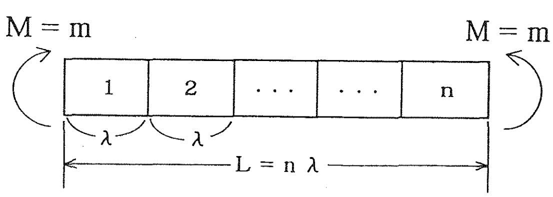

Fig. 1. Application of the series linking model to beams

$$P_n(m)=1-\{1-P(m)\}^n \quad (1)$$

3 Test method and results

3.1 Test method

In order to make probabilistic evaluation of the size effect of changes in the bending span of a beam on its flexural resistance, it is necessary to repeat a test on many specimens under the same test conditions.

In the four-point bending test, two types of prismatic concrete specimen were used: those having a 4.5 x 4.5 cm cross section (Series A) and those having a 8.5 x 8.5 cm cross section (Series B). For the Series A specimens, a fixed shear span of 8 cm and pure bending spans L of 5, 7, and 9 cm were used, and a total of about 150 specimens (about 50 each for the three bending spans) were used. For the Series B specimens, a fixed shear span of 20 cm and pure bending spans L of 5, 10, and 20 cm were used, and a total of about 150 specimens (about 50 each for the three bending spans) were used. A closed-loop hydraulic servo testing machine was used for loading. The Series A specimens were loaded by load-control at a bending moment in the pure bending span of 0.6 Nm/s, and the Series B specimens were loaded by load-control at 1.6 Nm/s. A Teflon sheet was sandwiched at the support points and the loading points. The specimens were positioned so that the cast concrete surface faced forward. In view of the cross-sectional dimensions of the specimens, coarse aggregate having the maximum size of 10 mm (Series A) or 20 mm (Series B) was used for the concrete. The compressive strength of the concrete used was 48.2 N/mm^2 (Series A) and 49.1 N/mm^2 (Series B).

In this study, it was necessary, for the purpose of evaluating the variation of flexural resistance properly from the viewpoint of probability theory, to minimize the influence of factors other than the size effect of the bending span in each series. For factors that could not be controlled, it was also necessary to see to it that such factors would not affect only specimens of certain configuration. To this end, each series of specimens was made of concrete from one batch and kept under identical environmental conditions. The specimens were also arranged in such a manner that specimens of different bending spans were always placed side by side. In order to eliminate the influence of differences in the time preceding the loading test, 28-day-old specimens were taken out of the water curing tank and then air-dried for 7 or more days before the test, and the test was carried out so that no two specimens used consecutively were of the same bending span.

Only the test results that were obtained from a loading process in which (1) breaking occurred within the bending span, and (2) four-point loading was carried out symmetrically so that, judging from the load-deflection curve, the bending span experienced pure bending was deemed valid.

The test results and the discussion that follow include only those valid results.

3.2 Test results

Figure 2 shows histograms showing flexural resistances for different bending spans of each series of specimens. In both Series A and Series B specimens, the longer the bending span L, the smaller the flexural resistance distribution as a whole becomes, and so does the mean value. These results clearly indicate the existence of size effect.

Figure 3 shows histograms at the breaking points in different bending tests. The histograms for the experiments in which the bending span of L=5 cm was used show whether breaking occurred in the right or the left half of the bending span. The histograms for the other experiments show the third of the bending span in which breaking occurred. As the histograms indicate, breaking points are distributed more or less evenly within the bending span.

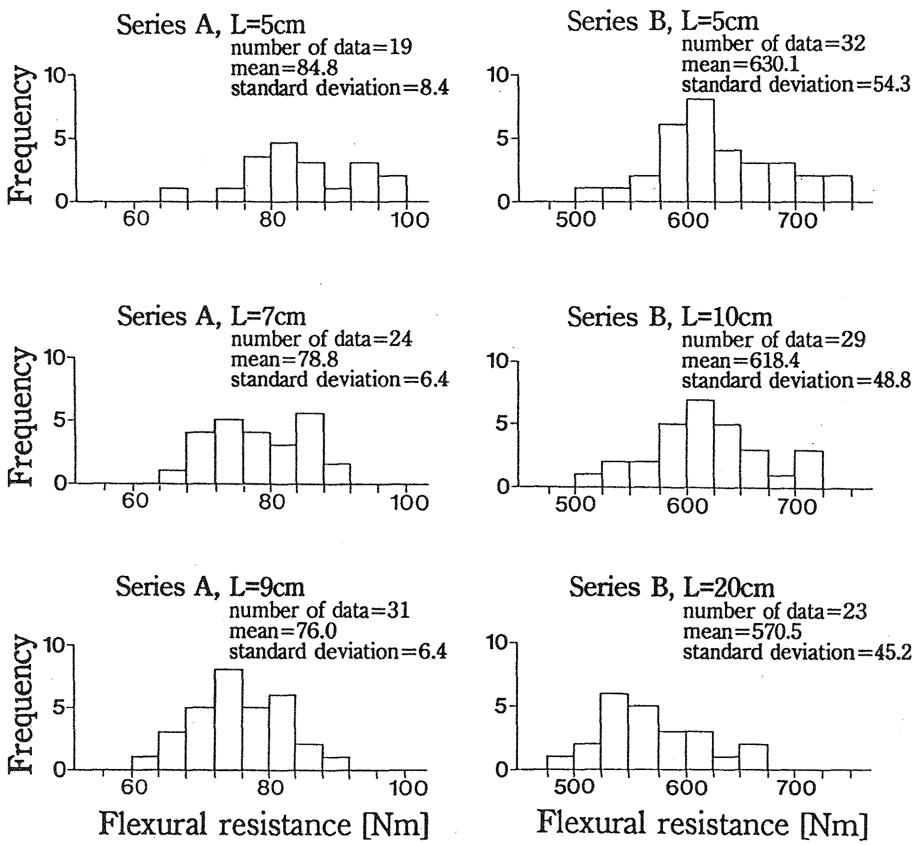

Fig. 2. Flexural resistance distribution

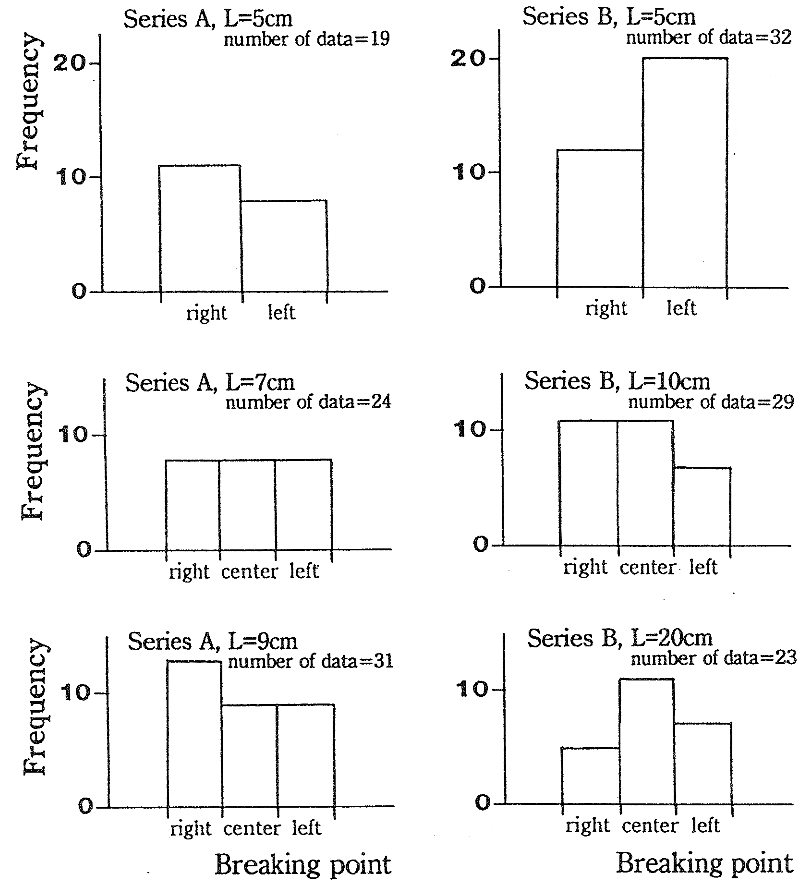

Fig. 3. Distribution of breaking points in test specimens

4 Discussion

4.1 Normal distribution modeling based on test results

Figure 4 shows the probability distribution functions, plotted on normal probability paper, of flexural resistance for different bending spans of each series of specimens, which have been derived from the test results shown in Fig. 2.

From Fig. 4, as a first step, the minimum pure bending span L of 5 cm in each series was assumed to be a standard-beam as described in section 2, and a normal distribution model was constructed using the straight broken line shown in Fig. 5. For the Series B specimens, a normal distribution model was constructed using a measured mean of 630.1 Nm and a standard deviation of 54.3 Nm because the probability distribution functions obtained from the test were close to a straight line. For the Series A specimens, a normal distribution model was formulated using the slope of the probability distribution function of the measured values that are close to the mean value because the number of data sets was as small as 19 and because the results for the Series B specimens indicated high validity of normal distribution modeling.

4.2 Application of the series linking model to size effect

Validity of the series linking model defined as Eq. (1) is evaluated using the normal distribution of the flexural resistance of a beam having a pure bending span of L=5 cm, which has been defined as a standard-beam in section 4.1.

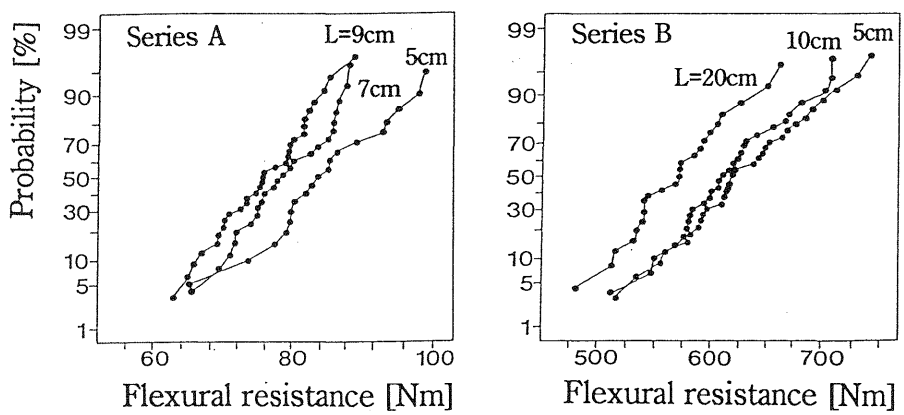

Fig. 4. Experimentally determined probability distribution functions

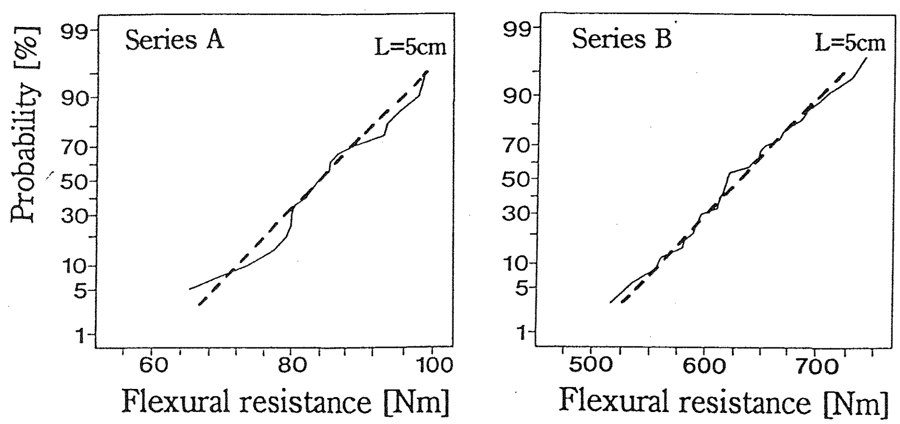

Fig. 5. Normal distribution modeling

Figure 6 shows a comparison between the probability distribution function (theoretical values) of flexural resistance in each bending span derived from Eq. (1) and the measured values. The broken lines *a* and *b* in the figure represent theoretical values of the flexural resistance of the Series A specimens for L=7 and 9 cm. The broken lines *c* and *d* represent theoretical values of the flexural resistance of the Series B specimens for L=10 and 20 cm. It can be seen from Fig. 6 that in the Series A specimens which have the smaller cross section (4.5 x 4.5 cm), measured values of flexural resistance for both L=7 and 9 cm are smaller than the theoretical values, and that the size effect is greater than in the case where the series linking model is assumed. As for the Series B specimens which have the larger cross section (8.5 x 8.5 cm), the results for L=10 cm show that the broken line *c* which assumes the series linking model show values of flexural resistance lower than the measured values. This indicates that in the Series B specimens L=5 cm is too short as the length of the standard-beam, and that there is a need to redefine the length of the standard-beam.

For the Series B specimens, therefore, L=10 cm was assumed for the standard-beam, and a normal distribution as represented by straight broken line *e* in the right graph of Fig. 7 for the Series B specimens was formulated. Since the experimentally determined probability distribution function was close to a straight line, a normal distribution was formulated using a measured mean of 618.4 Nm and a standard deviation of 48.8 Nm. The broken line *f* in the same graph shows theoretical values of flexural resistance for L=20 cm of the Series B specimens. The graph indicates that, as in the case of the Series A specimens, measured values of flexural resistance are smaller than the theoretical values. This indicates that the size effect is greater than in the case where the series linking model is assumed.

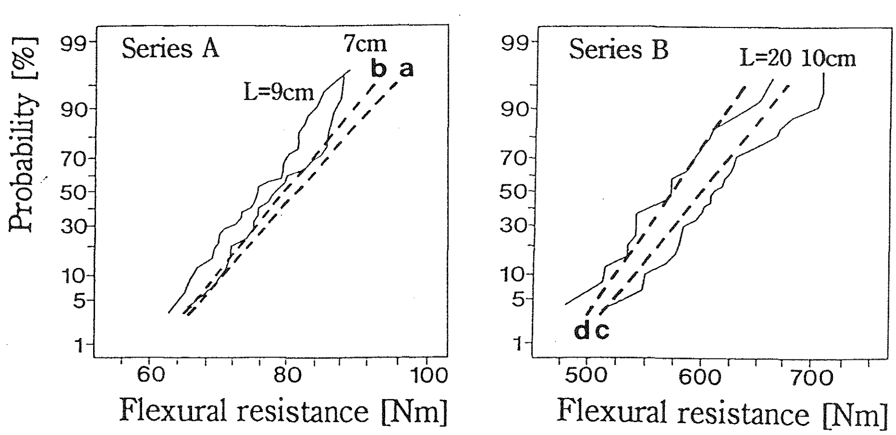

Fig. 6. Comparison between theoretical values obtained from the series linking model and measured values

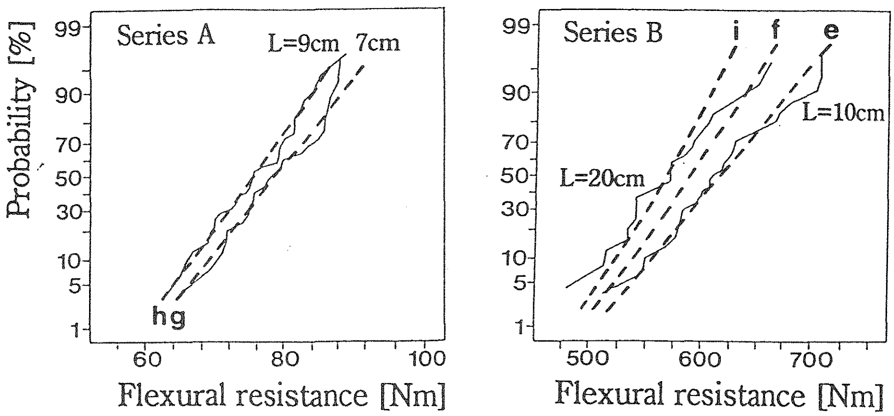

Fig. 7. Comparison between theoretical values obtained from the proposed model and measured values

From above, it was confirmed that the conventional series linking model is not adequate for the evaluation of the size effect due to beam length on flexural resistance, and that there is a need to develop a new model.

5 A new model for evaluation of size effect due to bending span

5.1 Proposed model

On the basis of the discussion in section 4, the authors propose the probability model defined by Eq. (2), though it has not yet been explained theoretically, as a model of size effect capable of expressing decreases in flexural resistance more accurately than the series linking model.

$$P_n(m)=1-\{1-P(m)\}^{n^2} \qquad (2)$$

The relations $\lambda=x$ and $\lambda=bx$ are assumed here as the length λ of the standard-beam. For each of these two relations, let $P^1(m)$ and $P^2(m)$ represent the probability distribution functions of flexural resistance for $L=\lambda$. From Eq. (2), $P^2(m)$ can be expressed, using $P^1(m)$, as Eq. (3). Let $P^1{}_a(m)$ and $P^2{}_{a/b}(m)$ represent the probability distribution functions of flexural resistance for L=ax derived, using the above relations $\lambda=x$ and $\lambda=bx$, respectively, from Eq. (2). From the relation defined by Eq. (3), the two probability distribution functions become identical, as shown in Eq. (4). It can be said, therefore, that the proposed probability model of Eq. (2) does not show any inconsistency due to the length λ of the standard-beam.

$$P^2(m)=1-\{1-P^1(m)\}^{b^2} \qquad (3)$$
$$P^2{}_{a/b}(m)=1-\{1-P^2(m)\}^{(a/b)^2}=1-\{1-P^1(m)\}^{a^2}=P^1{}_a(m) \qquad (4)$$

5.2 Discussion

Validity of the proposed model defined by Eq. (2) is evaluated using the normal distribution of the flexural resistance of beams which have the pure bending spans L of 5 cm (Series A) and 10 cm (Series B) assumed for the standard-beam in section 4.

Figure 7 compares the probability distribution function (theoretical values) of flexural resistance for each bending span derived from Eq. (2) with measured values. The broken lines *g* and *h* in the figure represent theoretical values of the flexural resistance of the Series A specimens for pure bending spans L of 7 and 9 cm. The broken line *i* represents theoretical values of the flexural resistance of the Series B specimens for the pure bending span L of 20 cm. As shown in Fig. 7, these theoretical values show good agreement with the measured values. From these results, it can be said that the proposed probability model defined by Eq. (2) closely approximates the size effect due to beam length on flexural resistance.

6 Conclusion

In this study, four-point bending tests were carried out on a large number of beams with an identical cross section and different pure bending spans in order to investigate the size effect due to the length of concrete beams on their flexural resistance. In the tests, flexural resistance decreased considerably as the bending span increased. The tests showed, for the types of specimens used in the tests, that such decreases in flexural resistance cannot be expressed by the conventional series linking model defined by Eq. (1), and that the probability model of Eq. (2) proposed by the authors is more accurate than the series linking model in expressing decreases in flexural resistance.

The authors intend to carry out similar tests on concrete specimens which have larger cross sections and longer pure bending spans to evaluate the usefulness of the proposed probability model. Believing that the reason the series linking model does not hold true has something to do with the mechanism of concrete fracture, the authors also intend to pursue this study to present a theoretical explanation of the proposed model.