Fracture Mechanics of Concrete Structures
Proceedings FRAMCOS-3
AEDIFICATIO Publishers, D-79104 Freiburg, Germany

SIZE EFFECT OF CONCRETE COMPRESSIVE STRENGTH FOR THE NON-STANDARD CYLINDRICAL SPECIMENS

J. K. Kim
Department of Civil Engineering, Korea Advanced Institute of Science and Technology, Korea
S. T. Yi
Department of Civil & Structural Engineering, Korea Power Engineering Company, Inc., Korea
S. H. Eo
Department of Civil Engineering, Changwon National Univ., Korea

Abstract
The reduction phenomena of concrete compressive strength with the size of cylinders is very interesting, but till now an adequate analysis technique is not available. Based on the existing research results, the bigger the member size, the smaller the strength. However, real test results reveal that the reduction rate becomes blunt and there are considerable differences between size effect law and test results.

The purpose of this paper is to propose a model equation which can predict the compressive strength of nonstandard concrete cylinder specimens with varying height to diameter ratio. The effects of maximum aggregate size on the microcrack zone are considered and the concept of characteristic length is newly introduced.

Proposed models can be practically used to predict the compressive strength of various sized concrete cores sampled from existing structures.
Key words: Size effect, fracture mechanics, non-standard compressive strength

1 Introduction

The reduction phenomena of concrete compressive strength with the size of cylinders was found earliest by Gonnerman in 1925. After that this was experimentally supported by many researches, but by this time a consistent analysis was impossible due to the variety in the selection of experimental variables to prove the size effect.

Moreover, there have been several theoretical studies(for example, Weibull - Weakest Link Theory, Tucker - Strength Summation Theory, and Nielson - Surface Theory etc.), but they did not consider the stable crack growth after the propagation of initial crack and the effects of stress redistribution due to the crack growth.

To solve the discrepancy of Weibull-Type size effect, many studies using fracture mechanics have been performed and now the research results have accumulated to considerable levels. Nowadays the concern is concentrated in applying these results to the analysis and design of concrete structures.

In the previous studies, a model equation for predicting the compressive strength of concrete was proposed. But the equation is theoretically valid for geometrically similar specimens such as the standard cylinder specimens with height to diameter ratio of 2. In this study, a generalized equation will be derived for predicting the compressive strength of concrete from nonstandard cylinder specimens with varying height to diameter ratio. For this purpose, the effects of the maximum aggregate size on the fracture process zone are considered and the concept of characteristic length is newly introduced. With the derived equation, regression analyses are carried out with the extensive test data obtained from literatures. Based on the results of regression analyses, a practical prediction equation is suggested and verified through statistical analysis.

On the other hand, the ASTM standard C42-94 is used in practice to predict the strength of concrete core specimens sampled from the existing structures through correction factors. Comparisons of the predicted values from ASTM standard and size effect model suggested in this study show that the modified size effect model provides considerable correctness and generalization.

2 Theoretical investigation on the size effect law

2.1 Theoretical review of the size effect law

Considering the energy balance at crack propagation in concrete, Bazant derived the size effect law from the dimensional analysis for geometrically similar members as follows :

$$\sigma_n = \frac{P}{bd} = \frac{Bf_t'}{\sqrt{1+\frac{d}{\lambda_o d_a}}} \tag{1}$$

Where, σ_n = nominal stress at failure, P = load or loading parameter, b = thickness, d = characteristic dimension, f_t' = direct tensile strength of concrete, d_a = maximum aggregate size, B and λ_o = empirical constants.

In derivation of Eq. 1, the hypotheses include that total energy release is proportional to the area of the fracture process zone, $nd_a a$ where n is a constant and a is the length of crack band. From the existing experimental results, however, it seems to be reasonable to assume that the fracture process zone width does not vary linearly with the maximum aggregate size, d_a, but $d_a^m (m = \text{constant}, 0 < m < 1)$. Because cracks occur at a narrowly strain concentrated region, the differential amount of crack is not exactly proportional to the aggregate size.

Using the crack band theory and introducing the size independent strength σ_o (= $\alpha f_t'$) in Eq. 1, a modified size effect law considering the effect of maximum aggregate size on the fracture process zone is obtained.

$$\sigma_n = \alpha f_t' + \frac{B f_t'}{\sqrt{1 + \dfrac{d}{\lambda_o d_a^m}}} \tag{2}$$

It was also proposed by Bazant in different approach. In Eq. 2, λ_o may be, of course, a function of the strength of concrete, f_c', and the maximum aggregate size, d_a, and α may be a function of f_c'. So, the nominal compressive strength of cylindrical specimens can be expressed as follows :

$$f_o = \alpha(f_c')f_t' + \frac{B f_t'}{\sqrt{1 + \dfrac{d}{\lambda_o(f_c', d_a) d_a^m}}} \tag{3}$$

In this study, however, major emphasis is given on the maximum aggregate size due to the minor effect of f_c'.

2.2 Derivation of modified size effect law for non-standard cylinder specimens

In order to apply to more general test specimens of which height/diameter ratio is not 2, the equations should be modified to reflect the width of microcrack zone and the characteristic length which provides the main crack zone.

In Fig. 1 the characteristic length is represented by $(h_i - \beta d_i)$. It can be replaced by h_i or d_i especially when the specimens are geometrically similar since the ratios of the characteristic length $(h_1 - \beta d_1)/(h_2 - \beta d_2)$, h_1/h_2 and d_1/d_2 have the same value. But $(h_1 - \beta d_1)/(h_2 - \beta d_2)$ is not equal to h_1/h_2 if the specimens have the same diameter ($d_1 = d_2$) as shown in Fig. 1(b). In other words, the specimen which exhibits the size effect when the size is twice the size of the specimen denoted ABCD, is not the specimen denoted A'''B'''C'''D''' which satisfies $h_2 = 2h_1$, but the specimen denoted A'B'C'D' or the specimen denoted A"B"C"D" which satisfies $(h_2 - \beta d_2) = 2(h_1 - \beta d_1)$ or $(h_2' - \beta d_1) = 2(h_1 - \beta d_1)$ respectively. This conclusion results from the condition that only the effects of the microcrack zone and the characteristic length are considered as factors on the size effect. On the other hand, it is considered that end restraints(denoted by inclined area given in Fig. 1(b)) and energy release zone(denoted by dotted area given in Fig. 1(b)) affect the size effect in uniaxial compressive strength.

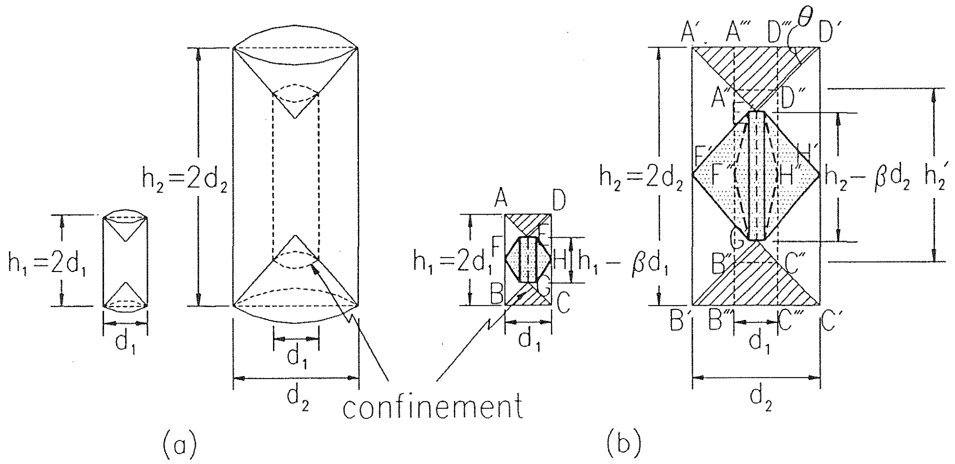

Fig. 1. Characteristic length of general cylindrical specimens subjected to uniaxial compressive load

Unless the confinement effect and the energy release zone are considered the specimens A'B'C'D' and A"B"C"D" show the same size effect. The areas denoted A'E'D' and A"E'D" represent the confinement effects for specimens A'B'C'D' and A"B"C"D" respectively. Thus the specimen A'B'C'D' has the greater load resistant capacity than the specimen A"B"C"D" has, as the confinement is related to the volume, i.e., $(d_2/d_1)^3$ while the stress is related to the area, i.e., $(d_2/d_1)^2$. But if the energy release zones are considered for the specimens, the specimen A'B'C'D' has the more energy per unit volume, that is, the lower load resistant capacity per unit area (i.e., stress), than the specimen A"B"C"D" has since the same energy is required for the unit crack to be created. The ratio h/d is also considered to be a factor on the size effect since the larger the ratio h/d the smaller value of θ is expected, which determines the degree of confinement effect.

As a result, the effects of confinement and energy release zone are considered to act contradictory to each other on the size effect of uniaxial compressive strength. Furthermore, it is difficult to consider them for derivation of a size effect model as well as they have minor importance within practical size range compared with the effects of microcrack zone width and the characteristic length. The effect of the maximum aggregate size also can be minor effect on the microcrack zone width within a practice range. So, to determine the empirical formula of Eq. 4 with Eq. 2, the uniaxial compressive strength f_t' can be replaced with f_c'. Where, η means the strength size factor defined as a f_o/f_c'.

$$\eta = f_o / f_c' = \alpha_1 + \frac{B}{\sqrt{1+\lambda_1(h-\beta d)}} \tag{4}$$

If maximum aggregate size d_a is considered, Eq. 4 can be written as follows :

$$\eta = f_o / f_c' = \alpha_1 + \frac{B}{\sqrt{1+\frac{1}{\lambda_o d_a^{\,m}}(h-\beta d)}} \tag{5}$$

In chapter 3, the validity of Eq. 4 and Eq. 5 is also illustrated through regression analyses with extensive experimental data obtained from literature. In analysis of test data, it should be noted that the application of Eq. 4 and Eq. 5 is limited for cases $h \geq \beta d$ as shown in Fig. 2(b) and (c).

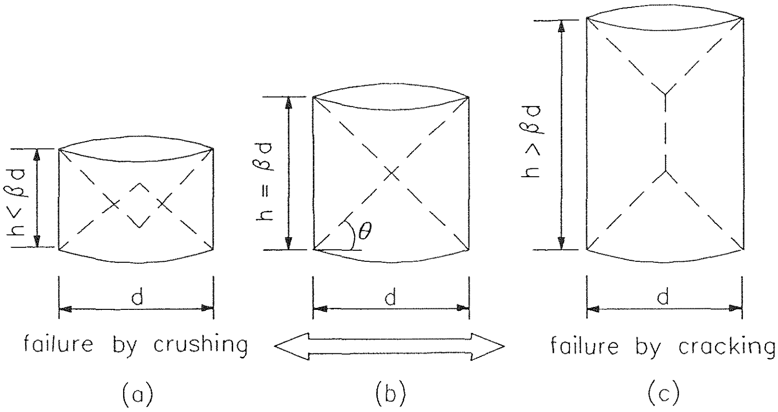

Fig. 2. Failure modes according to the specimen geometry

If $h < \beta d$ as shown in Fig. 2(a), the confinement zone extends through the specimen to lead failure by crushing not by cracking.

3 Regression analysis of existing test data

3.1 Not considering the effect of maximum aggregate size

In order to obtain a prediction model for general cylinder specimens, regression analyses are carried out with 678 test data of non-standard cylinders, including 222 test data of standard cylinders. Eq. 6 is obtained from the analyses, the results are given in Fig. 3, where the relationship between $1 + (h - d)/50$ and f_o/ f_c' are represented by the solid curve. From the figure it can be seen that a major portion of data are assembled in a certain particular range since the diameters of most cylinders used in tests were 7.5, 10.0 and 15.0 cm. It ought to be noted that, when the value of *h/d* approaches 1.0, the scatter of data will be increased due to the effects of end confinement and energy release zone.

$$f_o = 0.8 f_c' + \frac{0.4 f_c'}{\sqrt{1+(h-d)/50}} \tag{6}$$

Where the units of f_o and f_c' are in MPa and the unit of *h* and *d* are in mm. From the figure it is observed that, when the value of *h/d* is smaller than 1.0, η diverges to infinite. But considering the limited condition of Eq. 6, the curve belonging to this range has no meaning.

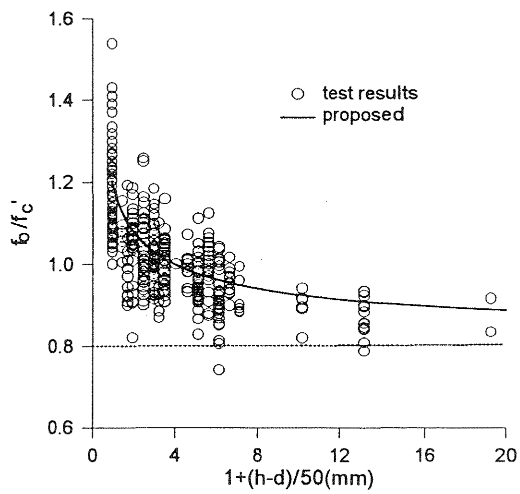

Fig. 3. Relationship between *1+ (h-d)/50* and strength size factors

Thus, for any specimens satisfying the condition $h \geq \beta d$, the compressive strength of the cylinder can be obtained by substituting the values of *d*, *h* and compressive strength of standard cylinder(f_c') into the Eq. 6. If there are no confinement effects due to forcing plate, that is, the ratio of *h/d* is very large, actual compressive strength of concrete would be 80% of laboratory test results

Fig. 4 shows the comparison of the analytical and experimental values of compressive strength of cylindrical concrete specimens. In the figure, horizontal and vertical axes represent values calculated from Eq. 6 and obtained from experimental results respectively. The standard deviation and the correlation coefficient are 1.4 MPa and 0.991 respectively. Another important comparative parameter which be can used to check the accuracy of the formulas is a slope of the regression line between the calculated and measured values. The slope of the regression line for the proposed formula is about 0.97, the solid line in Fig. 4 represents the linear equation of Y = 0.97 X + 0.3. If the correlation is perfect between the calculated and measured values, the slope of the regression line would be 1. The comparisons with data indicate that the proposed formula give a good prediction.

Most of the experiments were performed about 50 years ago, and at that time experimental techniques, testing machines and quality assurance were very poor, so there can be little reliability. Considering the papers used in this study, conditions of tests were not described in detail and every paper had its own conditions. But, in this study a new theory is introduced to obtain a prediction model and satisfactory results are obtained through regression analyses on the experimental data.

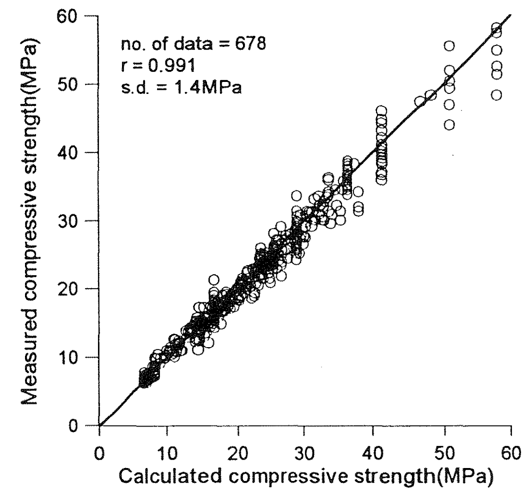

Fig. 4. Comparison of the measured and calculated values of concrete strength

3.2 Considering the effect of maximum aggregate size

The regression analyses were also performed to determine the model equation which can predict the reduction phenomena of compressive strength with the size of cylinder considering the effect of maximum aggregate size. From the regression analysis based on Eq. 5, it can be observed that the power of d_a is 0.00055 and the value of $d_a^{0.00055}$ approaches to 1.0. Since the effect of maximum aggregate size on the compressive strength is negligible, Eq. 6 can be also used in case of considering the maximum aggregate size. The reason is because most of maximum aggregate size used in the test is 2.54 cm and the range of size is also 1.27 ~ 5.08 cm.

3.3 Comparison of the results with ASTM requirements

When the non-destructive testing(NDT) of concrete structure is performed, generally, strength correction factors are used based on ASTM C42-94 to predict the strength of sampled cores with varying ratio of length/diameter. The experimental data of Gonnerman, Kesler and Murdock & Kesler are used to compare the prediction results with the suggested model equation and ASTM standard.

In Fig. 5, the thick and thin solid lines are results from Eq. 6 and ASTM requirements respectively. Specially, the thin line was obtained from the correction factors given in ASTM standard. Where the data point are selected from each author's paper at random and the line from Eq. 6 shows good agreements with the data point. In Fig. 5 vertical axis represents the strength correction factor and this is an inverse value of strength size factor, η. In the case where the length/diameter ratio exceeds the ASTM limit(1.94) the values from Eq. 6 can also be used with a sufficient correctness. It should be noted that Eq. 6 has a theoretical basis in fracture mechanics of concrete, while ASTM standard comes from the pure empirical bases.

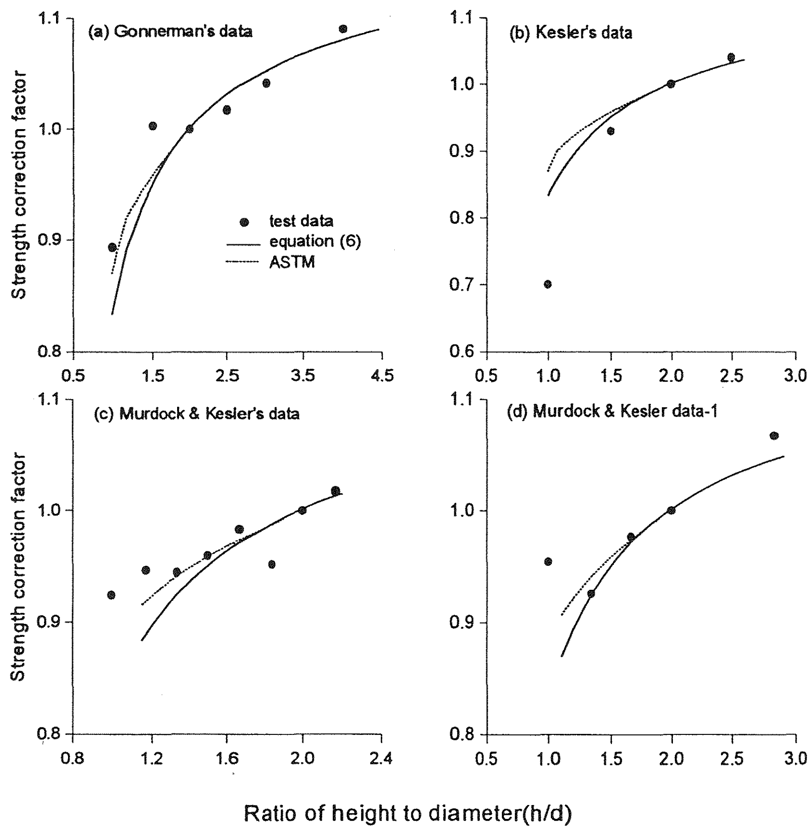

Fig. 5. Comparison of strength correction factors obtained based on ASTM C42-94 and Eq. 6

4 Conclusions

In this paper, a model equation which can predict the compressive strength of general cylinder specimens considering size effects is suggested based on collapse mechanism of non-linear fracture mechanics. To qualify this, regression analyses are performed on the existing uniaxial compressive strength data obtained from literature.

The following results are drawn from this work.

1. Strength reduction phenomena with increasing concrete member size are logically represented by using the size effect model equation based on non-linear fracture mechanics.
2. Where the compressive strength of concrete and diameter of cylinders are fallen in the range of 7～50 MPa and 3.81～91.44 cm respectively, the model equation based on size effects which can predict the concrete compressive strength of general height/diameter ratio is suggested.
3. The effect of maximum aggregate size on the size effect of compressive strength for general cylinders is negligible. The reason is that the effect of maximum aggregate size on the microcrack region can be ignored compared with the effect of characteristic length in the practical range.
4. According to suggested Eq. 6, the actual compressive strength of concrete is approximately 80% of laboratory standard cylinder strength when the confinement effects are disregarded.
5. The prediction values from Eq. 6 is less than those of ASTM standard but the difference is not great. And , if the height/diameter ratio exceeds the ASTM limit (1.94) the Eq. 6 can also be used with sufficient correctness.

5 Acknowledgment

The Authors would like to thank the Korea Science and Engineering Foundation for the partial financial support to ERC-STRESS(1996).

6 References

Bazant, Z. P., (1987) Fracture Energy of Heterogeneous Material and Similitude, **SEM-RILEM International Conference on Fracture of Concrete and Rock**, 390-402.

Bazant, Z.P., (1984) Size Effect in Blunt Fracture ; Concrete, Rock, Metal, **Journal of Engineering Mechanics, ASCE**, 110, 518-535.

Bazant, Z. P., (1993) Size Effect in Tensile and Compressive Quasibrittle Failures, **JCI International Workshop on Size Effect in Concrete Structures**, 141-160.

Bazant, Z.P., and Oh, B.H., (1983) Crack Band Theory for Fracture of Concrete, **Materials and Structures**, (RILEM, Paris), 16, 155-177.

Bazant, Z. P., and Xiang, Yuyin, (1997) Size Effect in Compression Fracture: Splitting Crack Band Propagation, **Journal of Engineering Mechanics, ASCE**, 123, 162-172.

Benjamin and Cornell, (1970) **Probability, Statistics, and Decision for Civil Engineers**, McGraw-Hill, New York, Section 4.3.

Blanks, R.F., and McNamara, C.C., (1935) Mass Concrete Tests in Large Cylinders, **ACI Journal**, 31, 280-303.
Dept. of the Interior, Bureau of Reclamation, Boulder Canyon Project, Final Report (1935), **Part VII-Cement and Concrete Investigations**, Bulletin 4, Mass Concrete Investigation, 105.
Gonnerman, H.F., (1925) Effect of Size and Shape of Test Specimen on Compressive Strength of Concrete, **Proc., ASTM**, 25, 237-250.
Gyengo, T., (1938) Effect of Type of Test Specimen and Gradation of Aggregate on Compressive Strength of Concrete, **ACI Journal**, 33, 269-282.
IMSL, Library, Edition 8, IMSL, Inc.
Johnson, R.F., (1962) Strength Tests on Scaled-Down Concrete Suitable for Models, **Magazine of Concrete Research**, 14, 47-53.
Kesler C.E., (1959) Effect of Length to Diameter Ratio on Compressive Strength-An ASTM Cooperative Investigation, **Proc., ASTM**, 59, 1216-1229.
Kim, J.K., and Eo, S.H., (1990) Size Effect in Concrete Specimens with Dissimilar Initial Cracks, **Magazine of Concrete Research**, 42, 233-238.
Kim, J.K., Eo, S.H. and Park, H.K., (1989) Size Effect in Concrete Structures without Initial Crack, Fracture Mechanics ; **Application to Concrete, SP-118, ACI**, Detroit, 179-196.
Kim, J.K. Eo, S.H., Moon, Y.H., and Cho, S.Y., (1987) The Size Effect for the Compressive Strength of Concrete, **J. of the Architectural Inst. of Korea**, Dec., 225-234.
Kuczynski, (1960) La Resistance du Beton Etudiee sur des Eporouvettes de Diffentes Formes et de Diveres Dimension, **RILEM Bulletin**, No.8, 77-95.
Murdock, J.W., and Kesler C.E., (1957) Effect of Length to Diameter Ratio of Specimen on the Apparent Compressive Strength of Concrete, **ASTM Bulletin**, No.221, 68-73.
Neville, A.M., (1955) The Influence of Size and Concrete Tests Cube on Mean Strength and Standard Deviation, **Magazine of Concrete Research**, 7, 121-132.
Nielson, K.E.C., (1954) Effect of Various Factors on the Flexural Strength of Concrete Test Beams, **Magazine of Concrete Research**, 15, 105-114.
Powers, T.C., (1956) Concrete Studies at the Bull Run Dam, City of Portland, Oregon, U.S. Department of the Interior, **Bureau of Concrete Research**.
Smadi, M.M., and Slate, F.O., (1989) Microcracking of High and Normal Strength Concretes under Short- and Long- Term Loadings, **ACI Materials Journal**, 86, 117-127.
Standard Test Method for Obtaining and Testing Drilled Cores and Sawed Beams of Concrete, (ASTM C42-94), 1995 Annual Book of ASTM Standards, Section 4, **American Society for Testing and Materials**, Philadelphia, 24-27.
Tucker, J., (1941) Statistical Theory of the Effect of Dimensions and Method of Loading on the Modulus of Rupture of Beams, **Proc., ASTM**, 41, 1072-1088.
Weibull, W., (1939) A Statistical Theory of the Strength of Materials, **Proc., Royal Swedish Inst. Eng. Res.**, No.151-152.