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Plasticity and damage mechanics models for concrete with alkali-aggregate reaction

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ABSTRACT: This paper describes two constitutive models for concrete affected by alkali-aggregate reaction. One is plasticity based model and the other damage mechanics model. Examples of prediction using a finite element program incorporate the models also presented.

1 INTRODUCTION

Alkali-aggregate reaction (AAR) in concrete produces a hygroscopic gel, which absorbs moisture and expands. The expansion of the gel causes cracking throughout the concrete.

AAR has the following simultaneous and interactive effects on the reinforced concrete structures.

- Variable deterioration of strength due to cracks inserted by the formation and expansion of the gel
- Variable deterioration of modulus of deformation
- Deterioration of bond between reinforcement and concrete
- Expansion, which induces stresses in the structure. The value of the expansion is highly variable within the structure due to variable level of restraint.

Many researchers/investigators found that the traditional approach for structural appraisal based purely on the properties of cores does not predict the performance of structures with AAR. This is mainly due to the prestressing effect of AAR expansion, which altered the mechanism on which the ordinary reinforced concrete structures work (Clayton 1990, Cope 1992, Imai 1986, Clark 1992, Fujii 1986, Bach 1992, Alexander 1992). More sophisticated approach taking into account of all the major effects, including material deterioration and AAR expansion, should be adopted

A finite element (FE) computer program for the analysis of AAR affected concrete structures was developed in 1993, in which a plasticity-based constitutive model was adopted for the concrete (Cope et al 1994). As a further development of the computer program, a damage mechanics model was also developed (Wen et al 2000). In his paper, the finite element computer program is first briefly outlined followed by a brief description of the two constitutive models, which are illustrated trough examples.

2 OUT LINE OF FE PROGRAM

The computer program is running in three stages as shown in Figure 1.

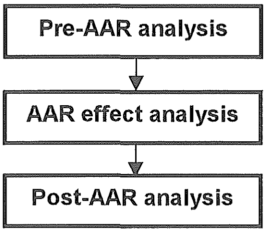

Figure 1. The FE program structure.

The first stage, pre-AAR analysis, is a non-linear finite element analysis under given service loading before AAR occurs in concrete. This stage is necessary for concrete with AAR because of the stress dependent nature of AAR expansion.

The second stage is AAR effect analysis simulating the formation and propagation of AAR cracks and expansion in the concrete. An incremental approach is adopted in this stage allowing for the interaction between AAR expansion, which is stress-dependent, and the stresses, which change with the growth of expansion. Materials degradation due to cracking and expansion is also evaluated in this stage and this is further discussed in Section 3.

The third stage is the analysis of the affected structure to assess its post-AAR performance. In this stage the affected structure is loaded to a desired level or up-to failure.

3 MODEL FOR AAR EXPANSION

The expansion of the concrete is a result of expansion of a hygroscopic gel when the gel takes in moisture under osmotic pressure. The expansion is, therefore, sensitive to the restraining pressure. Figure 2 shows longitudinal expansion curves of plain and reinforced cylinders with variable reinforcement ratio. The expansion was significantly reduced by a moderate amount of reinforcement.

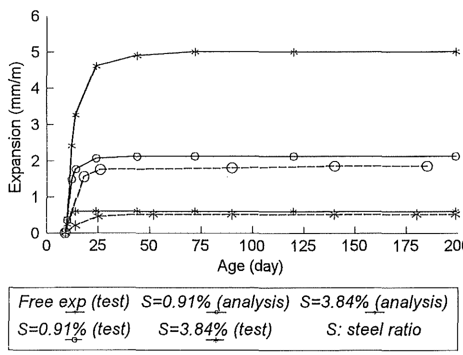

Figure 2 Longitudinal expansions of plain and reinforced concrete cylinders, analytical results (Wen 1993) compared with experiment (Hobbs 88)

Due to the stress-dependent nature of the expansion, an incremental expansion analysis must be adopted in order to reflect the interaction between induced stress and expansion. Based on available data from UK and Japan, an expansion model was proposed (May et al 1991, Wen 1993). In this model, the process of expansion is traced not in terms of real time, but in terms of the value of the so-called "free-expansion". Free-expansion is the expansion a concrete would experience if there were not any restraint to the concrete. For a given increment of free-expansion, the restrained expansion is determined according to the following equation.

$$\Delta\varepsilon_{exi} = F(\sigma_i)\Delta\varepsilon^{f}_{ex} \tag{1}$$

where $\Delta\varepsilon^{f}_{ex}$ = free-expansion increment; $\Delta\varepsilon_{exi}$ = actual expansion in ith principal stress direction; $F(\sigma_i)$ = relation between free expansion and restrained expansion which is a function of principal stresses. An example of simplified relationship is given in Equation (2).

$$F(\sigma_i) = \begin{cases} 1 & 0 < \sigma_i \\ \left(1 - \dfrac{\sigma_i}{\sigma_{co}}\right) & \sigma_{co} \le \sigma_i < 0 \\ 0 & \sigma_i < \sigma_{co} \end{cases} \tag{2}$$

where σ_{co} is the value of compressive stress at which the expansion is suppressed completely. From the research on AAR concrete, the value of σ_{co} is around between 4 to 6 N/mm^2.

The incremental analysis is carried out in a large number of incremental step to simulate AAR cracking and expansion growth. In each step, a small increment of free-expansion is given to each integration (sampling) point of concrete element and restrained expansion is determined. Material deterioration is also evaluated at each step. The expansion analysis stops when a pre-assumed value of total free-expansion is reached.

Figures 3 and 4 show an example of the expansion analysis. Figure 3 shows a beam tested by Cope et al (Cope 1990). In this experiment, horizontal expansions along top and along reinforcement and vertical expansion on the side surface of the beam were investigated. Plain concrete cylinders made of the same concrete were put aside the beams to measure the free-expansion of the concrete. The measured expansions on the beam were compared with the free-expansion of the cylinders. Figure 4 shows comparison of the results with the predictions using the expansion model.

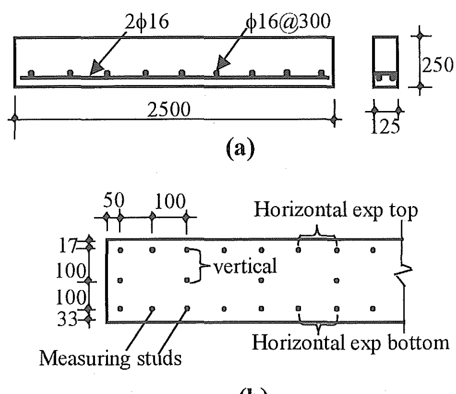

Figure 3. (a) Simply reinforced RC beam tested for AAR expansion by Cope et al (Cope 1990). (b) Arrangement of expansion measurement.

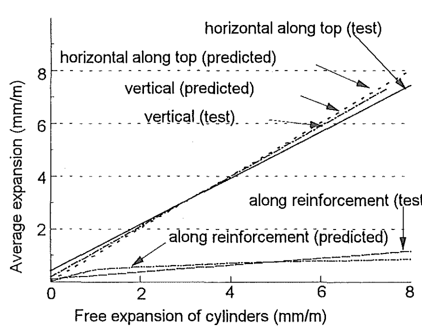

Figure 4. Measured expansion compared with prediction using the FE expansion analysis for the beam tested by Cope et al (Cope 1990, 1994)

4 MODELING OF DETERIORATION OF CONCRETE

As described in Section 3, the expansion analysis is carried out in a large number of steps to simulate growth of AAR expansion. At the end of each step, the properties of affected concrete are modified according deterioration model for the concrete.

Deterioration of concrete properties is a function of equivalent expansion. The following is a set of equations governing the change of material properties due to AAR expansion.

In the plasticity model, the effective stress-strain relation for concrete in compression takes the form suggested by Sturman et al (Sturman 1965), Equation (3).

$$\sigma = E\left(1 + C|\varepsilon|^{\alpha-1}\right)\varepsilon \tag{3}$$

where σ = effective stress at a sampling point; ε = effective strain; E = initial tangent modulus of effective stress-strain curve. E is determined as follows.

$$E = \frac{E_c}{1 + \varepsilon_{ex} E_c / E_g} \tag{4}$$

where E_c = initial tangent modulus of elasticity of concrete before AAR occurring; ε_{ex} = equivalent AAR expansion; E_g = nominal modulus of elasticity of the gel which is usually in the range of 60 - 120 N/mm^2.

α and C in Equation (3) are determined by Equations (5) and (6).

$$\alpha = \frac{1}{1 - \dfrac{f'_{ao}}{E\varepsilon_{ao}}} \tag{5}$$

$$C = -\varepsilon_{ao}^{1-\alpha}\left(1 - \frac{f'_{ao}}{E\varepsilon_{ao}}\right) \tag{6}$$

In Equations (5) and (6), f'_{ao}, ε_{ao} are peak compressive stress and strain of AAR affected concrete. They are functions of AAR expansion and determined as follows.

$$f'_{ao} = f'_c - A\varepsilon_{ex} \tag{7}$$

$$\varepsilon_{ao} = \varepsilon_o + [\beta + (1-\beta)e^{-1000\varepsilon_{ex}}]\varepsilon_{ex} \tag{8}$$

Ultimate crushing strain, ε_{au}, of AAR affected concrete is defined in Equation (9).

$$\varepsilon_{au} = \varepsilon_u + [\beta + (1-\beta)e^{-1000\varepsilon_{ex}}]\varepsilon_{ex} \tag{9}$$

In Equations (7) to (9), f'_c = strength (peak stress) of concrete before AAR occurring; ε_o = strain at peak stress of concrete before AAR occurring; A = strength reduction factor (normal range is between 1200 - 1500 N/mm^2); β = strain factor (about 0.5); ε_u = ultimate crushing strain of concrete without AAR.

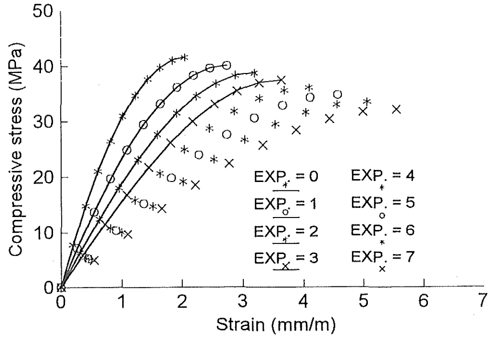

Figure 5. The model for deterioration of concrete with AAR expansion.

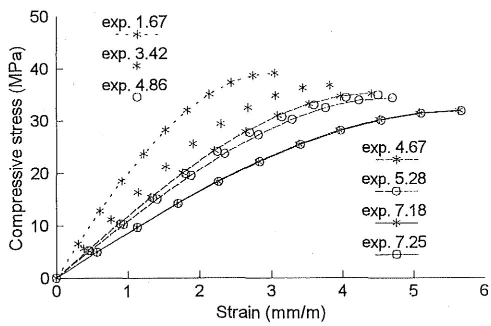

Figure 6 Stress-strain curves of concrete with AAR expansion obtained from laboratory experiments (Wen 1993).

5 PLASTICITY BASED CONSTITUTIVE MODEL

In plasticity-based approach, crack due to direct tensile stress (Type I crack) was represented using the smeared crack model. Strain softening behavior is assumed for concrete after cracking. Closing of cracked concrete follows the path of secant modulus. For concrete under compression, the behavior is governed by the effective stress – plastic strain relationship, which is obtained from Equations (3) to (9) by subtracting the elastic strain.

$$\underline{\sigma} = E\left(\frac{\underline{\sigma}}{E} + \underline{\varepsilon}_p\right)\left[1 + C\left(\frac{\underline{\sigma}}{E} + \underline{\varepsilon}_p\right)^{\alpha-1}\right] \tag{10}$$

where $\underline{\sigma}$ = effective stress; $\underline{\varepsilon}_p$ = effective plastic strain; E, C and α are determined fro Equations (4) to (6).

Associated flow and work hardening rules are adopted for the concrete model.

6 EXAMPLE OF BEAM FOR PARAMATRIC STUDIES

The program has been used to predict a number of AAR affected RC structural members and these are

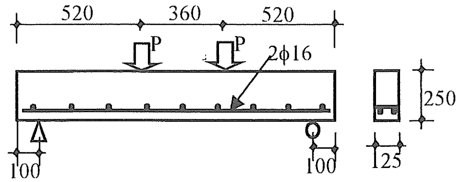

Figure 7. Beam for parametric studies.

presented elsewhere (Cope 1994, Wen 1993). The predictions and actual behavior of the members generally agreed well.

In this section, a parametric study on a hypothetical beam is presented as a demonstration. This beam imitates the beams tested by Cope et al (Cope 1990). The details of the beam and loading position are given in Figure 7. In this study, the influence of loading applied during the expansion growth and the value of free-expansion on structural performances were investigated. Figure 8 shows the load-deflection curves for four beams: one control beam without AAR and other three beams with variable value of free-expansion from 1mm/m to 5mm/m. The beams were subject to a constant load, about 30% of their load capacity during expansion period. The horizontal line in Figure 8 indicates the change of deflection due to the growth of AAR expansion for beams subject to constant load during AAR expansion. The deflections shown in Figure 8 were measured on the top of the middle section of the beams.

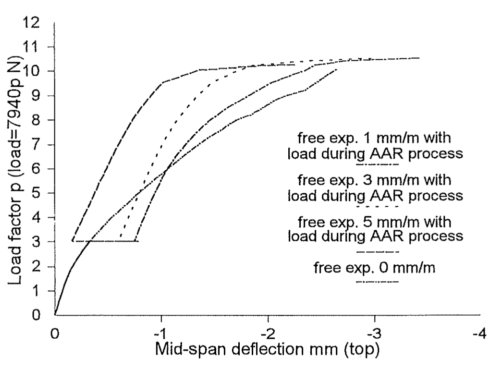

Figure 8 Load deflection curves for beams subjected to different AAR expansion. The beam with AAR expansion was loaded with about 30% of its ultimate load.

Figure 9 shows the crack patterns of three beams when they approach to failure. Figure 9(A) is for the control beam without AAR. The beam is failed in a shear mode. The bunch of "smeared" cracks indicates a major shear crack.

Figure 9(B) is for a beam without loading during AAR expansion (not shown in Figure 8). After AAR has reached to a free-expansion of 5mm/m, the beam was loaded up to failure. The failure mode, however, changed to that of bending due to the reinforcement reach its yield stress.

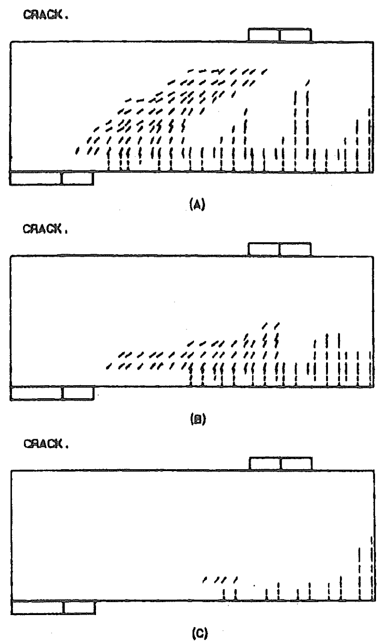

Figure 9 Predicted crack pattern at failure. The figures show only half of the beams elevation. (A) is control beam without AAR; (B) is the beam without load applied during AAR expansion period; (C) is the beam subject to 30% ultimate load when AAR takes place.

Figure 9(C) is for a beam subject to 30% its ultimate load during the period of AAR. The analysis showed that the beam failed in a bending mode with a major bending crack at the middle section of the beam.

7 DAMAGE CONSTITUTIVE MODEL FOR CONCRETE WITH AAR

In a damage mechanic model material deterioration is measured by a damage index, D.

$$\underline{\sigma} = E_c(1-D)\underline{\varepsilon} \tag{11}$$

where D = damage index; E_c = initial modulus of concrete before subject to loading; $\underline{\sigma}$ and $\underline{\varepsilon}$ are equivalent stress and strain respectively.

To derive for the damage index, Equation (3) can be rewritten as

$$\begin{aligned}\sigma &= \frac{E_c}{1+\varepsilon_{ex}E_c/E_g}\left(1+C|\varepsilon|^{\alpha-1}\right)\varepsilon \\ &= E_c\left(\frac{1}{1+\varepsilon_{ex}E_c/E_g}+\frac{C|\varepsilon|^{\alpha-1}}{1+\varepsilon_{ex}E_c/E_g}\right)\varepsilon\end{aligned} \tag{12}$$

Compare Equations (11) and (12), we have

$$D = 1-\frac{1}{1+\varepsilon_{ex}\,E_c/E_g}-\frac{C|\varepsilon|^{\alpha-1}}{1+\varepsilon_{ex}\,E_c/E_g} \tag{13}$$

For concrete affected by AAR subject to further loading, the damage index, D, is expressed as follows.

$$D = \begin{cases} D_I + C_1 |\varepsilon|^\gamma & \text{for } 0 \le \varepsilon \le \varepsilon_{a0} \\ D_{a0} + C_3(\varepsilon - \varepsilon_{a0}) & \text{for } \varepsilon_{a0} \le \varepsilon \le \varepsilon_{au} \end{cases} \quad (14)$$

where D_I is initial damage of AAR affected concrete immediately before further loading; D_{a0} is the damage of affected concrete at $\varepsilon = \varepsilon_{a0}$ when subjected to further loading.

C_1, C_3 and γ are constants determined from initial conditions when AAR affected concrete is subjected to further loading. Comparing Equations (13) and (14) the constants are obtained as follows.

$$D_I = 1 - \frac{1}{1 + \varepsilon_{ex} E_c / E_g} \quad (15)$$

$$C_1 = \frac{-C}{1 + \varepsilon_{ex} E_c / E_g} \quad (16)$$

$$C_3 = \frac{1 - D_{a0}}{\varepsilon_{au} - \varepsilon_{a0}} \quad (17)$$

$$\gamma = \alpha - 1 \quad (18)$$

Equation (14) is shown in Figure 10.

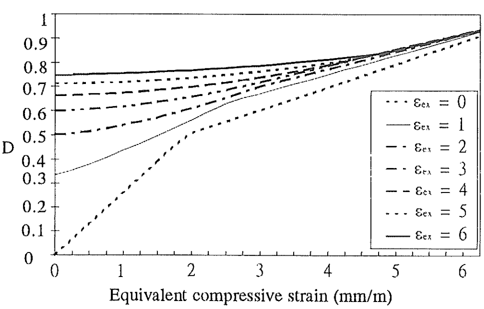

Figure 10 Damage index for concrete affected by AAR and subject for further loading.

Figure 11 plots stress – strain relation obtained by substituting D in Equation (14) in to Equation (11). As expected the pre-peak part of the curves are the same as those shown in Figure 5.

In the damage mechanics model, equivalent strain takes the form proposed by Mazars (Mazars 1991). The total damage is expressed as follows.

$$D = \alpha_T D_T + \alpha_C D_C \quad (19)$$

where D_T and D_C are damage index of concrete subject to tensile and compressive stresses under uniaxial condition; α_T and α_C are factors measuring the contributions of damage due to tension component

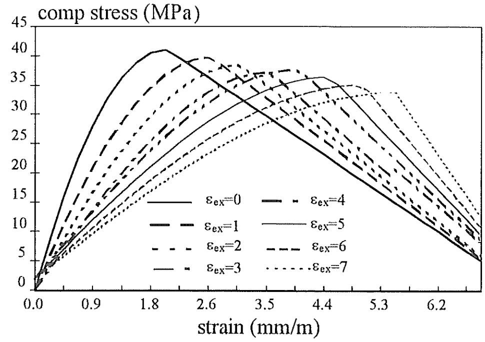

Figure 11 Equivalent stress – strain relations of concrete using the damage mechanics model.

and compressive component respectively. α_T and α_C are determined as follows.

$$\alpha_T = \frac{1}{\tilde{\varepsilon}^2}\left[(1-q)\sum_{i=1}^{3}\varepsilon_{Ti}(\varepsilon_{Ti} + \varepsilon_{Ci}) + q\sum_{i=1}^{3}\varepsilon_{Ti}\langle\varepsilon_{Ti} + \varepsilon_{Ci}\rangle\right] \quad (20)$$

$$\alpha_C = \frac{1}{\tilde{\varepsilon}^2}\left[(1-q)\sum_{i=1}^{3}\varepsilon_{Ci}(\varepsilon_{Ti} + \varepsilon_{Ci}) + q\sum_{i=1}^{3}\varepsilon_{Ci}\langle\varepsilon_{Ti} + \varepsilon_{Ci}\rangle\right] \quad (21)$$

In Equations (20) and (21), $\tilde{\varepsilon}$ is equivalent strain, which is evaluated by Equation (22).

$$\tilde{\varepsilon} = \sqrt{(1-q)\sum_{i=1}^{3}\varepsilon_i^2 + q\sum_{i=1}^{3}\langle\varepsilon_i\rangle^2} \quad (22)$$

In Equations (20) to (22), $\langle\cdot\rangle$ is Macauley bracket. $\langle x\rangle = x$ when $x \ge 0$; $\langle x\rangle = 0$ when $x < 0$.

ε_{Ti} is the positive strain in principal stress direction, i, due to direct tensile stress in that direction plus positive strain due to compressive stresses in other principal directions (Poisson's effect). ε_{Ci} is the negative strain in principal stress direction i due to direct compressive stress in that direction plus negative strain due to tensile stresses in other principle directions. It is obvious that

$$\varepsilon_i = \varepsilon_{Ci} + \varepsilon_{Ti} \quad (23)$$

Factor q is a weighting factor for tensile strain in the calculation of equivalent train. By changing q value in Equations (20) – (22), the contribution of tensile and compressive strain to the damage of concrete can be adjusted. When q = 1,

$$\tilde{\varepsilon} = \sqrt{\sum_{i=1}^{3}\langle\varepsilon_i\rangle^2} \quad (24)$$

Equation (24) is equivalent strain proposed by Mazars (Mazars1991), which relates damage of concrete only to tensile strain. Appropriate q value is better determined through experiment.

It may be noted, since

$$\tilde{\varepsilon}^2 = (1-q)\sum_{i=1}^{3}\varepsilon_{Ci}(\varepsilon_{Ti}+\varepsilon_{Ci}) + q\sum_{i=1}^{3}\varepsilon_{Ci}\langle\varepsilon_{Ti}+\varepsilon_{Ci}\rangle + (1-q)\sum_{i=1}^{3}\varepsilon_{Ti}(\varepsilon_{Ti}+\varepsilon_{Ci}) + q\sum_{i=1}^{3}\varepsilon_{Ti}\langle\varepsilon_{Ti}+\varepsilon_{Ci}\rangle \tag{25}$$

from Equations (20) and (21), it is obvious that $\alpha_T + \alpha_C = 1$.

In the case of AAR affected concrete subject to compression

$$D_C = 1 - \frac{1}{1+\varepsilon_{ex}\,E_c/E_g} - \frac{C_2|\tilde{\varepsilon}|^{\alpha-1}}{1+\varepsilon_{ex}\,E_c/E_g} \tag{26}$$

8 EXAMPLE OF DANAGE MECHANICS MODEL

A concrete cube is subjected to three dimensional compression, Figure 12, with equal lateral pressures, i.e. $\sigma_1 = \sigma_2 = \beta\sigma_3$. Consider the cases when β varies between 0 and 0.25. Poisson's ratio $\nu = 0.2$.

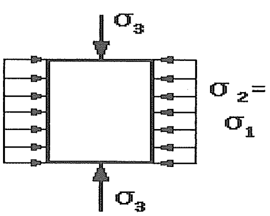

Figure 12. A concrete cube subject to three dimensional compression.

The following q values proposed by Wang et al (Wang 1985) are used for calculation

$$q = q(\beta) = \begin{cases} 12.85\beta; & 0.00 \le \beta < 0.05 \\ 0.6425 + 3.915(\beta - 0.05); & 0.05 \le \beta < 0.15 \\ 0.934 + 0.44(\beta - 0.15); & 0.15 \le \beta < 0.25 \\ 1.0; & 0.25 \le \beta < 1.00 \end{cases} \tag{27}$$

For this example

$$\varepsilon_3 = [\sigma_3/E(D)](1-2\nu\beta) \tag{28}$$

$$\varepsilon_1 = \varepsilon_2 = [\sigma_3/E(D)](\beta - \nu(1+\beta)) \tag{29}$$

Since strain in all three dimensions are negative, thus,

$$D = D_C = 1 - \frac{1}{1+\varepsilon_a\,E_c/E_g} - \frac{C_2|\tilde{\varepsilon}|^{\alpha-1}}{1+\varepsilon_a\,E_c/E_g} \tag{30}$$

Stress (σ_3) - strain (ε_3) curves for β = 0, 0.05, 0.10, 0.15 and 0.20 are plotted in Fig. 13. σ_3-ε_3 curve for concrete affected by AAR for β = 0.10 and β = 0.20 with variable expansion are shown in Figs. 14 and 15 respectively.

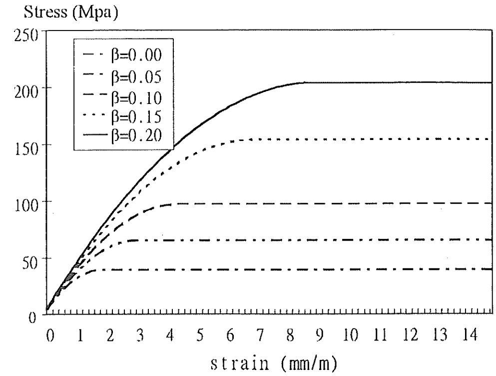

Figure 13. Concrete without AAR under lateral restraint ratio up to 0.2.

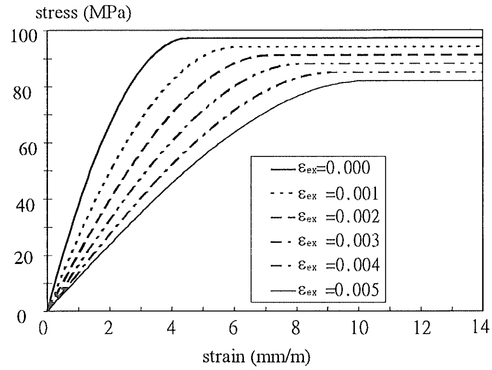

Figure 14. Post-AAR stress-strain curves for concrete with AAR expansions up to 0.005. The lateral compression ratio is 0.1.

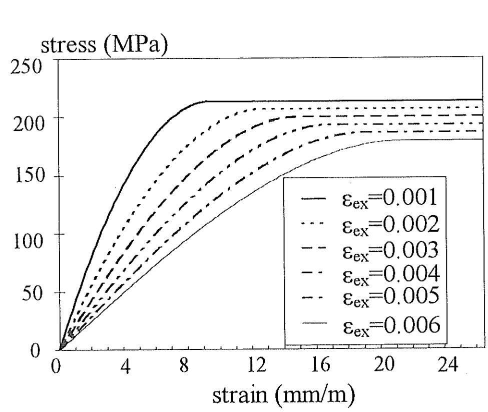

Figure 15 Post-AAR stress-strain curves for concrete with AAR expansions up to 0.006. The lateral compression ratio is 0.2.

9 CONCLUDING REMARKS

In this paper two constitutive models for concrete with AAR are introduced. In the plasticity model, deterioration of concrete due to AAR is reflected by the degradation of effective stress – strain relation governing the progression of yield surface and hardening of the concrete. This is not happening in EF analysis for normal concrete without AAR. The variation of governing stress-strain relation increased the numerical instability of the analysis.

Taking into account the interaction between AAR expansion and stress, an increment analysis must be adopted for the analysis of the expansion. Research also showed that it is appropriate to consider the loading condition during the period when AAR expansion growth for the assessment of affected structures.

The deterioration of concrete in damage mechanics model is reflected through the total damage index. Damage index is divided in tow separate items. The damage index reflecting the effect of AAR is accumulated during the AAR growth in a similar manner with the plasticity model. Damage index for post AAR concrete was in principle similar to those for normal concrete but derived from stress-strain relation of affected concrete.

Preliminary running of the FE computer program shows both models incorporating damage mechanics model reviewed that the model naturally reflects the characteristics of concrete affected by AAR and is easy to implement in finite element computer program.

In both models, only the cracks caused by direct tension (type I crack) are explicitly expressed using a smeared crack model. The cracks due to gel expansion are considered as an integral part of the affected concrete and included in the expansion of the concrete.

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