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Fast Fourier One-dimentional analysis of concrete crack surface

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ABSTRACT: The 3-dimensional shape of a concrete crack surface results from the crack-forming process under a certain stress condition, and the properties of the shape provide important data for elucidation of formation and extension mechanisms. We previously presented analysis functions to represent the shape properties of a concrete crack surface based on the 3-dimensional coordinates obtained by a measurement system using the structured light projection method. Fourier analysis are widely used methods for analyzing shapes of objects, but these methods have not been applied to the analysis of concrete. It is thought, however, that Fourier analysis can be useful to the determination of the shape of a concrete crack surface. As the first step to achieving our final goal of the establishment of accurate constitutive law of a concrete crack surface, we show here the usefulness of Fourier transform as a method for determining the shape properties of a concrete crack surface.

1 INTRODUCTION

We have proposed methods for 2-dimensional and 3-dimensional analyses of the shape properties (e.g., inclination, depth and surface area) of concrete crack surfaces, and we have presented and discussed results of 2-dimensional and 3-dimensional analyses of the surfaces of cracks formed by four different stress modes (tension, splitting, bending and shear). These methods have enabled evaluation of each shape property of a concrete crack surface, and aspects of similarities among crack shapes and shape properties specific to cracks that had not been known have been revealed.

In the field of applied physics, Fourier transform and fast Fourier transform (FFT) have been widely used to analyze shapes of objects, and many aspects concerning object shapes have been revealed using these methods. The types of objects analyzed by these methods are limited to those for which shape changes are relatively small. The entire shape of the object can be described by only sine and cosine functions from Fourier analysis, enabling an irregular shape to be represented by a frequency spectrum distribution.

There have only been a few studies in which Fourier analysis has been applied to concrete material, such as studies on cross-sectional profiles of concrete joint surfaces with a metal trowelling finish using FFT one-dimensional analysis, but, to the best of our knowledge, there have been no studies in which concrete crack surfaces have been analyzed by Fourier analysis.

Fourier analysis of concrete crack surfaces would enable the shape properties of a crack surface (e.g., inclination, depth and surface area) to be all analyzed together as a Fourier spectrum. However, since a concrete crack surface is a "rough surface", it would be necessary to first investigate how the degree of surface uneveness affects the analytical results in order to systematically perform Fourier analysis of a concrete crack surface.

In this study, in order to perform FFT one-dimensional analysis of 3-dimensional coordinate data obtained by using the structured light projection method, described in our previous report, we first investigated how differences in the values of measurement sensitivity (SN) and sampling interval (SI), parameter for measurement and analysis, change the analytical results, and we obtained appropriate parameter values. Using these parameter values, we then performed FFT one-dimensional analysis on specimens of concrete cracks formed by tension, the basic crack-forming stress mode. Finally, we examined the applicability and problems concerning the applicability of FFT one-dimensional analysis to evaluation of the shapes of concrete crack surfaces.

2 CRACK SURFACE MODEL FOR FFT ONE-DIMENSIONAL ANALYSIS

There are two types of Fourier analysis of the shape of an object: 1) analysis of the surface of an object only in the direction of an arbitrary straight line (Fourier one-dimensional) and 2) analysis of the entire surface of an object (Fourier two-dimensional analysis). Both of these methods are thought to be applicable to analysis of concrete crack surfaces, although the applicability of Fourier analysis to concrete crack surfaces has not actually been tested.

Therefore, in this study, as a first step toward determining the applicability of Fourier analysis to concrete crack surfaces, we tested the applicability of FFT one-dimensional analysis for determining the shape of a concrete tensile crack surface. The method we used for measurement of three-dimensional coordinate values of the concrete crack surface, which are essential for the analysis, was the same as that used in our previous study and has been described in detail in a previous paper.

2.1 *Method for measuring concrete crack surface coordinates*

The structured light projection method was used for measurement of three-dimensional coordinate values of the concrete crack surface. In this measurement system, a cross-sectional profile of the crack surface is inputted into a computeras an image and converted into numerical values. Three-dimensional coordinate values of the concrete crack surface are obtained by composite image processing. An example of a coordinate system obtained by this measurement method is shown in Figure 1,where the x-y plane is the measurement reference plane and the z-axis shows the uneven surface of the crack surface.

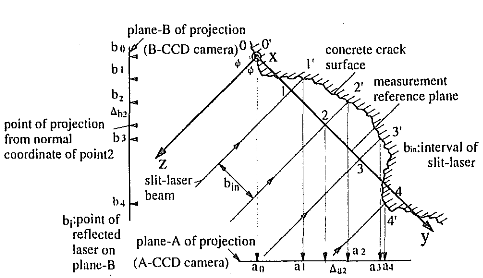

Figure 1 Coordinate axes in the measurement system

2.2 *FFT cross-sectional modeling of a crack surface*

The basic equation used in this study for FFT one-dimensional analysis of a concrete crack surface is the following equation for calculating Fourier coefficient C of discrete data at point N:

$$c_k = \frac{1}{N}\sum_{j=0}^{N-1} z_j w^{-kj} \tag{1}$$

where $k = 0, 1, \cdots, (N-1)$

$$w = e^{\frac{2\pi i}{N}} \tag{2}$$

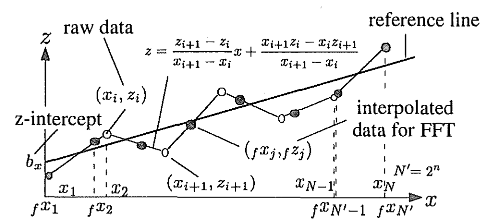

Figure 2 Cross-sectional model of concrete crack surface for one-dimentional analysis

A cross-sectional profile model (shown in Figure. 2) is needed to apply *Eq.*(1) to the three-dimensional coordinate values of a crack surface. This model requires that 2^n sampling points be set in a straight line on the plane to be analyzed; however, there are not necessarily 2^n raw data points on the crack surface cross-sectional profile. Thus, for the analysis in this study, we divided the crack cross-sectional profile into 2^n equal parts, and we derived a formula for a straight line passing through the adjacent raw data points to calculate the coordinate values in the 2^n-equally-divided parts.

3 ESTABLISHMENT OF MEASUREMENT AND ANALYTICAL PARAMETERS FOR FFT ONE-DIMENSIONAL ANALYSIS

The basic stress modes causing concrete cracking are tension, bending and shear. Among these three basic modes, the mode that is the most stable under the condition of uniformly distributed load and for which the concrete cracking process is the most obvious is tension. In order to investigate the applicability of Fourier analysis to an object surface that is divided into rough surfaces such as a concrete crack surface, a crack surface that has relatively few rough parts is desirable. Considering the stability of tension as a mode of concrete cracking under the condition of a uniformly distributed load and based on our previous experimental results showing that tensile specimens produced the flattest surfaces on cracking, we decided to use tensile test specimens for analysis in the present study.

3.1 *Outline of the tensile crack experiments*

A schematic of a tensile test specimen is shown in Figure 3,and data on the test specimens are shown in Table 1. The composition of a concrete specimen, which is the same composition as that used for steel-reinforced concrete in buildings, is shown in Table 2. Table 3 shows the properties of the concrete.

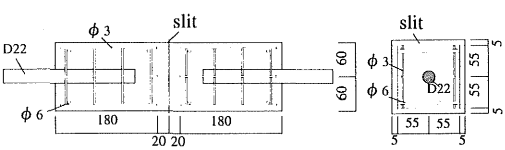

Figure 3 Concrete specimen used in the tensile experiment

Table1 The data on the test speciments

notation	size	stress mode	number of speciments
Tn-1~Tn-5	130x130x400	tension	5

Table2 Mix design of concrete

cement	nominal strength	slump	Maximum size of coarseaggregate
normal	30N/mm^2	18cm	20mm

coarse aggregate	cement	water cement ratio	sand-coarse aggregate ratio
crashed stone	N	48.00%	47.70%

Table3 Concrete properties

name	measured slump	compressive strength	splitting strength
S18-	21.7cm	36.2N/mm^2	2.64N/mm^2

3.2 *Differences in the results of FFT one-dimensional analysis due to differences in measurement sensitivity and sampling interval*

In this study, FFT one-dimensional analysis using a total of nine combinations of three different measurement sensitivities (SN) (0.114mm/dot, 0.125mm/dot and 0.159mm/dot) and three different sampling intervals (SI) (0.5mm, 1.0mm and 2.0mm) (see Table 4) was applied to the tensile crack surface plane of specimen Tn-3 to determine how changes in measurement sensitivity and sampling interval affect the results of Fourier analysis. The measurement areas on the measurement reference plane are about 56mm×60mm at the measurement sensitivity of 0.114mm/dot and about 58mm×60mm at the other measurement sensitivities.

Table4 Measurement condition for crack surface (Tn-3)

angle of incident (rad)	SN (mm/dot)	interval of laserbeam (mm)	number of composited images	SI (mm)
$\pi/4$	0.114	0.5	121	0.5
	0.125			1
	0.159			2

Figure 4 shows a 3-dimensionally reconstructed model of the tensile crack surface of specimen Tn-3 (SN: 0.125mm/dot, SI: 0.5mm).It can be seen that the 3-dimensional coordinate values of the crack surface sufficiently represent the unevenness of the crack surface. FFT analysis using the abovementioned nine combinations of measurement sensitivity and sampling interval was applied to this model of the Tn-3 specimen crack surface, and the analytical results in only the x-axis direction of the coordinates on the measurement reference plane are shown as a histogram with frequency on the horizontal axis and intensity on the vertical axis.

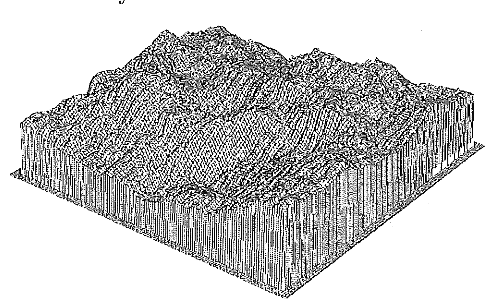
Figure 4 3-dimensionally reconstructured model of tensile crack surface of Tn-3

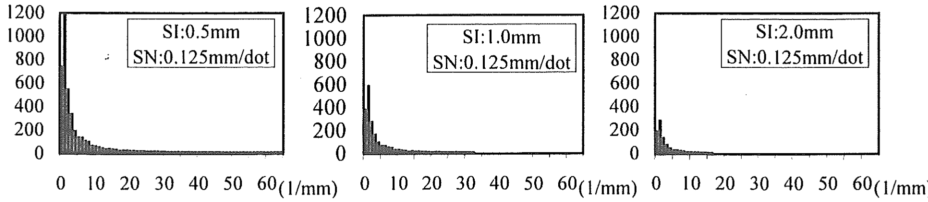

Figure 5 Difference in the result of FFT due to difference of sampling interval (x-axis direction)

Figure 5 shows the results of analysis with only one fixed measurement sensitivity of $0.125mm/dot$ and three sampling intervals ($0.5mm$, $1.0mm$ and $2.0mm$). According to Fourier analysis theory, the smaller the number of sampling points (i.e., the larger the sampling interval) is, the larger is the analytical maximum frequency (hereafter called fm), which can cause great changes in the intensity spectrum distribution. Such changes can actually be seen in Figure 5. These differences in the intensity spectrum distributions make comparison of the analytical results difficult. This problem can be resolved by "normalizing" the analytical results. The normalized analytical results of FFT one-dimensional analyses using the abovementioned nine combinations of measurement sensitivity and sampling interval are shown in Figure 6. The f_m valuescorresponding to sampling intervalsof $0.5mm$,$1.0mm$ and $2.0mm$ are $65/mm$, $33/mm$ and $17/mm$, respectively.

The FFT one-dimensional spectra of the Tn-3 specimen crack surface all show the same characteristics: a downward sloping distribution with high intensity at low frequency and a sudden decline in intensity when frequency exceeds $10/mm$. A comparison of the results of FFT one-dimensional analyses with measurement sensitivites of $0.114mm/dot$, $0.125mm/dot$ and $0.159mm/dot$ shows that the intensity peaks in the minimum frequency region in the case of the highest measurement sensitivity ($0.114mm/dot$), whereas it peaks in the next-highest frequency re-

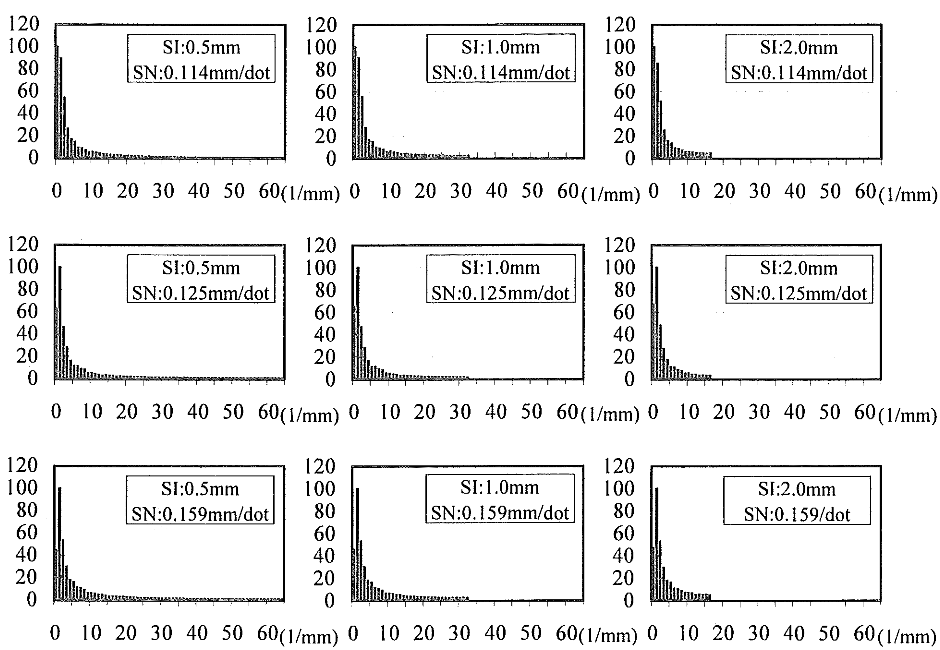

Figure 6 Normalized analytical results of FFT One-dimentional analyses using the 9 combinations of SN and SI(x-axis direction)

gion to the minimum frequency region in the case of measurement sensitivities of $0.125mm/dot$ and $0.159mm/dot$. This slight difference in the measurement sensitivities is thought to be due to the previously mentioned difference in the areas of FFT one-dimensional analysis. This effect of the area of analysis was also seen in the authors' previous study1) on two-dimensional analysis of the shape properties of concrete crack surfaces. The results also showed that there is almost no differences in the results of analyses using measurement sensitivities of $0.125mm/dot$ and $0.159mm/dot$. Next, we compared the results of analyses using different sampling intervals ($0.5mm$, $1.0mm$ and $2.0mm$). Since the number of sampling points naturally decreases as the sampling interval isincreased, it becomes impossible to analyze the high frequency region of the intensity spectrum obtained by FFT one-dimensional analysis of a crack surface. If a large proportion of the intensity distribution of the crack surface shape is confirmed to exist in the frequency region that has been cut due to an increase in the sampling interval, it will be necessary to increase the number of sampling points for Fourier analysis of the crack surface. The intensity distribution in the high frequency region in Figure 6 in the case of a sampling interval of $2.0mm$ can not be sufficiently evaluated, indicating that a sampling interval of less than $1.0mm$ was appropriate for the present analysis. The above-mentioned characteristics of the results of FFT one-dimensional analysis of a crack surface in the x-axis direction were also found in the case of analysis in the y-axis direction, but the difference in the intensity peak region in the case of maximum measurement sensitivity ($0.114mm/dot$) seen in the results of analyses in the x-axis direction was not seen in the results of analyses in the y-axis direction. Thus, more stable results were obtained from analyses in the y-axis direction than from analyses in the x-axis direction. The results of both FFT one-dimensional analysis in the x-axis direction (shown in Figure 5) and FFT one-dimensional analysis in the y-axis direction indicated that the measurement sensitivities and sampling intervals selected in this study are sufficient for FFT one-dimensional analysis of a crack surface and that a measurement sensitivity of $0.114mm/dot$ and sampling interval of $1.0mm$ are appropriate for analysis of the entire intensity spectrum distribution, including the distribution in the high frequency region.

4 FFT ONE-DIMENSIONAL ANALYSIS OF A TENSILE CRACK SURFACE

As was observed in our previous study, the location of a concrete crack surface is sometimes different to that expected from the experimental conditions (hereafter called estimated crack surface). There are two methods for dealing with this difference in the locations of estimated and actual crack surfaces for Fourier analysis: 1) using the raw data of the 3-dimensional coordinates of for Fourier analysis (method A) and 2) using translated coordinate data obtained by translating and rotating the measurement sampling points by using the concepts of measurement crack plane and measurement crack line for Fourier analysis (method B). The results obtained by these two methods should be compared.

Two different techniques can be used in method B. In one technique (B-1), as shown in Figure 2, a cross-sectional profile of the crack surface is extracted from the raw data of the measured coordinates, and the measurement crack line is calculated by applying the method of least squares to the sampling points on the profile. The measurement sampling points are then translated and rotated from the z intercept of the measurement crack line and the angle of inclination of the measurement crack line with the measurement reference plane (θ_{s0}) These translated values are used for the FFT one-dimensional analysis. In the other technique (B-2), the measurement crack plane is calculated by applying the method of least squares to the raw data of the measured 3-dimensional coordinates of the crack surface, and the transformed 3-dimensional coordinate data of the crack surface are calculated from the direction cosine vector and position vector of the measurement crack plane so as to match the measurement reference plane to the measurement crack plane. Next, the cross-sectional profile is extracted from these 3-dimensional translated data, and FFT one-dimensional analysis is then carried out by the same procedure as that used in the B-1 technique.

Although the procedure in the B-2 method for translating crack surface coordinate values using the measurement crack plane should improve the accuracy of the results of Fourier analysis, the procedure is very complicated. Thus, if satisfactory results of FFT one-dimensional analysis can be obtained by using the simple B-1 method, use of the B-2 method should be avoided. In the present study, we therefore used only the B-1 method for simple FFT one-dimensional analysis of a concrete crack surface. We examined the applicability of Fourier analysis to a concrete crack surface, and we also examined the usefulness of using correction by the measurement crack line (method B) for resolving problems that we found as a result of applying Fourier analysis to a concrete crack surface. Thus, in the present study, using the five tensile test specimens shown in Table 1, we carried out FFT one-dimensional analysis of a concrete crack surface 1) using measurement reference x-axis and y-axis coordinates by the above-described method A and 2) in the x-axis and y-axis directions using raw coordinate data translated using the measurement crack line by method B-1.

4.1 *FFT one-dimensional analysis using coordinates on the measurement reference plane*

The measurement conditions are shown in Table 5.As shown in the table,the 3-dimensional coordinate values of each tensile crack surface were measured with the measurement sensitivityand sampling interval set to $0.125 mm/dot$ and $1.0 mm$,respectively.The angles of inclination of the measurement crack lines with the crack surfaces are given in Table 6.Figure 7 (a) and (b) show the non-normalized results of FFT one-dimensional analyses of the tensile crack surfaces in the x-axis and y-axis directions. Due to space restrictions, only the results for specimens Tn-1, Tn-4 and Tn-5 are presented here.

Table5 Measurement condition in the tensile experiment

angle of incident	slit width	interval of laserbeam	SN	SI	number of composited images
$\pi/4$	0.2	1	0.125	1	61

Table6 The angle of inclination of measurement crack lines

notation	Tn-1	Tn-2	Tn-3	Tn-4	Tn-5
x-axis direction	0.081	-0.074	0.029	-0.142	-0.135
y-axis direction	0.311	0.346	0.02	0.002	-0.087

In the results of FFT one-dimensional analysis of the tensile crack surface in the x-axis direction based on the measurement reference plane coordinates shown in Figure 7 (a), the intensity spectrum are all characterized by a downward sloping distribution, with the intensity being high in low frequency region, gradually decreasing as the frequency increases, and reaching almost zero at a frequency of about $15/mm$. Although the results are not shown in Figure 7 (a), the distributions of the intensity spectrum for specimens Tn-2 and Tn-3 were the same. Figure 7 (b) shows the results of FFT one-dimensional analysis in the y-axis direction of coordinates on the measurement reference planes of the specimens. Similar to the results in the x-axis direction, these intensity spectra are also all characterized by a downward sloping distribution, with the intensity being high in low frequency region and decreasing as the frequency increases.The distributions of the intensity spectra for specimens Tn-2 and Tn-3 were the same.Moreover,a comparison of the FFT one-dimensional spectra peaks for the tensile

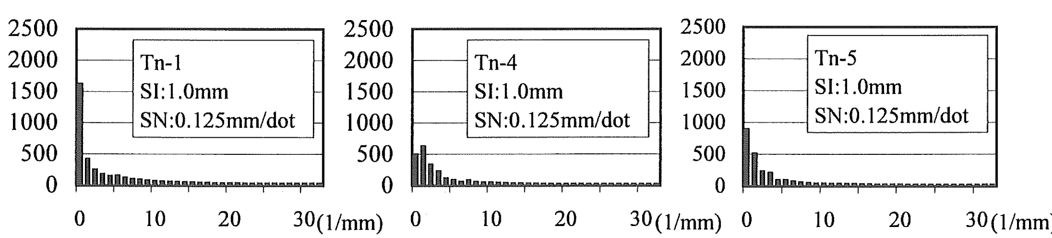

Figure 7(a) Result of FFT one-dimensional analysis of the tensile crack surface on the measurement reference plane (x-axis direction)

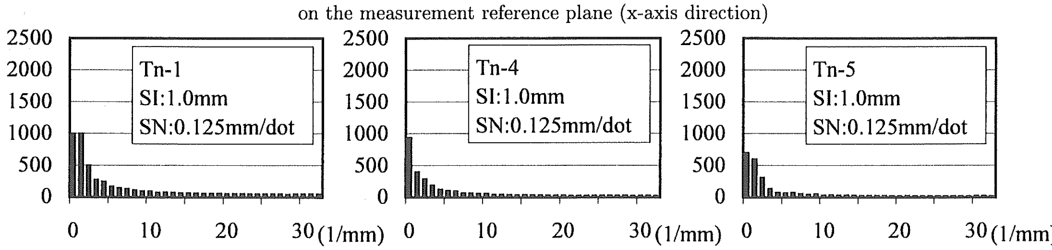

Figure 7(b) Result of FFT one-dimensional analysis of the tensile crack surface on the measurement reference plane (y-axis direction)

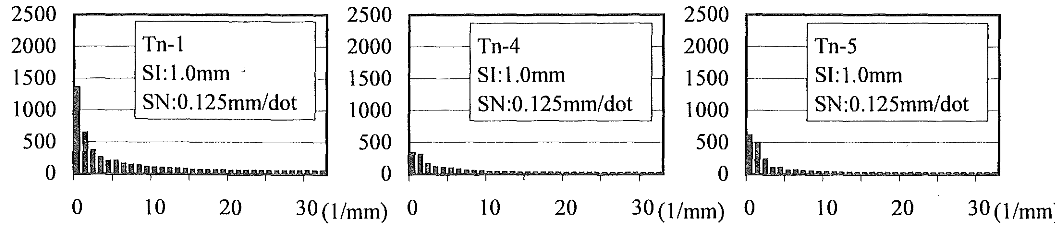

Figure 8(a) Results of FFT One-dimentional analyses of the tensile crack surface on the measurement crack line (x-axis direction)

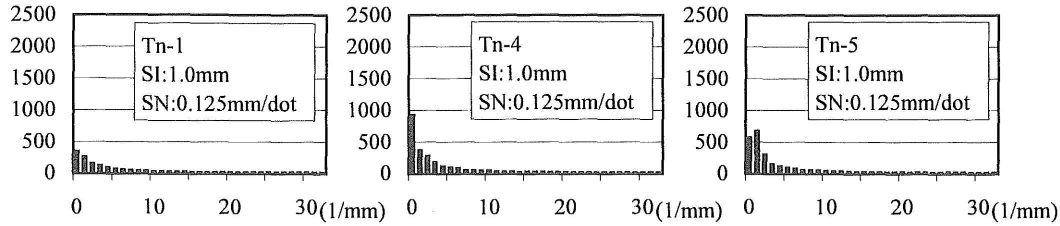

Figure 8(b) Results of FFT One-dimentional analyses of the tensile crack surface on the measurement crack line (y-axis direction)

crack surfaces of all five specimens showed that the peak heights and intensity distributions varied greatly among the specimens.Although it is specculated that the angle of inclination of the measurement crack line with the crack surface greatly affects the overall shape of the cross-sectional profile of the crack surface, a correlation was not found between the inclination angle and the FFT one-dimensional intensity spectrum distribution.

4.2 *FFT one-dimensional analysis using coordinates on the measurement crack line*

Figure 8 (a) and (b) show the results of FFT one-dimensional analysis of the crack cross-setional profiles on the measurement crack line. The spectra distributions in Figure 8 (a) and (b) show almost the same constant rates of decline, but the peaks are much smaller than those obtained from analysis using measurement reference plane coordinates. It is thought that if the inclination angle of the measurement crack line is used as a parameter, it would be possible to recognize similarity in the properties of two concrete crack surfaces from the results of FFT one-dimensional analysis.However, the accuracy of this assumption must be tested by further analysis using the abovementioned B-2 method and a comparison of the results with the results obtained in the present study.

5 CONCLUSIONS

In order to carry out FFT one-dimensional anal- ysis of a concrete tensile crack surface,we first obtained a cross-sectional profile from the 3-dimensional coordinate data of the crack surface and then constructed a model by linear interpolation of the data for Fourier analysis. When one tensile crack surface to which this modeling method had been applied was analyzed using combinations of three different measurement sensitivities and three different sampling intervals, it was demonstrated that the best results of FFT one-dimensional analysis can be obtained with the measurement sensitivity set to $0.125mm/dot$ and the sampling interval set to $1.0mm$. FFT one-dimensional analysis using these parameter values was then carried out on tensile crack surfaces of five concrete test specimens. Great differences were found between the results obtained by analysis using measurement reference plane coordinates, but the differences between the intensity spectrum distributions obtained from analysis using measurement crack line coordinates were not so great, indicating that the shape of a crack surface could be more accurately evaluated by the angle of inclination of the measurement crack line and the intensity spectrum along the measurement crack line. However, further study is needed to establish a method for applying FFT one-dimensional analysis to a crack cross-sectional profile calculated by translating measured 3-dimensional coordinate values of a crack surface to measurement crack plane coordinate values.

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