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Self-induced lateral X-tension in concrete under compression and constitutive equation of its integrity

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Since the mode of failure of brittle solids under compression is splitting, a suggested inference, deduced from this phenomenon, is that *gradient strains and internal thrust* generate *transverse X-tension* in concrete, from the very outset. *X-tension* constitutes a *new kind of stress state* and is the only origin of the loss of concrete integrity *under compression.* X-tension has its maximum in the middle of the stress field of compression and increases in accordance with the longitudinal strains. The *equation for X-tension* is presented, based on the parameters of the concrete components. The release of energy, accumulated by X-tension, explains the explosion-type failure of high-strength concrete under compression. The presented model explains the decrease in the ratio between strength-in-tension to the strength-in-compression with increase in the strengths.
The constitutive equation of the nonlinear behavior of concrete in compression is given and it includes the *Semi-Gaussian* of the *integrity* of concrete, decreased beyond the limit of linearity due to microcracking. The term: 'plasticity' is not used here, since it has nothing in common with this nonlinearity.

1 PHENOMENON OF MICROCRACKING AND SPLITTING

It is clear now that the traditional description of the limiting states of concrete under compression in terms of shear stresses does not work. The central point in the problem remains open: what is the reason for degradation and failure of concrete under increasing compression? For a long time the two-pyramid pattern of failure of a concrete cube under compression served as proof of the central role of shear stresses in the failure. But when paraffined paper was placed between the cube and the press platens to eliminate the friction between them, it suddenly turned out that the failure pattern is not one of shear *but of splitting,* and there is no explanation for this phenomenon, Gvozdev (1949), Jaeger (1979), Avram (1981).

Without friction the splitting of cubes and prisms under compression is usually expressed by two vertical macrocracks, which appear in the middle of the specimen perpendicular to each other. In wide specimens several parallel splitting cracks can appear, while in cores 4" in diameter there are five – six radial cracks extending to about 2/3 of the core radius, without reaching the center. There is no need for special analysis to conclude that splitting cannot appear without *internal self-induced transverse tension,* strong enough to rupture the material in this unusual direction.

Paradox: In homogeneous materials there are no forces which can induce splitting under compression. They do exist in heterogeneous material like concrete, Blechman (1995), but it is usually smeared into a homogeneous body. *Intermediate conclusion:* When brittle solids are modeled as homogeneous or quasi homogeneous – the link with reality is lost.

Microcracking under compression

As a prelude to our analysis of the problem, some fundamentals on the behavior of heterogens under compression should be recalled, Blechman (1988, 1995, 1997a)

⋆ Concrete is a matrogen, meaning a heterogeneous brittle solid consisting of a matrix and particles of aggregate floating in it.

⋆ In concrete local gradient strains are induced by the differences in Poisson extension of its components and by the thrust between the matrix and the particles due to differences in their elastic modulus. These gradient strains appear from the very beginning of loading.

⋆ Gradient strains only are the origin of microcracking. Microcracks appear as thin "discs" parallel to the direction of maximal (principal) compression stress, when gradient strains exceed the resistance of concrete to local microrupture, Carrasquillo et al(1981), Slate et al (1986).

⋆ The microcracks reduce the integrity of concrete in the lateral directions and, accordingly, its bearing capacity.

⋆ Appearance of microcracks is reflected in the stress-strain curve by its deviation from linearity, Berg (1950), (1961).

⋆ The reduction of integrity, not plasticity, is the origin of the nonlinearity of the stress-strain curve under monotonic load.

⋆ Concrete under compression always fails in splitting. High strength concrete and ceramics explode.

2 LOCAL GRADIENTS

As described in Blechman (1995),(1997), a number of micromechanisms can induce lateral strain gradients in parallel. Their action is expressed by the gradient factor – δ, which is a function of:

(*a*) the differences in Poisson's ratio of the concrete components

$$\delta_\nu = (k_a \nu_a - k_m \nu_m) K_{sm}. \tag{1a}$$

Here: ν_a, ν_m = Poisson's ratio for aggregate and matrix, resp;

k_a, k_m = coefficients;

K_{sm} = aggregate/matrix stiffness ratio.

An alternative expression for the Poisson gradient is:

$$\delta_\nu = \nu_o - \nu_m,$$

where ν_o is the measured Poisson's ratio in concrete.

(*b*) the differences in the elastic moduli of aggregate – E_a and of matrix – E_m

$$\delta_E = k\frac{E_a}{E_m} - 1, \tag{1b}$$

where k is a coefficient.

The complicated gradient factor for crystalline materials in general, and in high-strength concrete in particular, is described in Blechman (1997).

Gradient strains and dispersed tension

The equations of lateral gradient strains as a function of the longitudinal strain ε_c of compression and of the gradient factor in general – δ for the above mechanisms are as follows

$$\epsilon^* = \delta\varepsilon_c, \tag{2}$$

where δ is given by equations (1a),(1b).

As shown in Table 1, the gradient strains in concrete can explain about 60% of the lateral tension self-induced in concrete under compression. The remainder could be explained by dispersed tension since from the well-known tests it can be concluded that:

⋆ The density of microcracks induced under compression increases towards the middle of the cross-section, where they reach maximum, Jaeger(1979), meaning that there is transverse dispersed tension, increasing toward the middle.

⋆ Under compression this lateral tension is strong enough to split the element in the middle.

⋆ The dispersed tension is assumed to be an overlap of the stresses induced by lateral local gradients and by internal thrust.

To describe the *strains* of dispersed tension ϵ_{ds} in the middle of the specimen, we assume them proportional to the gradient ϵ^* and by the factor ρ and to the width of the element by a factor – w

$$\epsilon_{ds} = w\rho\delta\varepsilon_c. \tag{3}$$

The w-factor can be described as $w = (b/b_o)^m$, where b_o is the width of the specimen used to calibrate the equation, b is the width of the loaded element and m is the exponent. For the current state of data we assume that the experiments were made on specimens, with $b = b_o$, which gives $w = 1$.

3 EQUATION OF X-TENSION FOR UNIAXIAL COMPRESSION

The self-induced strain of lateral tension under *uniaxial compression* can be defined as the sum of the local tension, generated by thrust and Poisson gradients, and of the dispersed tension. It is called *X-tension* and denoted – ϵ^X. Its equation is the sum of (2) and (3)

$$\epsilon^X = (1 + w\rho)\delta\,\varepsilon_c. \tag{4}$$

Lateral external forces can induce lateral shortening of concrete. We will use the term *widening* to define the state, when the extension of concrete exceeds the lateral compression. Therefore any confinement, and first of all, the external lateral stresses, will shift the onset of gradient strains and, consequently, the moment of splitting – to higher load levels.

The limiting strain under X-tension – ϵ_p^X is reached at the peak point of the SSc, where concrete splits. Eq.(4) for $\varepsilon_c = \varepsilon_p$ gives

$$\epsilon_p^X = (1 + w\rho)\delta\,\varepsilon_p. \tag{5}$$

4 CONSTITUTIVE EQUATION OF INTEGRITY

In Blechman (1988) and (1992), it was shown that the nonlinearity of the ascending branch of the stress-strain relationship is a result of decrease in concrete *integrity* due to microcracking. The *constitutive equation* of this change under compression, called *central function*, was obtained in the form

$$\sigma = \varepsilon E G^+. \tag{6}$$

Here G^+ is the following Semi-Gaussian of positive values of random variable, $(\varepsilon - \varepsilon_a > 0)$

$$G^+ = \exp\left[-0.5\frac{(\varepsilon - \varepsilon_a)^2}{d^2}\right] \leq 1, \qquad (7)$$

which expresses the stochasticity of the process of microcracking and corresponding decrease in integrity, where ε = loading strain, ε_a = the limit of linearity. The scattering factor – d (corresponding to the standard deviation in statistics, denoted there as σ), is linked with ε_a and ε_p, (Blechman, 1989), as follows

$$d^2 = \varepsilon_p(\varepsilon_p - \varepsilon_a). \qquad (8)$$

The parameter complementary to the integrity is "reduction" – A (called also atrophy): $A = 1 - G$, and it reflects the loss of the capability of concrete to absorb the loading energy.

A very important feature of reduction is that it *does not change* the elastic modulus – E of concrete in the direction of maximum compressive stress, when $\varepsilon_a < \varepsilon < \varepsilon_p$, and E changes very smoothly and slowly even after the peak point of stress-strain curve, Karsan (1969). What do actually change in concrete under load are its lateral stiffness *to tension* and the bulk modulus.

5 STRAIN CONVERTER AND THE LINKAGE WITH MACRO

We will define a *strain converter*:

$$\theta_X = \frac{\epsilon^X}{\varepsilon_c}, \qquad (9a)$$

as the ratio between the strains of X-tension and those of compression, which induce them. From eq.(4) we have

$$\theta_X = (1 + w\rho)\delta. \qquad (9b)$$

The *linkage* between the critical strains of X-tension and the macroparameters of the constitutive equation can be described as follows, Blechman (1997b)

$$\varepsilon_a = \frac{\epsilon^R_{min}}{\theta_X}. \qquad (10)$$

$$d = \frac{d_\epsilon}{\theta_X}. \qquad (11)$$

Here ϵ^R_{min} = the threshold of the local resistance of concrete to microrupture and d_ϵ = the scattering factor in the Gaussian of the *pdf* (probability density function) of the above mentioned resistance.

Estimation of strain converter

To estimate the strain converter for the uniaxial stress-state, denoted θ_{X1}, we assume $\varepsilon_{pt} \approx \varepsilon^X_p$, then $\theta_{X1} = \varepsilon_{pt}/\varepsilon_{ct}$. As shown in Table 1, θ_{X1} goes down from 0.12 for concrete C12 to 0.06 for C80, the commonly known value of strain converter θ_{X1}=0.1 being that of concrete C20.

Now we can find ρ from eq.(9b)

$$\rho = \frac{1}{w}\left(\frac{\theta_{X1}}{\delta} - 1\right). \qquad (12)$$

For $w = 1$, we get $\rho = \theta_{X1}/\delta - 1$.

Table 1: Estimates of ρ

Concrete grade according to CEB	C12	C20	C40	C80
$\theta^X_1 = \theta_s$	0.12	0.10	0.08	0.06
δ	0.075	0.06	0.05	0.04
ρ	0.60	0.67	0.60	0.67
Initial elastic modulus, MPa$\cdot 10^{-3}$	14	17	26	36
Specific rupture energy, $\mathcal{E} \cdot 10^3$	56	82	150	282
Second estimation θ_s	0.12	0.10	0.08	0.063

The data of Table 1, based on commonly accepted values of the strength ratio and on evaluation of the gradient factors, gives $\rho = 0.60 - 0.67$, (almost constant).

6 STRENGTH CONVERTER

The constititive equation (6) is an universal expression of the pattern of stress-strain curve and is common to tension, torsion, shear and even to load-deformation curves in bending. Using it we can define the *strength converter* – θ_s as follows

$$\theta_s = \frac{\sigma_{pt}}{\sigma_{pc}} = \frac{\varepsilon_{pt}E_tG^+_{pt}}{\varepsilon_{pc}E_cG^+_{pc}}. \qquad (13a)$$

Here σ_{pc}, σ_{pt}, $\varepsilon_{pt}, \varepsilon_{pc}$ – the strength and peak strains in compression and tension, resp., p, t, c – subscripts of the peak point, tension and compression, respectively.

As is well known, $E_t = E_c$ and due to the above mentioned similarity, we can take $G^+_{pt} = cG^+_{pc}$. Our analysis shows that c is close to unity and the strength converter, being approximated by the ratio of the two peak strains, is equal the strain converter θ_{X1}

$$\theta_s = \varepsilon_{pt}/\varepsilon_{pc}. \qquad (13b)$$

The values of strength converter – θ_s, are shown in Table 1. The large quantity of energy accumulated in the lateral direction by X-tension, explains the explosive-type failure of specimens of high-strength concrete, with fragments blown to the sides.

Thrust vs. Poisson gradients

In *low-stress* concrete the thrust is the main mechanism generating the local gradients of lateral tension. But usually they are combined with tension generated by Poisson gradients between the components. These two kinds of gradient tension vary differently during the advance in microcracking. Cracking of a spot induced by Poisson gradients, eliminates the tension in it, but only a small part of these spots are cracked. In contrast, when microcracks cut the thrust spots, their mutual neutralization is weakened, and the (lateral) thrust increases.

When vertical macrocracks split the low-strength concrete they curve around the aggregate particles, while in high-strength concrete the split is almost flat, passing through the aggregate, Carrasquillo et al (1981). In both cases the gradient factor - δ decreases with arising of the concrete strength, because the difference in elastic modulus and in Poisson's ratio between matrix and aggregate decreases.

Decrease of strength convereter

The origin of the tension-to-compression strength ratio in concrete and its decrease with increase of both strengths is an important time-honored question, and the gradient models can offer a good answer. Since the strength in tension is the *primordial feature* of concrete, we can rewrite eq.(13a), using the strain convereter, as follows

$$\sigma_{pc} = \frac{\sigma_{pt}}{(1+w\rho)\delta}. \qquad (14)$$

The increase in both strengths is equally proportional to the improvement in the strength of the matrix due to increase of its density and its adhesion to the aggregate, but is distorted by the gradient factor δ.

The Poisson's ratio of the matrix – ν_m increases with its density, which results in decrease of the gradient factor $\delta = \nu_a - \nu_m$, since Poisson's ratio of the aggregate ν_a does not change. Thus the decrease of the numerator in (14) *adds its own contribution* to the grows of the compressive strength *above* its simple proportionality with tensile strength!

Converter vs. tensile strength

On the evidence of extensive data, Avram et al (1981), Nordijk (1989), and Raphael (1984), it is common to express the tension-to-compression strength ratio in concrete by the regression

$$f_t' = k_1 f_c^{2/3}. \qquad (15)$$

where $f_t' = \sigma_{pt}$, and $f_c' = \sigma_{pc}$.

But the tensile strength is primordial, and inverting eq.(15), we have

$$f_c' = k_2 (f_t')^{3/2}. \qquad (16)$$

At the same time

$$f_c' = \frac{f_t'}{\theta}. \qquad (17)$$

Equating the two above expressions gives

$$\theta = \frac{k_3}{\sqrt{f_t'}}. \qquad (18)$$

Calibration of eq.(18), with $\theta = 0.1$ for C20, gives $k_3 = 0.15$, then

$$\theta = \frac{0.15}{\sqrt{f_t'}}. \qquad (19)$$

The results of calculation of the converter from eq.(19), given in Table 1 as θ_{s2}, are in good conformity with the commonly used values of θ_{s1}. Since θ is an axes-related parameter, where the strength reflects the influence of the cross-section, the square root in eq.(19) shows the decrease of converter vs. increasing matrix density.

It should be noted that eq.(19) derives from eq.(15), which is a regression and cannot be transformed in this simple algebraic way without an error. Therefore the relations demonstrated above are qualitative only.

7 BRAZILIAN TEST

In the splitting tensile test, known as the Brazilian test, the compression is transmitted through narrow strips with area 10 % or less of the cross-section of the specimen. In the theory of elasticity the splitting effect in this test is explained by lateral tension induced through the vertical cross-section between the two strips, except for the small contact zone. From this test the tensile strength – $\sigma_{t,spl}$ is calculated as

$$\sigma_{t,spl} = \frac{2F_{spl}}{\pi dl}, \qquad (20)$$

where F_{spl} - is the limiting force of fracture and d, l - are the width and length of the specimen, resp.

Since $\sigma_{t,spl} = \theta\sigma_{comp}$ we have $F_{spl} = 0.5\pi dl\theta\sigma_{pc}$. Then the compressive stress under the strip – σ_{strip} for $\theta = 0.09$, will be

$$\sigma_{strip} = \frac{F_{spl}}{0.1dl} = \frac{\theta\sigma_{pc}\pi dl}{0.2dl} = 1.4\sigma_{comp}.$$

Obviously, when $\sigma_{strip} = 1.4\sigma_{comp}$, we will find microcracking and serious X-tension under strip, which will alter radically our notion of the failure in the Brazilian test. Nordijk (1989) in his review of 78 works, found the average ratio between the tensile strength of concrete in Brazilian splitting and its strength in direct tension (p.37) as 1.14, with the range (Table 13.2, p.95) from 0.88 to 1.40.

8 MODES OF TENSION

We are familiar with the plain tension produced by axial action of an external tensile force. X-tension is different, being generated internally by the gradients and thrust among the heterogen's components. As is commonly known, the stress state of torsion can be transformed into the principal stresses of compression and tension, but then X-tension, generated by compression, is actually added to this "classical" tension. The same question applies for shear! And it is obvious that addition of X-tension in these stress-states is crucial.

Another mode of tension appears in restrained elements under cooling, differs from plain tension, being induced internally at every piece of the element instead of coming from the ends. As a result the micro-distribution of the strains and stresses in the cross-section can be different.

Tension in the *Brazilian* test is some combination of transformed plain tension and X-tension with internal thrust, and is not identical with plain tension at all. The tension *under bending* is also different due to gradients in the stress field itself.

The resistance of concrete under these modes of tension is different and it is of practical importance to study and model each of them as distinct phenomena. In a large number of cases, especially in shear, the failure of concrete is a result of combined action of some modes of tension and the role of X-tension in splitting the concrete under shear is not recognized.

9 CONSTITUTIVE EQUATIONS

The fact that under compression the real limiting state of concrete is a function of self-induced lateral tension implies a crucial phenomenon: all eleven principal stress states in concrete can be described in terms of its limting states *of plain- and X-tension only!*

In the gradient models of mesomechanics, the behavior of a brittle heterogen under compression is described in terms of *constitutive equation of integrity – CEI* in terms of principal strains and stresses as follows, (Blechman 2001), (the influence of time is not taken under consideration here)

I. The CEI for the stresses read

$$\sigma_1 = +\varepsilon_1 E_1 G_2^+ G_3^+ - \nu_{21}\varepsilon_2 E_2 G_1^+ G_3^+ - \nu_{31}\varepsilon_3 E_3 G_1^+ G_2^+. \qquad (21)$$

$$\sigma_2 = -\nu_{12}\varepsilon_1 E_1 G_2^+ G_3^+ + \varepsilon_2 E_2 G_1^+ G_3^+ - \nu_{32}\varepsilon_3 E_3 G_1^+ G_2^+. \qquad (22)$$

$$\sigma_3 = -\nu_{13}\varepsilon_1 E_1 G_2^+ G_3^+ - \nu_{23}\varepsilon_2 E_2 G_1^+ G_3^+ + \varepsilon_3 E_3 G_1^+ G_2^+. \qquad (23)$$

In these equations the two Gaussians in any term express the integrity of concrete (heterogen) along the two axes, complementary to the axis under consideration, where

ε_k, σ_k = loading strains and stresses at axis k, resp.;
ν_{ij} = Poisson ratio from axis i to axis j;
ϵ_j = lateral strain induced on axis j.

II. CEI for the strains read

$$\varepsilon_1 = +\frac{\sigma_1}{E_1 G_2^+ G_3^+} - \nu_{21}\frac{\sigma_2}{E_2 G_1^+ G_3^+} - \nu_{31}\frac{\sigma_3}{E_3 G_1^+ G_2^+}, \qquad (24)$$

$$\epsilon_2 = -\nu_{12}\frac{\sigma_1}{E_1 G_2^+ G_3^+} + \frac{\sigma_2}{E_2 G_1^+ G_3^+} - \nu_{31}\frac{\sigma_3}{E_3 G_1^+ G_2^+}, \qquad (25)$$

$$\epsilon_3 = -\nu_{13}\frac{\sigma_1}{E_1 G_2^+ G_3^+} - \nu_{23}\frac{\sigma_2}{E_2 G_1^+ G_3^+} + \frac{\sigma_3}{E_3 G_1^+ G_2^+}. \qquad (26)$$

When the strains are below the limit of linearity: $\varepsilon_k < \varepsilon_{ak}$, the Semi-Gaussians in corresponding directions are of unity and the CEI are replaced by Hooke's law.

10 SUMMARY

$\star$ Since the mode of failure of concrete under compression is splitting, a suggested inference, deduced from this phenomenon, is that gradient strains and internal thrust generate *transverse tension* in concrete *under compression,* from the very outset.

$\star$ Mesomechanics of concrete deals with the problem of its integrity and splitting under compression in terms of lateral local microtension combined with dispersed tension, which constitute a *new kind of stress state,* called X-tension, self-induced under compression and distinguished from uniaxial tension by its origins and its action.

$\star$ The *equation for X-tension* is presented and is based on the mechanical characteristics of the components.

$\star$ Lateral X-tension is the *only origin* of the reduction (degeneration) of concrete integrity under compression. It has its maximum in the middle of the stress field of compression and increases in accordance with the longitudinal strains. The energy accumulated by X–tension can explain the explosion-type failure of high-strength brittle solids under compression.

⋆ Under uniaxial compression due to X-tension, concrete is atrophied in two lateral directions, resulting in reduction of its integrity and the consequent *decrease in its bearing capacity.*
⋆ The specific quality of this reduction lies in the fact that it does not change the elastic modulus of concrete in the direction of maximum compressive stress, when $\varepsilon < \varepsilon_p$.
⋆ The gradient models explain the appearance of microcracks, their stochasticity and stability in heterogeneous materials, in good agreement with experiments. The mechanisms of X-tension *are universal,* affecting any brittle material: concrete, rock, ceramics. They *do not need initial microcracks* to initiate the process of cracking and degeneration and suffice to exhaust the bearing capacity of concrete under compression *without recourse to shear stresses.* The X-tension model is *descriptive* and is based on measurable parameters of concrete.
⋆ The highly important and surprising part in the presented theory resides in the fact that *neither maximum stresses not maximum strains* are responsible for the strength of brittle solids (concrete) under any state of loading, *but the limiting reduction only.*
⋆ As a consequence, only the totality of tension modes: ordinary uniaxial tension, X-tension or tension in mixed states – are responsible for the failure of brittle solids under *any state* of loading. This crucial fact leads to a *unified description* of the behavior and fracture of Brittle Solids in general and of concrete in particular for all types of stress states.
⋆ The existence of X-tension has been on the surface for a long time, but was overlooked because of the superiority of the shear ideology.

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