Fracture Mechanics of Concrete Structures, de Borst et al (eds) © 2001 Swets & Zeitlinger, Lisse, ISBN 90 2651 825 0

Importance of Multiple Damage Model for Analysis of RC Structures

Supratic Gupta
Gunma University, Kiryu, Japan

ABSTRACT: Gupta & Tanabe(1997,1998) had proposed unified concrete plasticity model that can simulate stress-strain in both tensile and compressive region appropriately. It is realized that this type of model, where classical plasticity approach is implemented with only one damage parameter, cannot simulate stress-strain under cyclic conditions. In this paper, it is proposed to analyze the possibility of use of multiple damage parameter to simulate stress-strain under such cyclic conditions. Few cases of uniaxial and biaxial cyclic cases of stress-strain situations are studied to demonstrate the possibility of the development of multiple damage model.

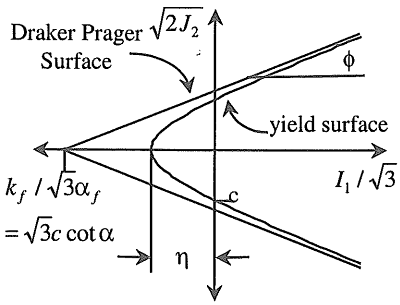
1 INTRODUCTION

Development of analytical models for analysis of reinforced concrete structures is a very complicated subject. Different researchers have attempted to simulate the behavior of reinforced concrete member using one dimensional models, like beam theory, etc. to two or three dimensional models. In the case of two dimensional analysis, various researchers have adopted discrete crack approach. This approach is very successful in simulating RC members failing in shear mode. However this model has its own limitation of the requirement of defining the crack beforehand and is rarely adopted in the three dimensional analysis. Except in the one-dimensional models and some simplified two and three dimensional analysis, most of the researchers have restricted their research to the simulation of RC members under monotonic loading conditions.

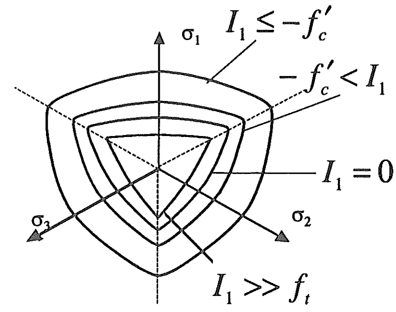
Gupta & Tanabe(1997,1998) had presented unified concrete plasticity model that can simulate behavior of concrete in three dimension condition using smeared crack model. This model was basically a classical plasticity model where Drucker-Prager model was modified such that we have a more triangular cross-section in tensile zone and a more circular cross-section in compression zone(Fig. 1). In this model, the parameters of cohesion C and friction angle ϕ are the most important parameters. By controlling the variation of these parameters appropriately in tensile and compressive zone, this model could satisfactorily simulate stress-strain of condition in all biaxial conditions without changing the model parameters in tensile and compression zone. However, limitation of this model was realized when attempting to simulate the stress-strain behavior of concrete under cyclic condition. It was realized that even though it is possible to simulate stress-strain of concrete in a unified manner without changing the adopted parameters, it is impossible to simulate the cyclic stress-strain relationship with a model with one damage parameter only.

In this paper, the necessity of development of a model with multiple damage parameters is presented. The main problems faced in the development of this type multiple damage are presented to initiate a debate in this line. Though it is realized that implementation of multiple damage parameter adopting classical plasticity is itself a big problem, it was thought that it might be worth to investigate the number of damage parameters that is required and the inter-relationship of the different damage parameters. Most important problem in the development of appropriate model simulating stress-strain of concrete under cyclic condition is the question of what should be the stress-strain in such conditions. Very few experimental results exist in these conditions because it is extremely difficult to carry out such experiments and requires special experimental setup. Interpretation of these experimental results is also an important question.

In this paper, different basic stress-strain situations that show the relationship between different damage parameters are adopted, the experimental or

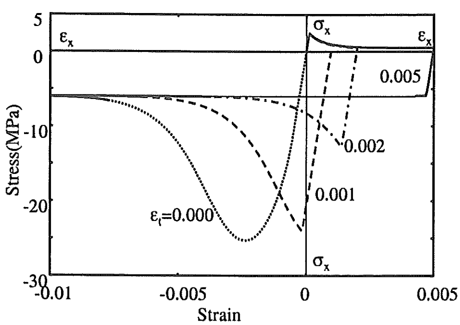


a) Meridian Plane

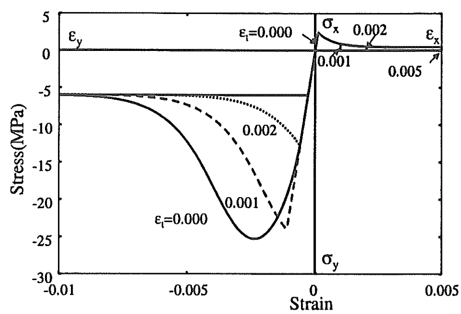


b) Deviatoric Plane

Fig. 1: The Unified Concrete Plasticity Model



a) Uniaxial case



b) Biaxial case

Fig. 2: Stress-strain using one damage parameter

numerical models that exist for such conditions are summarized and finally attempt is made to simulate such behavior by the unified concrete plasticity model adopting inter-dependent multiple damage parameter.

This paper shows that the initial stages of the development of the multiple damage parameter model and shows the requirement of additional experimental work that might be necessary to fully understand the inter-relationship between these damage parameters..

2 THE UNIFIED CONCRETE PLASTICITY MODEL AND ITS LIMITATIONS

Gupta & Tanabe(1997,1998) presented the unified concrete plasticity model for the simulation of concrete stress-strain in three dimensional condition. This model adopted modified Draker-Prager model as shown in Figure 1 with appropriate variation of cohesion C and friction angle ϕ□ in tensile and compressive zone(Eq. 1).

$$C = C_0\left[\gamma \exp\{(-m_1\omega)p_1(X) + (-m_2^2\omega^2)p_2(X)\} + (1-\gamma)\right]$$
$$\phi = \begin{cases} \phi_0 + (\phi_f - \phi_0)\sqrt{(\omega+k)(2-\omega-k)}\,p_2(X) & \omega \leq 1 \\ \phi_0 + (\phi_f - \phi_0)p_2(X) & \omega > 1 \end{cases} \quad (1)$$

where $X(=I_1/\sqrt{3J_2})$ was adopted to define the variation in the transition zone. For further details can be found in Gupta, 1997.

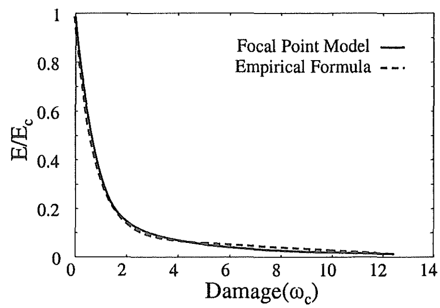
This model could satisfactorily simulate stress-strain in different biaxial conditions without adopting different set of parameters in tensile and compressive zone(Sec. 4.2.3).

Figure 2 shows the limitation of this model in simulation of the cyclic stress-strain conditions. This is because this type of approach adopts a single damage parameter for the calculations. Once damage accumulates in a particular path, this value remains in memory. If we unload and load is some other loading path, this model would reflect the damage accumulated in previous path. This is contrary to the experimentally observed facts. For example, if we take a cracked RC specimen and load it compressive loading perpendicular to the crack orientations, we do not expect much reduction of compressive stress. This condition is similar to case where tensile stress is applied followed by compressive stress in same direction. Experiments with cyclic tensile stress and compressive stress in perpendicular direction, shows that the damage are interrelated. Maekawa & Okamura(1982) have performed experiments of plain concrete and Hsu (1993) and his research group(Bekarbi, 1991) and other researchers have performed such experiments on RC specimen which show that damages in this two directions are interrelated. This is because of the fact that both cases produce cracks in similar direction. Though Gupta & Tanabe(1997,1998) have argued that unified concrete plasticity model can simulate stress-strain under various proportional loading conditions from tensile to compressive region without adopting different set of parameters, it can be realized that model adopting single damage parameter can not satisfactorily simulate stress-strain under cyclic conditions satisfactorily.

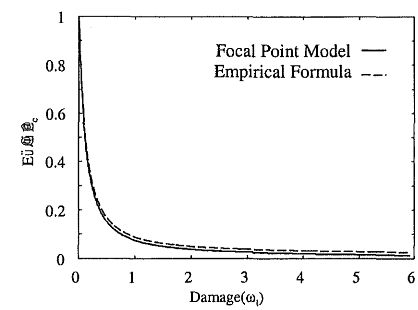
3 DEVELOPMENT OF MULTIPLE DAMAGE PARAMETER MODEL

In previous section, it was clear that it is important to develop a strategy and model that can take care of multiple damage parameters. It is assumed that we need ***6 damage parameters, one each in tensile and compressive region in all the three directions.*** Now as explained in previous sections, it is expected that these damage parameters will be different, however interrelated. This development has two sets of problems: a) Development of a strategy to implement multiple damage parameter, b) determine the relation between different damage parameters. Though the first is important, it is possibly practical to pursue the later in the initial stages. After the relationship between different damage parameters are clear, we can possible think of the strategy to integrate the 6 parameters in a logical manner.

One more severe problem exists in this development. Experimental data are very rare in this field. Hence whatever data exists in this field is very important and we have to interpret the data carefully. For example, we have experiments of RC and plain concrete members. Whereas experiments on plain concrete end at peak strength at cracking or crushing point, experiments on RC specimens go much beyond the peak. Hence experimental results have to be carefully considered in developing this multiple damage parameter model.



a) Compression



b) Tension

Fig. 3: Stiffness degradation

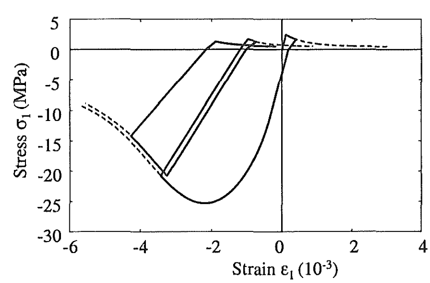


Fig. 4: Cyclic stress-strain in uniaxial condition

This paper presents few case studies, which show clearly that it might be fruitful trying to develop this type of model. Further experimental and careful analytical consideration is necessary before the full establishment of this type of model.

4 DITERMINATION OF RELATION BETWEEN VARIOUS DAMAGE PARAMETERS

Few case studies in determining the possible relation of damage parameters are presented here. In classical plasticity, one damage parameter can be implemented. Hence, we switched the damage parameters when we adopted a different stress path. The other parameters adopted in this analysis are: C_0=28.25, ϕ_0=5, ϕ_f=22, f'_c=25.2N/mm2, f_t=2.52.N/mm^2, E_c=21700 N/mm^2, μ=0.22, k=35, ω_1=2.5, ω_2=1.0, β=0.82.(Gupta & Tanabe,1997,1998, Gupta, 1997). This study intends to check the feasibility of implementation of multiple damage parameters. Hence, empirical formulas derived here may not be general.

4.1 *Cyclic tensile and compressive loading in same direction*

Experimental studies exist that shows the possible stress-strain relationship of concrete in both uniaxial tension and compression. While post peak tensile behavior is said to depend of fracture criteria or tension stiffness effect depending on the concrete is part of plain concrete or reinforced concrete member or zone, post peak behavior of concrete under compression is assumed to undergo gradual softening. However, what should be the exact nature of the post-peak softening is still a matter of further research. But whatever may be the softening slope in either of these cases, Gupta & Tanabe,1997,1998 have shown that the unified concrete plasticity model can simulate them to the satisfaction of the user by changing the rate of change of Cohesion C and Friction angle ϕ of Eq. 1.

There are various experimental work and analytical models about the unloading branches of concrete under uniaxial tension and compression(Yankelevsky & Reinhardt, 1987, Yankelevsky & Reinhardt, 1989, Kent& Park, 1971, Darwin & Pecknold, 1977, Karson, & Jirsa, 1969). It has been shown that unloading stiffness gradually undergoes degradation both in uniaxial tension and compression. There is hysteretic loop in unloading and reloading in both the cases. Though experiments exist showing the cyclic stress-strain in individual case of uniaxial tension and uniaxial compression, experiments are not available combining the two situations.

In this paper, well-known focal point model (Yankelevsky & Reinhardt, 1987, Yankelevsky & Reinhardt, 1989) is adopted as reference. In this model, stress unloads toward a focal point, ($-f_c$,$-f_c/E_c$) and ($-f_t$,$-f_t/E_t$) in tension and compression re spectively. The stiffness degradation could be simu lated quite easily based on the following assump tions.

a) Damage parameters ω_{1c} in compression and ω_{1t} in tension are independent parameters.
b) Stiffness degradation is achieved the simple formula [E]= α[D], where α is given in Eq. 2 to match the expected results of focal point model(Fig. 3). Eq. 2 implies that degradation is a direct function of the damage in respective condition

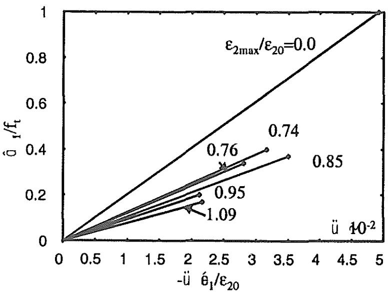


Fig. 5. Tensile strain after unloading of compressive stress[4]

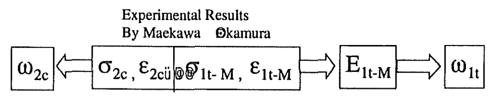


Fig.6: Flow Chart to relate ω_{1t} and ω_{2c}

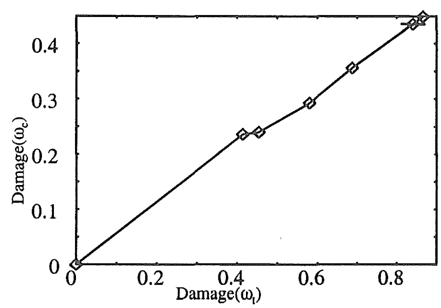


Fig. 7: Relation of damage parameters

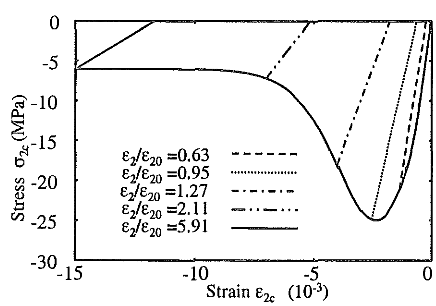


Fig. 8: First stage of compressive loading

$$\alpha_c = 0.97e^{-3.5\omega} + 0.03(1 - 0.06\omega) \qquad (2.a)$$

$$\begin{aligned} &\beta = f_t[0.18 + 0.82\exp(-m_1 m_2(\varepsilon - \varepsilon_t))] \\ &\alpha_t = (\beta + f_t)/(\omega / m_2 + 2\varepsilon_t) \quad m_1 = 2.5, m_2 = 603 \end{aligned} \qquad (2.b)$$

It was possible to determine exact relation for uniaxial tension (purposefully shown little differently in the figure), where as the empirical formula uniaxial compression is an approximate equation. Figure 4 shows the stress-strain under cyclic stress conditions. The dotted line show the stress strain if the particular stress path is followed in place of the reverse path in cyclic loading. The adoption of independent damage parameters is well justified at least in the initial stage because crack produced by the tensile stress does not create weakness for the compressive stress in same direction. In this analytical experiment, it is not yet possible to simulate the hysteretic loop.

4.2 *Cyclic tensile and compressive loading in perpendicular direction*

When compressive stress is applied, micro cracks and at later stages visible cracks appear in orthogonal direction. This is the same direction in which crack would appear if tensile stress is applied in the perpendicular direction. Hence it can be expected that damage parameters for tension and compression in perpendicular directions be interrelated.

In the experiment of tensile load applied by compressive load of RC member, Hsu(1993) and his research group(Belarbi, 1993) have shown through experiments of RC members that the relation depends on sequential or proportional loading. They have also shown that the case of sequential load where compressive loading is applied without unloading, the tensile load yields results comparable to the case of proportional loading. In case of proportional loading, softening of both peak stress and peak strain was observed, whereas only softening of peak stress was observed in case of sequential loading (where tensile load is unloaded more then 90% level). In this case of numerical experiment, the case of sequential loading after full unloading on initial loading path is considered.

4.2.1 *Compressive loading after by tensile load is unloaded*

Maekawa & Okamura(1982) had performed experiment on plain concrete member. This set of experimental results exists for various level of compressive load, where no experimental results exist for the post peak region (beyond 1.09 ε_0). Two important observations in this experiments are : a) Softening of the stiffness or slope of stress-strain curve $E_{1t\text{-}M}$, b) Considerable softening of peak stress $\sigma_{1t\text{-}M}$, where subscript $_M$ represent experimental result by Maekawa & Okamura(1982)

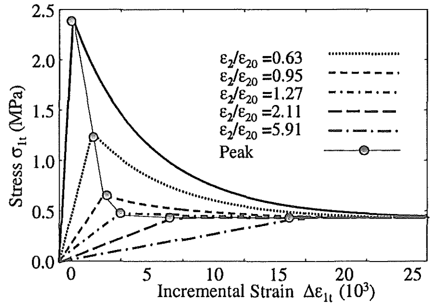


Fig. 9: Second stage of tensile loading

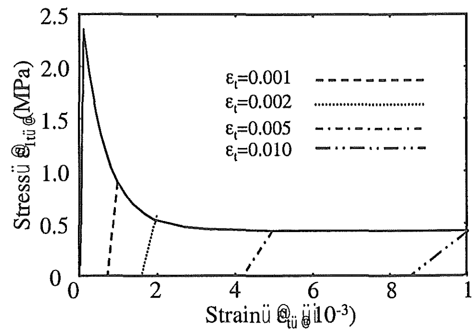


Fig. 10: First Stage of Tensile Loading

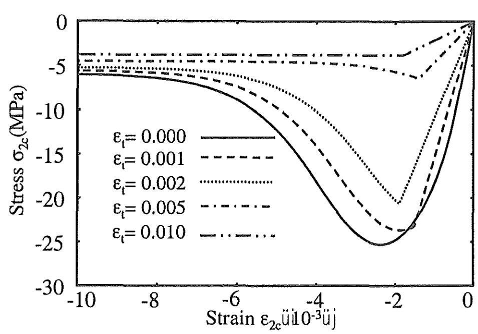


Fig. 11: Second stage of compressive loading

.The present unified concrete plasticity model implements tension stiffening effect for reinforced concrete member. In uniaxial tension, a particular point(σ_{1t},ε_{1t}) on the softening curve represent a particular value of damage ω_{1t}. The unloading slope E_{1t} depends on the ω_{1t} as shown in Eq. 2b. Hence if we assume ω_{1t} and ω_{2c} are interrelated, i.e. ω_{1t} develops due to development of ω_{2c} in compression, then the stress strain curve will start parallel to the unloading slope E_{1t} and the peak stress σ_{1t} depending on damage ω_{1t}. Hence we can understand that it is impossible to match both stiffness and peak stress and

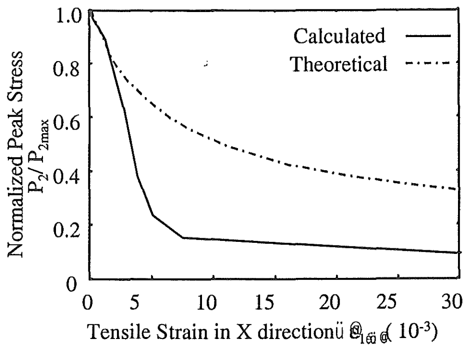


Fig. 12: Peak load softening in comparison to experimental results [5,6]

would depend on the assumed softening slope of the uniaxial tension curve.

To correlate ω_{1t} and ω_{2c}, we assume $E_{1t} = E_{1t\text{-}M}$. Figure 6 shows the flow chart for the calculation and plotted in Figure 7. The relation between ω_{1t} and ω_{2c} was found to be is quite linear and can be written as

$$\omega_{1t} = \alpha\, \omega_{2c} \quad \text{where } \alpha = 0.58 \qquad (3)$$

Figure 8-9 shows the stress strain behavior under such cyclic loading condition.

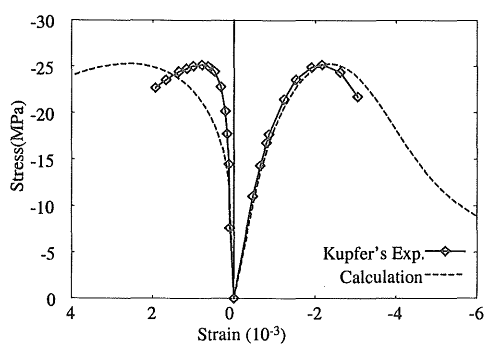
4.2.2 *Tensile loading followed by compressive load is unloaded*

We assume same relationship between ω_{1t} and ω_{2c} determined in previous section in Eq. 3 is also valid here. Figure 10-11 shows the stress strain behavior under such condition. Figure 12 shows the peak stress softening in comparison the experimentally derived relationship by Hsu(1993) and his research group(Belarbi, 1993). Though it does not match properly, both show downward trend. In this case we are trying to match results for RC specimen based on relationship derived from experimental results of plain concrete specimen. Hence more experimental study and in depth consideration is required.

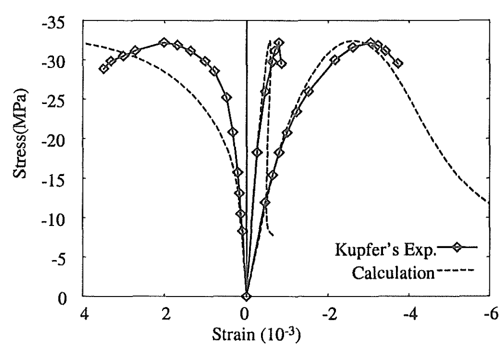
This clearly shows that quite logical results can be simulated using multiple damage model.

4.2.3 *Biaxial and Triaxial Compression*

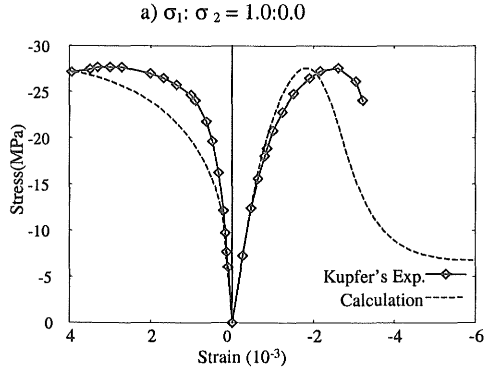
Figure 13 show the simulation of stress strain in proportional biaxial loading along with the relation-



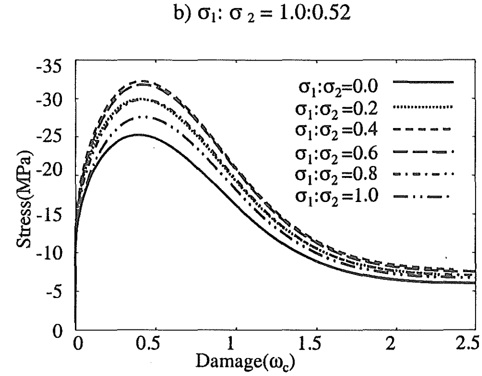
a) $\sigma_1 : \sigma_2 = 1.0:0.0$



b) $\sigma_1 : \sigma_2 = 1.0:0.52$



c) $\sigma_1 : \sigma_2 = 1.0:1.0$



d) Relationship with Damage

Fig. 13: Comparison with Biaxial Experiment by Kupfer et al. 1969

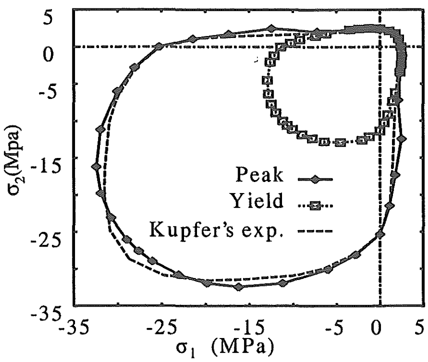
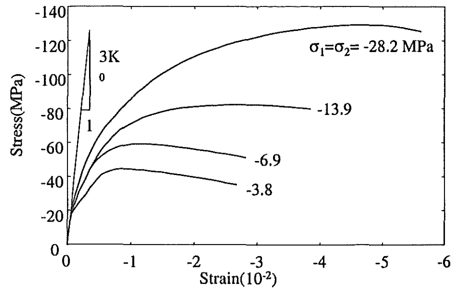


Fig. 14: Comparison with peak strength of Kupfer's experiment [3]

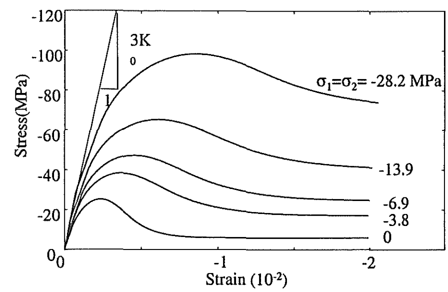
ship between the stress and damage parameter. Figure 14 shows the comparison of peak stress with Kupfer et al's(1969) experimental results. Fig. 15 shows the comparison in triaxial loading condition. In this case, triaxial load ($\sigma_1 = \sigma_2 = \sigma_3$) was applied up to a state, after which $\sigma_1 = \sigma_2$ are maintained as constant and stress of σ_3 was increased.

Numerical prediction by Gupta & Tanabe(1997,1998) in these simulations show reasonable good simulation trend in comparison to experimental results. However, correlation cannot be derived from these stress-strain relationships. Figure 13.d and Figure 15.c shows the relationship of stress and damage parameter w. We can see that peak stress in obtained at fixed value of damage ω in all these cases. Hence it looks logical if we take damage parameter ω as a measure of determining matters like unloading slopes, etc. as defined in Eq. 2a.

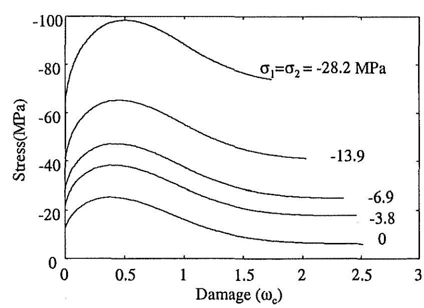
Author is now looking for experimental results for simulation of sequential biaxial loading to investigate the possible relationship between damage developed in compression in various directions.



a) Experimental Results (Richert et al.)



b) Results of Numerical Calculations



c) Relationship with Damage

Fig. 15: Comparison with Triaxial Experimental Results Richert et al.

5 CONCLUSION

To overcome the limitation of approaches using classical plasticity with only one damage parameter in describing the cyclic stress strain relationship, this research attempts to implement multiple damage parameters. Unified concrete plasticity model proposed by Gupta & Tanabe,1997,1998, a model that can simulate stress-strain properly in most proportional case is adopted in this analysis. Though it is important to find a methodology to implement the 6 (one each in tension and compression in all the three axis) parameters together, it was decided that it is more important to check the possible relations between the damage parameters.

In this paper, few case studies of monotonic and cyclic loading of tension and compression in uniaxial and biaxial condition are considered. The following conclusions were drawn from the case studies:

a) For uniaxial case, implementation of two independent damage parameters were found logical. After implementation of appropriate proportional softening of unloading stiffness, it was possible to simulate the stress-strain relation similar to the popular *focal point model.*
b) In biaxial case, direct relationship between the two damage parameters were derived from experiment of plain concrete specimen by Maekawa

& Okamura(1982). Logical stress-strain analysis was observed, after implementation of this model and the softened stiffness for unloading derived in the uniaxial case.

From the above analysis it was understood that it might be worth carrying out further reserach to implement the multiple damage model. The most important problem faced in this research is the scarcity of experimental results. Authors are at present looking for experimental work on plain or reinforced concrete members under sequential biaxial compressive load to understand the possible relationship between damages in compression in different directions.

6 ACKNOWLEDGEMENT

Author thanks Prof. Koichi Maekawa of Tokyo University, Japan for the discussion, advices and help in collection of relevant experimental data. Author would also like to thank Prof Tadaaki Tanaabe of Nagoya University Japan and Prof. Yukikazu Tsuji for valuable help and support in carrying out this research.

7 REFERENCES

1. Gupta, S. & Tanabe, T. 1997. Modification of the Unified Concrete Plasticity Model and it characteristics, *Journal of Materials, Concrete Structures and Pavements, JSCE*, No. 571/V-36, pp. 225-234.
2. Gupta, S. & Tanabe, T. 1998. Three dimensional analysis of reinforced concrete members by Unified Concrete Plasticity Model, *Journal of Materials, Concrete Structures and Pavements, JSCE*, No. 592/V-39, pp. 225-234. 1998.
3. Gupta, S 1997. *The development of Unified Concrete Plasticity Model for three dimensional analysis of reinforced concrete members*, Doctoral Thesis, Civil Engineering Department, Nagoya University, No. 9705.
4. Kupfer, H, Hilsdrof, H. K. & Rusch,H. 1969. Behavior of concrete under Biaxial stress, *ACI*, Vol.66,No.8, Aug., ,pp.656-636.
5. Maekawa, K. & Okamura, H., 1982. "Deformational Behavior of Concrete under Biaxial Compression-Tension Stress States", Concrete Journal, Vol. 21, No. 3., pp. 111-121(Japanese).
6. Hsu T. T. C. 1993. *Unified Theory of Reinforced Concrete*, CRC, ISBN 0-8493-8613-6.
7. Belarbi, A, 1991. *Stress-Strain Relationships of Reinforced Concrete in Biaxial tension-compression*, PhD dissertation, University of Houston, Houston, 501pp.
8. Yankelevsky, D. Z. & Reinhardt, H. W. 1987. Model for Cyclic Compressive Behavior of Concrete, *Journal of Structural Engineering*, Vol. 113, No.2.
9. Yankelevsky, D. Z. & Reinhardt, H. W. 1989. Uniaxial Behavior of Concrete in Cyclic Tension, *Journal of Structural Engineering*, Vol. 115, No.1, January, 1989.
10. Kent, D. C. & Park, R 1971. Flexural members with confined concrete, *Journal of Structural Division, ASCE*, ST7, Vol. 97, pp.1969-1990, 1971.
11. Darwin, D. & Pecknold, D. A. 1977. Analysis of Cyclic Loading of Plane R/C Structures, *Computers & Structures*, Vol. 7, Pregamon Press, pp.137-147.
12. Karson, I. D. & Jirsa, J. O. 1969. Behavior of Concrete under Compressive Loading, *Journal of Structural Division, ASCE*, Vol. 95, No. ST12, pp.2543-2563.