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Microplane model with relaxed kinematie constraint

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ABSTRACT: In the present paper the microplane model for concrete like materials is discussed. In the model the material is characterized by a relation between the stress and strain components on planes of various orientations. To realistically model quasi-brittle materials under compressive load, for each microplane the total strain tensor has to be decomposed into the normal (volumetric and deviatoric) and shear strain components. It is shown that for dominant tensile load the decomposition of the normal microplane strain together with the fact that the tensile strength is for an order of magnitude smaller than it's compressive strength leads to unrealistic model response. To keep the conceptual simplicity, the model is improved in the framework of the kinematic microplane theory, however, the kinematic constraint is relaxed. It is demonstrated that the improved model is able to realistically simulate complex fracture mechanisms of materials with and without initial anisotropy.

1 INTRODUCTION

Traditionally, the macroscopic models are formulated by total or incremental formulation between the σ_{ij} and ε_{ij} components of the stress and strain tensor, using the theory of tensorial invariants (Willam and Warnke, 1974; Ortiz, 1985). In the framework of the theory, there are various possible approaches for modeling of concrete, such as plasticity, plastic-fracturing theory, continuum damage mechanics, endocronic theory and their various combinations. Due to the complexity of the concrete, presently exists no model based on the stress and strain tensor and their invariants which is capable to realistically predict the behavior of concrete, not only for the three-dimensional monotonic loading, but for the general three-dimensional cyclic loading as well. For instance, the invariant type of the models have difficulties with correct modeling of concrete expansion at triaxial compressive load, which in some applications governs the failure mechanism and is a consequence of cracking (discontinuity). Such models are based on the continuum mechanics and are generally not capable to simulate complex stress-strain states, which involve cracking, using only a few available invariants. Moreover, based on the plasticity type of the flow rules, which is in these models most commonly used, it is difficult to model complex three-dimensional cyclic response of concrete. These, as well some other drawbacks of the constitutive laws based on the theory of tensorial invariants is main motivation for the use of the microplane theory as an alternative approach for macroscopic modeling of concrete.

The advanced kinematic constraint version of the microplane model for concrete was proposed by Bažant and Prat (1988) and later extended to general cyclic form with the rate sensitivity by Ožbolt and Bažant (1992). For dominant tensile damage (tensile softening) the model exhibits physically unrealistic behavior which is manifested by lateral expansion. To improve the model, different approaches have been used so far (Carol & Bažant 1997, Bažant et al. 2000). Principally, there are two possibilities to do so: (1) Imposing the static constraint approach at the microplane level or (2) keeping the kinematic constraint approach but modifying the microplane strain components by adopting additional constraints. The first approach would mean that the microplane stress components are calculated as the resolved components of the total macroscopic stress tensor. Because of the non-unique definition of the stress tensor in softening materials it is generally difficult to work directly with the static constraint approach. Therefore, in the present paper one of the possible improvements which is based on the relaxation of the kinematic constraint is discussed.

2 MICROPLANE MODEL

2.1 *General*

The microplane model is a three-dimensional macroscopic model in which the material is characterized by a uniaxial relations between the stress and strain components on planes of various orientations (microplanes - monitoring directions). In the model the tensorial invariance restrictions need not be directly enforced. They are automatically satisfied by superimposing the responses from all microplanes in a suitable manner. The basic concept behind the microplane model was advanced by Taylor (1938). Later the model was extended by Bažant and coworkers for modeling quasi-brittle materials which exhibit softening (Bažant & Gambarova 1984, Bažant & Prat 1988, Ožbolt & Bažant 1992, Bažant et al. 2000).

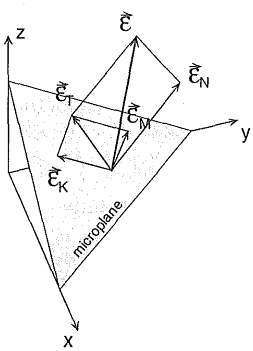

Figure 1. Microplane strain components.

The microplanes are defined by their unit normal vector components n_i (see Fig. 1). Normal and shear stress and strain components (σ_N, σ_{Tr}; ε_N, ε_{Tr}) are considered on each plane. They are calculated as the projections of the macroscopic strain tensor ε_{ij} (kinematic constraint). Based on the virtual work approach, the macroscopic stress tensor is obtained as an integral over all in advance defined microplane directions:

$$\sigma_{ij} = \frac{3}{2\pi}\int_S [\, \sigma_N n_i n_j + \frac{\sigma_{Tr}}{2}(n_i\delta_{rj} + n_j\delta_{ri})]\, \Omega(\mathbf{n})\, dS \tag{1}$$

where $\Omega(\mathbf{n})$ is a weight function of the normal direction **n** that introduces anisotropy of the material in it's initial state and S is the unit sphere surface.

The microplane model is principally similar to the discrete models (random particle, lattice, etc.), except that it works in the framework of continuum. The in advance defined monitoring directions (planes) are equivalent to the beam or truss elements of an discrete model. In the initial stage the model represents elastic continuum, however, as soon as damage localizes in a certain direction, the model should perform as a discrete model.

To model concrete like materials for dominant compressive load realistically and to control the initial elastic value of the Poisson's ratio, the normal microplane component has to be decomposed into volumetric and deviatoric part. ($\sigma_N = \sigma_V+\sigma_D$, $\varepsilon_N = \varepsilon_V+\varepsilon_D$; see Fig. 1). This leads to the following expression for the macroscopic stress tensor:

$$\sigma_{ij} = \frac{3}{2\pi}\int_S [\, n_i n_j \sigma_V + (n_i n_j - \delta_{ij} / 3)\sigma_D + \frac{\sigma_{Tr}}{2}(n_i\delta_{rj} + n_j\delta_{ri})]\, \Omega(\mathbf{n})\, dS \tag{2}$$

For each microplane component, empirical uniaxial stress-strain relations are in advance defined as:

$$\sigma_V = F_V(\varepsilon_V);\ \sigma_D = F_D(\varepsilon_D);\ \sigma_{Tr} = F_{Tr}(\varepsilon_{Tr}) \tag{3}$$

In the present model these relationships are of the secant form. They are based on the scalar damage theory:

$$\sigma_m = C_m \varepsilon_m ;\quad C_m = C_{m,0} f(\varepsilon_m) \tag{4}$$

where m denotes the microplane component, C_m is the secant stiffness moduli with the initial value $C_{m,0}$ and $f(\varepsilon_m)$ is a function of a scalar damage type. It is important to point out that for concrete the deviatoric compressive strength as well as the shear strength in (3) need to be set roughly 10 times larger than the volumetric and deviatoric tensile strength.

From known microplane stress-strain relations, the macroscopic stress tensor is calculated by introducing (3) into (2), whereas the integration over all microplane directions (in the present model 21 directions) is performed numerically. Furthermore, the stiffness tensor is calculated as the incremental form of (2).

As discussed by Jirásek (1993) the main reason for pathological behavior of the above model subjected to tension is related to the split of the normal microplane component into volumetric and deviatoric part and not to the kinematic constraint itself. To demonstrate why the problem arises only for tensile load let us consider a specimen loaded in uniaxial tension (plane stress state, see Fig. 2a). For simplicity reasons only two microplanes (**x** and **y**) are considered. Plane **x** is oriented in the load direction and plane **y** is perpendicular to it. The normal microplane stress and strain component in the plane **y** ($\sigma_{N,y}$, $\varepsilon_{N,y}$) is split into volumetric (σ_V, ε_V) and deviatoric ($\sigma_{D,y}$, $\varepsilon_{D,y}$) part:

$$\varepsilon_{N,y} = \varepsilon_V + \varepsilon_{D,y} ;\quad \sigma_{N,y} = \sigma_V + \sigma_{D,y} \tag{5a}$$

$$\sigma_y = \sigma_{N,y} = 0 \tag{5b}$$

The volumetric and deviatoric strain components on the plane **y** are for the linear elastic response (approximately point 1, see Fig. 2b):

$$\varepsilon_V = \tfrac{1}{3}(1-2\nu)\varepsilon_x \,; \quad \varepsilon_{D,y} = -\tfrac{1}{3}(1+\nu)\varepsilon_x \tag{6}$$

where ν = Poison's ratio and ε_x = total strain in the direction of the applied tensile load.

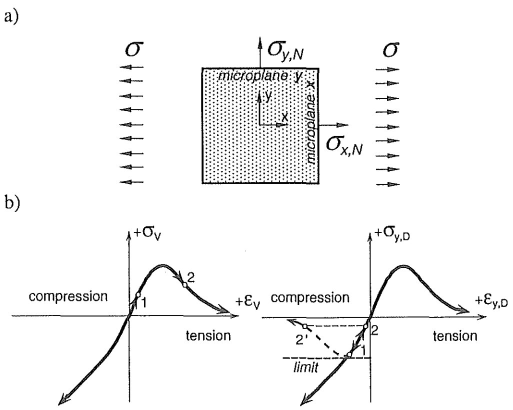

Figure 2. Volumetric and deviatoric microplane strain components for simple two-plane microplane model: (a) microplanes loaded in uniaxial tension (plane stress) and (b) volumetric and deviatoric stress-strain components.

From (6) follows that for $\varepsilon_x > 0$ the deviatoric stress and strain components of plane **y** are negative (compression, see Fig. 2b). For the crack state the load reduces to zero ($\varepsilon_x \to +\infty$; $\sigma = \sigma_x = 0$). Consequently, the volumetric stress reduces to zero as well ($\sigma_V = 0$). From (5) is then obvious that $\sigma_{D,y} = 0$. The volumetric and deviatoric stresses for virgin loading are calculated using (4):

$$\sigma_V = C_V \varepsilon_V \,; \quad \sigma_{D,y} = C_D \varepsilon_{D,y} \tag{7}$$

in which C_V and C_D are the secant stiffness moduli for volumetric and deviatoric components, respectively. The shape of these curves is qualitatively plotted in Figure 2b. With (7) and (5a) the condition (5b) is for the cracked state fulfilled either for: (i) $C_D = 0$ or for (ii) $\varepsilon_{D,y} = 0$. The first implies that the deviatoric component on plane **y** undergoes softening (see dotted line in Fig. 2b). As mentioned above, the deviatoric compressive strength is roughly 10 times large than the volumetric tensile strength. Consequently, (5b) is satisfied by the second condition ($\varepsilon_{D,y} = 0$), i.e. the deviatoric strain component $\varepsilon_{D,y}$ does not undergo softening. Therefore, from (ii) and (5a) follows that $\varepsilon_{N,y} = \varepsilon_V$ i.e. for uniaxial tensile fracture the model predicts lateral expansion.

Introducing (7) into (5) and denoting $\eta = C_{D,y} / C_V$ it follows:

$$\eta = -\varepsilon_V / \varepsilon_{D,y} \tag{8}$$

For the linear elastic stress-strain state η is obtained from (6) as $\eta = (1-2\nu)/(1+\nu)$. Analogously to the elastic solution, to fulfill the condition (i) for cracked state, η should be constant for any level of damage, and therefore:

$$C_{D,y} = \eta C_V \tag{9}$$

According to (9), the condition (i) is fulfilled only when $C_{D,y}$ is proportional to C_V during the entire load (damage) history. Only such stress-strain relationship for deviatoric component which satisfies (9) yields to the realistic solution for the uniaxial tension. If the relationship is different, the model predicts the pathological lateral expansion ((ii) is satisfied) or the normal stress component $\sigma_{N,y}$ for $\varepsilon_x = +\infty$ ($\sigma_x = 0$) does not reduce exactly to zero.

One of the consequences of the pathological model response observed in the finite element analysis is illustrated by two simple examples. First, the unit plane stress finite element with four integration

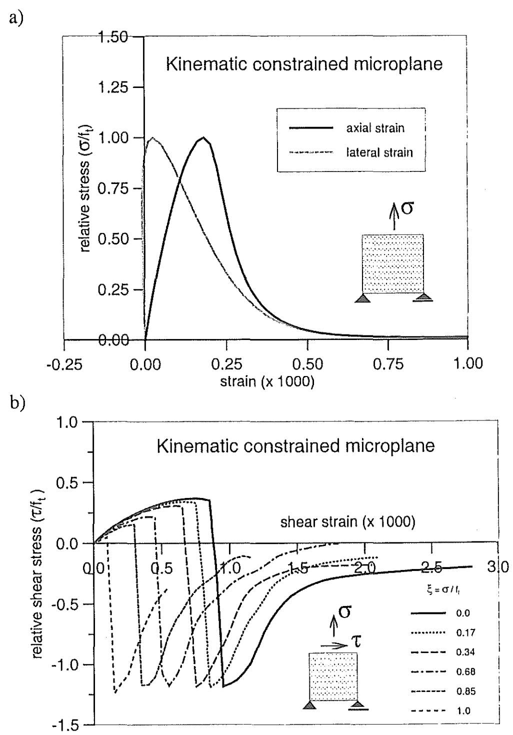

Figure 3. Kinematic constrained microplane for dominant tensile load: (a) Axial and lateral strains for uniaxial tensile load and (b) shear stress-strain curves for different levels of tensile load.

points is loaded in uniaxial tension. The calculated normalized stress-strain curves for axial and lateral strain components are plotted in Figure 3a. It can be seen that before the tensile strength is reached (hardening) the material correctly contracts in the lateral direction, however, after the onset of cracking the model predicts lateral expansion. At the end of the softening process the lateral and the axial strains are equal, i.e. the ratio between the axial and the lateral strain is equal to one. Second, the same element is loaded in tension and then, for a constant tensile stress smaller than the tensile strength, loaded in shear up to failure. The shear load is applied by controlling the horizontal displacements of two upper nodes of the element. The element is assumed to be unrestrained perpendicular to the direction of applied shear load. Figure 3b shows the calculated normalized shear stress-strain curves for different levels of tensile stresses. It can be seen that the model correctly predicts the maximum shear resistance, which decreases to zero when the tensile stress reaches the tensile strength. However, as soon as the shear resistance is reached the stress-strain curves exhibit unrealistic instability as a consequence of pathological expansion.

2.2 *Relaxation of the kinematic constraint*

At lower tensile stress concrete can be viewed as isotropic elastic continuum. With increase of the stress, in the weak zone a band of microcracks forms which by subsequent loading coalesce into a crack, i.e. debonding of the material particles (rupture) takes place. In the framework of the continuum theory, cracking is represented by the localization of strains. When the strain increases, the stress oriented in the same direction (damage direction, direction 1 in Fig. 4) decreases. The strain components which are oriented predominantly laterally to the direction of damage (direction 2 in Fig. 4) decrease approximately elastically, i.e. after damage evolution is completed (rupture), the three-dimensional stress-strain state at the beginning of loading shrinks into a uniaxial stress-strain state, i.e. the stresses around the crack are relaxed. This is essentially a structural effect which should be accounted for in any macroscopic material model. In the present model the effect is a main physical argument for the relaxation of the kinematic constrain which should reflect the relaxation of the stresses around the crack. To account for the relaxation, the total microplane strains in (3) are replaced by the effective strains components:

$$\varepsilon_{m,eff} = \varepsilon_m \psi \tag{10}$$

in which ψ stays for the so-called discontinuity function.

Before to decide how to calculate the effective microplane strains, an objective criterion for dominant tensile load has to be formulated. For direct tension in one, two or in all three directions, compression-tension, unrestricted shear (no restrains perpendicular to the shear direction) with or without tension, the volumetric strain as well as maximum principal stress are positive. Therefore, it is reasonably to assume that the dominant tensile load exists when:

$$\varepsilon_V > 0 \quad \text{and} \quad \sigma_I > 0 \tag{11}$$

where σ_I is the maximum principal stress. Both quantities (ε_V, σ_I) are invariant macroscopic properties. For fully cracked specimen the total volumetric strain is positive and the principal stress is close to zero. In the sense of the smeared crack approach, strains in direction of tensile load are large and those perpendicular to it are practically zero (unloading), i.e. the resulting volumetric strain is positive. Moreover, the total volumetric strain in cracked material is approximately the same as the non-elastic ('plastic') volumetric strain and, therefore, it can be used as an indicator for dominant tensile damage.

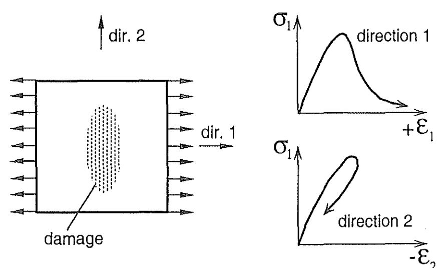

Figure 4. Macroscopic loading-unloading of the material as a function of damage orientation.

Unlike to the models proposed by Carol and Bažant (1995, 1997), the use of the effective strains (10) is not motivated by the theory of plasticity. Namely, the same as in the plasticity, the total microplane strain component can formally be decomposed into non-elastic (relaxed) part and the stress effective part. The actual motivation for the use of discontinuity function are physical arguments discussed above and Equation (9) which is related to a simple two-plane microplane model. According to (9), the deviatoric secant moduli C_D should be for the entire tensile load history proportional to the volumetric secant moduli C_V. Using (9) and the microplane stress-strain relationships (7), the deviatoric microplane stress component for virgin load is calculated as:

$$\sigma_D = C_D \varepsilon_D \psi \tag{12}$$

where $\varepsilon_D\psi$ denotes total effective deviatoric strain. Equation (12) can be rewritten as:

$$\sigma_D = C_D\,\varepsilon_{D,eff} \qquad or \qquad \sigma_D = C_{D,eff}\;\varepsilon_D$$
$$\varepsilon_{D,eff} = \varepsilon_D\,\psi\;; \qquad C_{D,eff} = C_D\,\psi \tag{13}$$

where $C_{D,eff}$ can be interpreted as a effective secant moduli chosen such that (9) holds.

According to (13), for virgin loading the deviatoric stresses can formally be calculated by the use of the effective strains or alternatively using total deviatoric strains ε_D multiplied by the effective secant deviatoric moduli $C_{D,eff}$, which varies from $C_{D,0}$ (initial state) to 0 (crack state) and is proportional to C_V. In both cases the final result is the same, i.e. η remains constant for the entire tensile load history what prevents pathological model response. The discontinuity function ψ has to be of the same type as a function $f(\varepsilon_V)$ which controls the volumetric secant moduli in (4). If one would use an arbitrary function, the pathological behavior could be eliminated, however, the macroscopic tensile stresses at crack state would generally not reduce to zero. Note that for $\psi = 1$ the material is treated as a smeared continuum.

2.2.1 *Normal microplane component*

The normal microplane strain component is decomposed into volumetric and deviatoric part. Although the volumetric strain component for individual microplanes could principally be split into the effective and non-effective part, because of the following reasons the volumetric strain is not split: (1) The volumetric strain is invariant with respect to the microplane orientation, (2) the static constraint on volumetric stress for tensile load is automatically fulfilled ($\varepsilon_V \rightarrow +\infty$, $\sigma_V \rightarrow 0$), (3) it serves as a macroscopic indicator for tensile dominant damage and (4) in the total (not decomposed) form, it is useful for monitoring of the stress-strain path of concrete under tensile cyclic loading.

As shown above, the negative deviatoric strain component means that the microplane normal is oriented close to the direction which is orthogonal to the damage direction. Therefore, when damage increases the deviatoric microplane stress and strain components for these directions should approximately relax (unload) to zero. To account for the above effect, the discontinuity function for the deviatoric strain is taken as:

$$\varepsilon_V > 0,\;\; \sigma_I > 0\;,\; \varepsilon_D < 0 \rightarrow \psi = f(\varepsilon_V) \tag{14a}$$

$$\varepsilon_D \geq 0 \rightarrow \;\; \psi = 1 \tag{14b}$$

The discontinuity function in (14a) has the same shape as the microplane stress-strain relationship for volumetric tension $f(\varepsilon_V)$. For the planes which are oriented predominantly into the damage direction ($\varepsilon_D > 0$), the total strain perpendicular to the crack surface yields to infinity and corresponding stress reduces to zero. Consequently, for these microplanes $\varepsilon_V \rightarrow +\infty$, σ_V, σ_D, $\sigma_T \rightarrow 0$ and $\varepsilon_D \rightarrow +\infty$ (softening) with $\psi = 1$.

2.2.2 *Discontinuity function for the shear microplane strain components*

The microplane shear resistance depends on the normal microplane stress. For positive normal stress (tension), after onset of cracking the shear resistance reduces to zero. On the contrary, for negative normal stress (compression), the microplane offers resistance over shear softening regime mainly through friction and it generally does not reduce to zero. Typical situation when this occurs is compression or shear-compression softening where microplane shear resistance predominantly relies on shear-friction due to the effect of normal compressive stresses. For these stress-strain states concrete is modeled as a cracked continuum. However, for dominant tensile damage, independent of the microplane orientation, the shear strain and stress components relax to zero. Consequently, the shear discontinuity function for individual microplanes reads:

$$\varepsilon_V > 0\;;\; \sigma_I > 0 \;\; \rightarrow \;\; \psi = f(\varepsilon_V) \tag{15a}$$

$$\text{else} \;\; \psi = 1 \tag{15b}$$

Function ψ is the same as for the normal strain component in (14a).

The discontinuity function for microplane shear stress component plays an important role by modeling of shear failure. When shear fracture is modeled by the three-invariant plasticity based models for concrete (Willam et al., 1999), the shear damage unrealistically spreads over a few elements. On the contrary, in the present microplane model ψ enables localization of damage in a row of single finite elements. Consequently, smeared fracture modeling of concrete becomes closer to the discrete crack approach and therefore more realistic.

2.3 *Unloading, reloading and cyclic loading*

To model unloading, reloading and cycling loading for general triaxial stress-strain states, loading-unloading rules for each microplane stress-strain component are introduced. In contrast to virgin loading, for cyclic loading the microplane stress-strain relations must be written in the incremental form:

$$d\sigma_m = C_m^T d\varepsilon_m \tag{16}$$

where C^T_m represents tangent moduli. For virgin loading it reads:

$$C_m^T = \frac{dC_m}{d\varepsilon_m}\varepsilon_m + C_m d\varepsilon_m \tag{17}$$

For unloading-reloading the tangent moduli is generally defined as:

$$C_m^T = C_{m,0}\alpha + \sigma_m\left(\frac{1-\alpha}{\varepsilon_m - \varepsilon_I}\right)$$

$$\varepsilon_m > \varepsilon_{m,P};\ \varepsilon_I = \varepsilon_{m,P} - \frac{\sigma_{m,P}}{C_{m,0}} + \beta(\varepsilon_m - \varepsilon_{m,P}) \qquad (18)$$

$$\varepsilon_m \leq \varepsilon_{m,P};\qquad \varepsilon_I = 0$$

In (18) $\sigma_{m,p}$ and $\varepsilon_{m,p}$ denote the positive or negative peak stress and the corresponding strain for each microplane component using values $\sigma_{m,p}^+$, $\varepsilon_{m,p}^+$ and $\sigma_{m,p}^-$, $\varepsilon_{m,p}^-$ for positive and negative peaks, α and β are empirically chosen constants between 1 and 0 and $C_{m,0}$ is initial elastic stiffness moduli for corresponding microplane component. More detail related to the modeling of cyclic loading are given in (Ožbolt et al. 2001).

2.4 *Initial anisotropy*

The initial anisotropy is a consequence of the material structure, i.e. by nature the material has different properties in different directions. The simplest possibility to account for initial anisotropy is setting function $\Omega(\mathbf{n})$ in (1) to be dependent on the orientation of the normal of each microplane relative to the in advance given weak direction(s) $\mathbf{w}_i$ or strong direction(s) $\mathbf{s}_i$ or both. Principally, when the microplane direction coincide with the weak direction $\Omega(\mathbf{n}) = 0$ and if it is perpendicular to it $\Omega(\mathbf{n}) = 1$. For the strong direction is opposite the true. Once the function is known the anisotropy is automatically taken into account by introducing $\Omega(\mathbf{n})$ into (2).

To better control the relations between dominant directions of the material (such as wood, composites, etc.) it is generally assumed that in the material a few dominant week directions $\mathbf{w}_i$ (for instance in wood, radial $\mathbf{w}_R$ and tangential $\mathbf{w}_T$) and one dominant strong $\mathbf{s}_i$ (in wood, parallel) direction exist. The resulting function $\Omega(\mathbf{n})$, which controls the initial anisotropy, is calculated as a product of the functions which control week and strong directions, respectively:

$$\Omega(\mathrm{n}) = \left(\sum_i \Omega(\mathrm{n})_{\mathrm{wi}}\right)\left(\sum_j \Omega(\mathrm{n})_{\mathrm{sj}}\right) \qquad (19)$$

The functions from (19) are controlled by the following empirical relations:

$$\Omega(\mathbf{n})_{\mathrm{wi}} = 1 - \left|(\cos\varphi_{\mathrm{wi}})\right|^{\gamma i} \qquad (20a)$$

$$\Omega(\mathbf{n})_{\mathrm{si}} = \left|(\cos\varphi_{\mathrm{si}})\right|^{\upsilon i} \qquad (20c)$$

where φ corresponds to the angle between the microplane orientation $\mathbf{n}$ and the week ($\mathbf{w}_i$) or strong ($\mathbf{s}_i$) orientation, respectively. The parameters γ ($0 < \gamma \leq 1$) and υ ($\upsilon > 1$) are empirical. Together with basic microplane parameters (Ožbolt et al. 2001) they control the relations between the initial stiffness and the strengths in the dominant directions of the material.

2.5 *Material model parameters*

Basic parameters of the present microplane model are Young's modulus and Poisson's ratio as well as a set of parameters which control the microplane stress-strain relationships (3). The resulting fracture properties (strength and fracture energy) are obtained implicitly by fit-back analysis on a single finite element or on the integration point (material model level). The material parameters need to be varied such that model response approximately corresponds to the known macroscopic properties of the material. For concrete a special code is written which for a given set of macroscopic properties (tensile strength, compressive strength and fracture energy) generates microplane model parameters. For more detail see Ožbolt et al. (2001).

3 VERIFICATION AND NUMERICAL EXAMPLES

In the present paper the model response is not compared with the typical test data. This comparison is present in (Ožbolt at al. 2001). To demonstrate that the improved model leads to realistic results for dominant tensile load, the examples shown in Figure 3 are re-analyzed with the new model. For this purpose, the same as for the original kinematic constraint microplane model, a single finite element (see Fig. 3) is first loaded in tension and subsequently, for different level of constant tensile load, loaded by shear. Moreover, it is demonstrated that model is able to realistically predict mixed-mode fracture of concrete as well as compressive failure of wood which exhibits strong initial anisotropy. In all numerical examples a standard eight-node solid finite elements with eight integration points were employed. As a localization limiter, the crack band method is used (Bažant and Oh 1983).

3.1 *Tension and tension-shear test*

Figure. 5a shows the calculated stress-strain curves (lateral and axial directions) for uniaxial tensile load. As can be seen, in contrast to the standard kinematic formulation of the model (see Fig. 3), the new model predicts in lateral direction contraction for the entire load history. By increase of damage in the axial di-a)

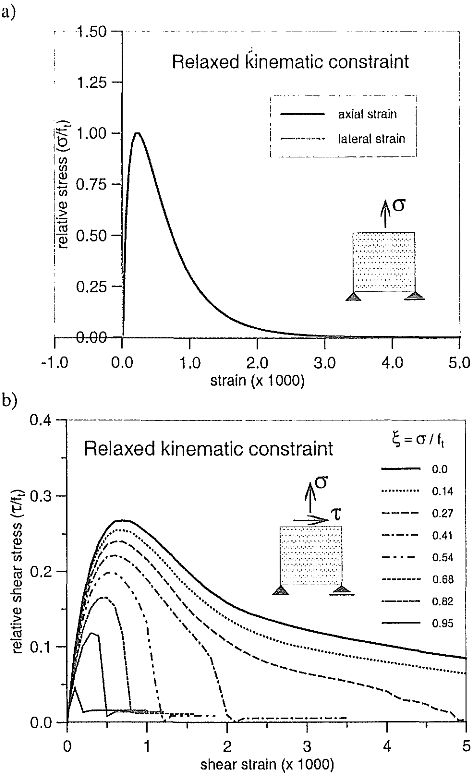

Figure 5. Uniaxial tension and tension-shear combination: (a) nominal stress-strain relationship and corresponding lateral strain and (b) shear resistance as a function of the tensile stress.

rection the lateral strain decreases asymptotically to zero.

In Figure 5b the calculated shear stress-strain curves are plotted for different levels of tensile stresses. In contrast to the original kinematic microplane model (see Fig. 3b), independent of the tensile stress level, the shear stresses are always correctly reduced to zero.

3.2 *Mixed-mode fracture of concrete*

In a number of numerical studies of the mixed mode fracture of concrete that have been recently performed by different authors, it has been concluded that the smeared fracture analysis based on a different local and nonlocal formulations is generally not able to correctly simulate this complex type of fracture (Di Prisco et al. 2000). To check whether the 3D finite element analysis (crack band approach) with the use of the present model can correctly predict mixed mode fracture of concrete, the tests performed by Nooru-Mohamed (1992) have been simulated (Ožbolt & Reinhardt 2000). In the tests, the double notched specimen were first loaded in shear and subsequently in tension by constant shear load. Figure 6 shows observed and calculated crack patterns (shear force: P = 5, 10 kN and $P = P_{max}$). As can be seen the calculated crack patterns are similar as the experimental. Obviously, the model is able to correctly deal with relatively complex fracture mechanisms in the frame-work of the local smeared crack theory.

a)

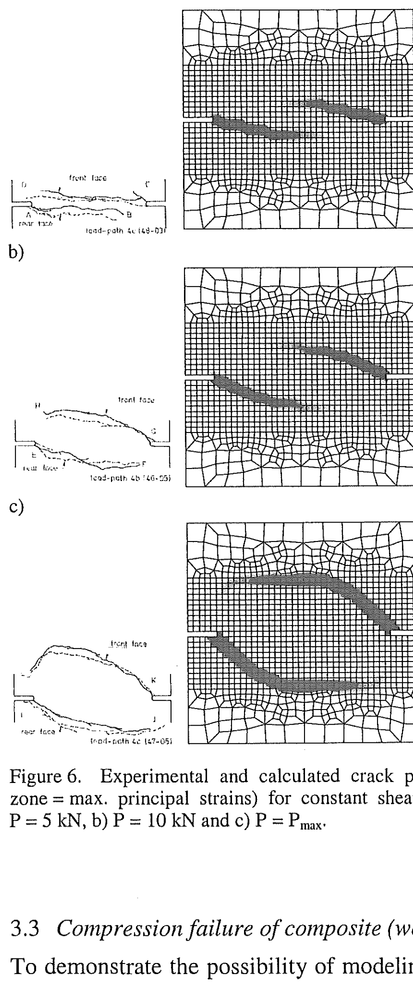

Figure 6. Experimental and calculated crack patterns (dark zone = max. principal strains) for constant shear load of: a) P = 5 kN, b) P = 10 kN and c) $P = P_{max}$.

3.3 *Compression failure of composite (wood)*

To demonstrate the possibility of modeling of materials that exhibit strong initial anisotropy, compressive failure of wooden specimen (50x50x100 mm) was simulated. The three dimensional finite element analysis of the specimen loaded in uniaxial compression was performed. In Figure 7 are shown the failure modes for different orientations of the week and the strong directions, respectively. Two week (tangential and radial) and one strong (parallel) directions were assumed. The predicted failure modes are in good agreement with experimental observations (Ožbolt & Aicher 2000).

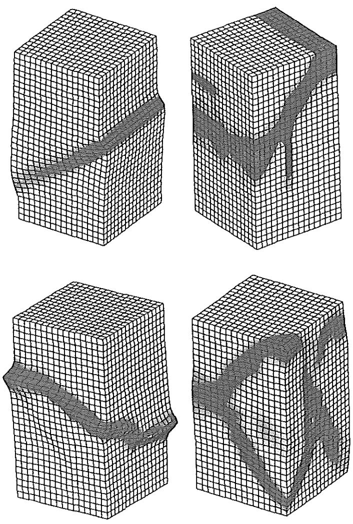

Figure 7. Different failure modes as a function of the orientation of the weak (strong) directions in the wood.

4 CONCLUSIONS

The present microplane model is the macroscopic three-dimensional material model for concrete like materials. The simplest and computationally most efficient form of the model is based on the kinematic constraint approach which, however, in combination with realistic stress-strain relationships for concrete leads to unrealistic model response for dominant tensile load. It is discussed that the main reason for this is due to the split of the normal microplane component into volumetric and deviatoric part and not to the kinematic constraint itself. To improve the model the kinematic constraint for dominant tensile load is relaxed by splitting of the microplane strain components into the effective and relaxed part. The split is controlled by the discontinuity function which accounts for the effect of discontinuity as a consequence of cracking. The conceptual simplicity of the new model is preserved. It is shown that the new model predicts physically correct results. Using relatively simple cyclic rules on the microplane level, the macroscopic cyclic response for any three-dimensional stress-strain history comes automatically out. Moreover, it is shown that the model is able to simulate complex fracture phenomena for materials with damage induced anisotropy as well as for the materials with strong initial anisotropy.

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