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Modeling of fatigue crack growth in concrete subjected to mode I crack opening

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ABSTRACT: Relying on previous observations and phenomenological studies presented in previous FraMCoS conferences, a model of mode I crack growth in concrete under fatigue is proposed. Based on the cohesive crack concept, the finite element computation, where the bulk material is assumed to be elastic is used. A standard finite element code CESAR is used to which a specifically developed module was added. This module allows to introduce cohesive forces on the crack surfaces, thus taking into account the non linear behaviour of the material. The cohesive force law is based on three characteristic parameters : the measured tensile strength, the fracture energy and an hysteresis law. The model is applied to simulate crack growth in notched mortar specimens subjected to three point bending tests and its results are compared with the experimental data.

1 INTRODUCTION

Fatigue of concrete is mainly due to the initiation of cracks and next their growth within the material. Crack growth gives rise to fracture surfaces where interacting phenomena hinder the propagation. These well known phenomena imply that non linear fracture mechanics is a suitable tool to describe the propagation of cracks. Considering the prediction aspect of crack propagation, the cohesive crack model, as originally conceived by Hillerborg (1976) is widely accepted as the constitutive model to describe the mechanical behaviour of quasibrittle materials like concrete. This model had been successfully used in numerical analysis by some researchers to calculate load-deflection curves of stable tests on notched specimens in a three point bending test ; see, e.g, Carpinteri et al (1986), Nallathambi & Karihaloo (1995).
Another feature is the concrete behaviour under cyclic loading, where the mechanism of crack growth is still complex ; and it is expectable that cyclic crack growth characteristics derive from the post-peak cyclic behaviour of concrete in tension. The post-peak cyclic behaviour of concrete in tension has been studied by Reinhardt et al (1984) and attempts were made to model the hysteresis loop under unloading-reloading. By way of examples, one can quote the Focal Point Model (FPM) by Wolinski, the Continuous Function Model (CFM) proposed by Hordijk (1991). All these models are based on a stress-crack opening relation and remain suitable for implementation in finite element codes. However, the numerical implementation of such models is not easy, as regard to the number of operations which must be executed in finite element analysis.
In this paper, simple analytical expressions are proposed to describe the unloading-reloading curve. The model is implemented in finite element code CESAR and applied to simulate the behaviour of notched mortar specimens in three point bending tests under static or fatigue loading.

2 PRESENTATION OF THE MODEL

2.1 *Hypotheses*

According to Hillerborg et al (1976), a crack in concrete is schematically composed of two zones (see figure 1).
The first one refers to the traction free crack, where both crack surfaces are wholly separated ; the second one (cohesive zone) refers to the extended part of the crack where forces are transmitted by frictional effect and other phenomenons such as aggregate interlock. The material outside the fracture zone is assumed to be linear elastic. The cohesive forces distribution along the cohesive zone derives

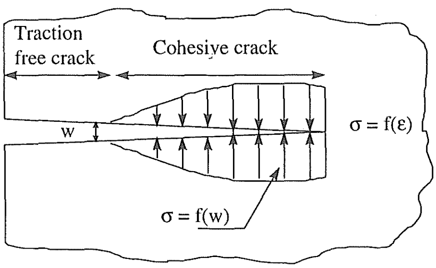

Figure 1. Sketch of the cohesive crack model.

from the post-peak behaviour of material in direct tensile test. Thus, under static loading, the post peak behaviour is characterized by the well known tension softening curve, noted also the σ-w relation. This assumption means that the cohesive forces are function of the crack width (w).

2.2 *Finite element implementation*

The implementation of the model is carried out on a notched beam in three point bending test. One half of the beam is considered assuming that a crack develops in the middle section of the beam along a straight line. Typical boundary conditions for a specimen at a particular external load are shown in figure 2. The tensile strength f_t of the material is required as a criterion for a crack propagation. Indeed, a crack is supposed to form when the normal tensile stress σ_x at the index node I2 reaches the level f_t. Then the released nodes are subjected to the cohesive forces.

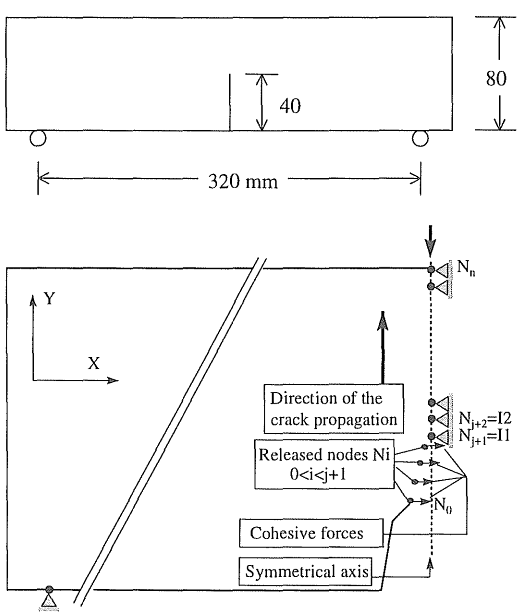

Figure 2. Modeling the beam. Boundary conditions.

Finite element analyses are realized using the finite element CESAR code developed at the LCPC and a specific module to compute the cohesive forces on the crack. For more information about the applied numerical techniques, it can refers to Toumi(1998).

2.3 *Model for crack cyclic growth*

The model is given by means of a stress-crack opening relation (figure 3).

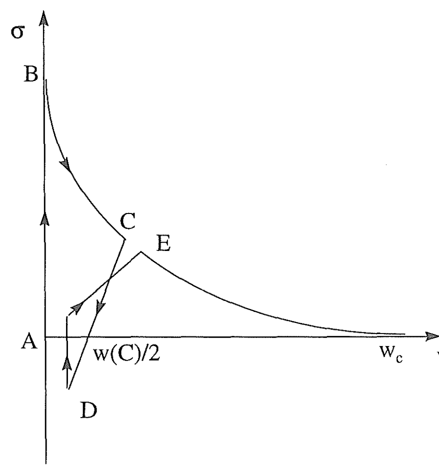

Figure 3. Description of the crack cyclic growth.

The model uses, on the one hand the envelope curve, on the other hand new expressions to describe the unloading-reloading cycle. The proposed expressions are chosen while being based on the experimental results of Cornelissen and Reinhardt (1984). The expression of the envelope curve as proposed by Hordijk is given by :

$$\frac{\sigma}{f_t} = [1+(c_1\frac{w}{w_c})^3]Exp(-c_2\frac{w}{w_c}) - \frac{w}{w_c}(1+c^3)Exp(-c_2) \quad (1)$$

where w = crack opening; w_c = critical crack opening; c_1 and c_2 = constants respectively equals to 3 and 6.93.

The critical crack opening w_c is given by :

$$w_c = 5.14\frac{G_F}{f_t} \quad (2)$$

where f_t = uniaxial tensile strength; G_F = fracture energy.

Starting from point C at the envelope curve, the unloading curve is determined by :

$$\sigma = \frac{\sigma(C)}{\lambda - 1}[\lambda \frac{w}{w(C)} - 1] \qquad (3)$$

where λ = constant (to be determined, see § 3.2).
Starting from point D at the lower stress level up to point E at the envelope curve, the reloading curve is given by :

$$\sigma = \sigma(E)[\frac{w}{w(E)}] \qquad (4)$$

The stress $\sigma(E)$ at the returning point on the envelope curve can be found with :

$$\sigma(E) = [1 - \mu]\sigma(C) \qquad (5)$$

where μ = degradation parameter, assumed to be constant and independent of the stress $\sigma(C)$ before unloading. For numerical computation, $\mu = 0.05$ according to Hordijk (1991).

3 COMPARISON BETWEEN EXPERIMENTAL AND NUMERICAL RESULTS

The tests presented in this section are related to a mortar whose mechanical characteristics are : f_t = 5.2 MPa; G_F = 34 N/m. The comparisons are carried out, first on static tests and, second on fatigue tests.

3.1 *Static tests*

Under static loading, the confrontation of the model with the experimental results relies on the measured load-deflection curve under continuous deformations and on the measured crack length during the loading. Concerning the load-deflection curve, figure 4 presents the confidence interval of the experimental curves with the average curve, as well as the simulated curve. In spite of a slate shift in the downward branch, one can note a fair fit between calculations and experimental results.

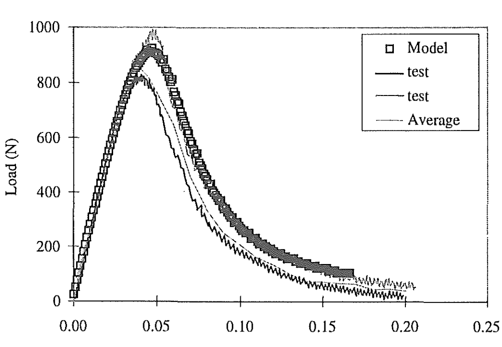

Figure 4. Measured and predicted load-deflection curves.

Besides the comparison of the load-deflection curve which reports the total behaviour of the sample, it were compared the computed crack length evolutions with those measured by means of the replica technique associated with Scanning Electron Microscopy (SEM) (Bascoul et al, 1994). The finite element calculated crack length is obtained by cumulating the distances between the released nodes. The results of this comparison are given on figure 5. It can be seen that the model gives a good approximation of the total crack length in the first period and tends to overestimate it when it exceeds 5 mm. This overestimation can be attributed to several factors. In particular, the non-linear behaviour of the specimen is limited to the tension softening in the plane fracture zone and it may that for the studied mortar some other damages are taking place outside of this fracture zone.

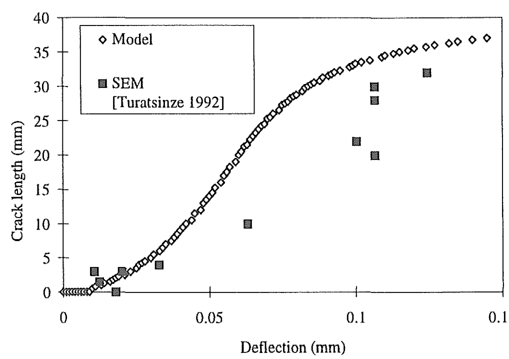

Figure 5. Observed and predicted crack length evolution.

3.2 *Fatigue tests*

Before extending simulation to the cases of the cyclic loading, the value of the constant λ in equation (3) must be determined. For this purpose, back analysis was performed. Indeed, λ is parameterized in order to better reproduce the load-deflection curve with unloading reloading loops in

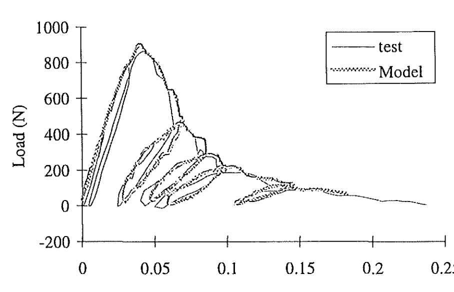

Figure 6. Experimental and predicted results for cyclic loops.

the post-peak phase. As one can note it in figure 6, λ=2 provides a good fit of the real response. Consequently in the following, the value of λ is fixed with 2.

Figures 7 and 8 show two examples of the simulated crack growth under fatigue loading. They correspond to 98 and 93 % as a maximum load rate respectively. In parallel, the experimental lengths measured by Scanning Electron Microscopy (SEM) are drawn.

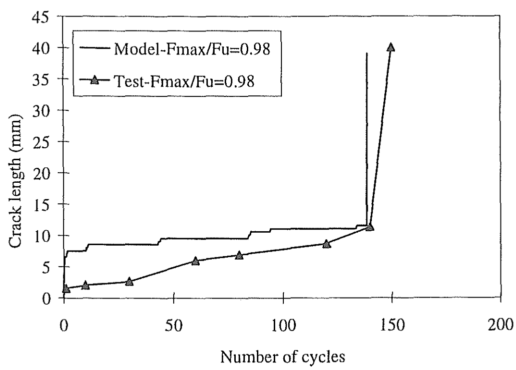

Figure 7. Crack length versus number of cycles. Comparison between the model and the experimental data : Example1.

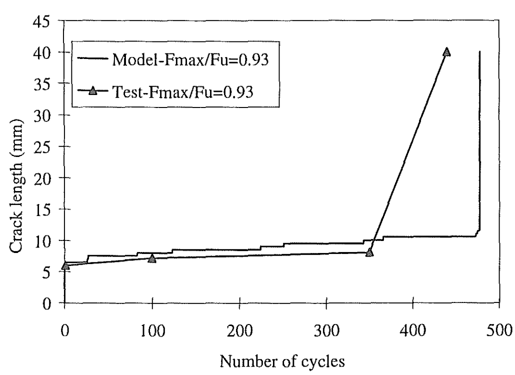

Figure 8. Crack length versus number of cycles. Comparison between the model and the experimental data : Example2.

For the two presented examples, the model gives good prediction of the crack cyclic propagation in concrete. In example 1, one can note that at the beginning of propagation, the model gives higher crack length values than the experimental ones. This difference vanishes when the crack growths up to give a total number of cycles almost identical. So one cannot conclude as for one overestimation of the model. However it would be necessary to undertake other simulations on other compositions of concrete in order to validate and, if necessary, to improve the model.

4 CONCLUDING REMARKS

A new simple model for crack cyclic growth based on the cohesive crack concept is proposed in this paper. Its implementation in finite element code CESAR permits to simulate the crack growth in concrete in three point bending tests both under static and fatigue loading. Under static loading, numerical calculations gives a good approach of the actual behaviour of the tested specimens. Calculations are extended to cyclic loading and the first results are in accordance with the experimental behaviour. The next step will be the optimisation of the computational tool from a numerical point of view. It would be also appropriate to extend the calculations for other concrete compositions in order to support if the selected parameters in the present model are the most determining.

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