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Meso-scopic concrete analysis with a lattice model

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ABSTRUCT: In order to perform the realistic simulation for concrete and reinforced concrete structures, we present a new analysis method in this paper, which employs lattice type numerical modeling and has the two distinct features. One of them is the introduction of four different types lattice members for interaction of each constituents; mortar, coase aggregate, steel and bond members, each of which is given a simple constitutive model. The other is the realistic representation of the meso-scopic geometrical configuration of concrete, which can be realized by the image-based geometry modeling technique. Since the latter incorporate the accurate meso-scopic morphology with the former modeling scheme, the proposed method enables us to capture the craking (or localozation) behavior induced by the meso-scopic heterogeneity. Several numerical example succesfully demonstrate the so-called compressive failure of concrete specimens under uniaxial loading and the dependency of the failure modes of reinforced concrete beams on height-span ratio.

1 INTRODUCTION

Concrete, which is known as a quasi-brittle material, consists of several distinct materials and its failure mechanisms are complex due to the local (or meso-scale) *heterogeneity* and *localization* behavior with cracking. In particular, the characteristics of its compressive and shear fracture observed prior to failure are still far from being fully understood. Inevitably therefore, the overall behavior of reinforced concrete structures is much more hard to analyze probably due to the lack of relationship between the overall mechanical behavior and meso-scopic heterogeneity generally represented by mortar and aggregates.

In this context, it has been widely recognized that the hierarchy of material characterization for concrete structures plays an important role especially in the area of computational mechanics. The hierarchical levels of numerical concrete analysis is attributed to Zaitsev and Wittmann (1981), who divided a class of numerical concrete models into four levels; macro-, meso-, micro- and nano-levels. These levels differ on the treatment of the *heterogeneity* as shown in Figure1. Although the micro- and meso-scopic levels provide important information about the material properties of mortar and coarse aggregates, it is usually understood that the mechanical behavior of concrete structures is governed mainly by the macro- and meso-scopic mechanical events. In general, the macro-scopic models is chosen for structural analysis of an overall concrete structure, and a meso-scopic model is utilized to characterize the material properties for the macro-scale ones, as can be anticipated from Figure1.

In a macro-scopic model, concrete is commonly regarded as a homogeneous material and the governing equations are provided often within the framework of continuum mechanics, which is relevant to the finite element method (FEM). The so-called smeared crack model is one of the typical examples of this type and often provides a realistic load-displacement diagram with low computational cost; see Bhatt and Kader (1998) and An and Maekawa (1998). It is, however, pointed out that, due to the inherent nature of FEM, the modeling of discontinuous displacement as well as stress and strain appears not straightforward; see the crack band model by Bažant and Oh (1983) and many others. Thus, the aim of macro-level analysis is to predict the main failure condition, like a ultimate load and load-displacement curve, without conducting experiments.

On the other hand, meso-level models are concerned mainly with local structures composed of coarse aggregates and a mortar matrix. Therefore, it can easily incorporate the meso-scopic heterogeneities into the complex mechanical responses such as a so-called size effect for un-notched specimens in tension. To be more specific, the fracture mechanism triggered by the meso-scopic heterogeneities is introduced in our numerical analyses without complex mathematical descriptions. A representative example of this type is a class of lattice type models; see Bažant et al. (1990) and Schlangen and van Mier

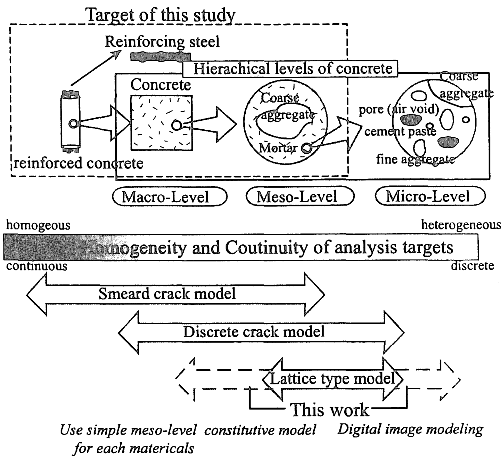

Figure 1 Structural scale and simulation models for concrete and concrete structures

(1992). Other meso-level models such as continuum-based and dicrete models are proposed in analogy with corresponding macro-scopic approaches; see, for example, Nagai et al. (1998) for the former model and Kwan et al. (1999) for the latter.

Although these models of different levels of hierarchy are capable of simulating the complex mechanical behavior of concrete structures to some extent, each model stays within its own usage. In particular, the meso-scopic characterization of concrete material has not been reflected on the simulations of the macroscopic mechanical behavior of concrete structures. There have been little attempts to combine both models of macro- and meso-scale levels.

Our goal of this study is to a new analysis method to realize realistic simulations for the macroscopic mechanical behavior of concrete structures with the meso-scopic model of concrete. In particular, our emphasis is placed on the simulation of reinforced concrete structures. We employ the lattice type model to represent the cracking behavior and bond-slip relationship between concrete and steel since distinct mechanical behavior in meso-scale is easily incorporated with analysis models. In order to activate such performance in a macro-scopic viewpoint, we annex the following two features to a lattice type numerical modeling:

1. Four types of lattice members are introduced to represent the material and interface responses in the reinforced concrete.
2. The digital images is utilized in the geometry modeling to reflect the effect of the actual meso-scopic *heterogeneity*.

The proposed numerical concrete model enables us to simulate crack propagation and debonding of concrete-steel interface without introducing any physical device. Two representative numerical examples are presented to demonstrate the performace of our model. One of them is the simulation of compression test of concrete specimens with emphasis on the effect of boundary conditions. The onter example is to demonstrate the well-known dependency of the cracking patterns in reinforced concrete beams on beam spans.

2 LATTICE MODEL

In order to realze the mechanical behavior of reinforced concrete structures as well as concrete itself, we develop a new analysis model in this section by enhancing the existing lattice type numerical models. After the general feature of the lattice type models are described, four types of lattice members are introduced to represent the quasi-brittle material response of concrete and the interface property between concrete and steel. Then the digital-image-based modeling technique, which realizes realistic meso-scopic geometry modeling, is presented as a key technology to complete the method.

2.1 *General feature*

The lattice type model to simulate the meso-scopic behavior of concrete was initiated by the distinct element method(DEM), which was proposed by Cundel (1979) to simulate the behavior of granular solids such as sand. In the lattice type numerical models, a macro-scopic continuum is discretized as a network of linear structural member such as truss or beam elements. This modeling strategy is extended to the simulation of crack growth with quasi-brittle material modeling for concrete; see, e.g., Zubelewicz (1980) and Zubelewicz and Mróz (1983).

The macro-scopic material nonlinearlity for concrete is usually introduced by reduction of stiffness, or removal of the members, whereas softening behavior in plasticity is often used to model one-dimensional material response of a meso-scopic concrete member. Note, however, that the introduction of softening to material modeling seems irrelevant because the mesoscopic treatment of the lattice type models can directly possess the brittle nature of concrete under tensile loading. Nonetheles, the discrete treatment of lattice members seems suitable for the modeling of bond-slip as well as cracking on the interface between concrete and steel.

2.2 *Meso-scopic constitutive model for each material*

We assume that a reinforced concrete structure consists of three separate materials in a macro-scopic viewpont; that is, concrete, steel and bond member. However, in a meso-scopic level, concrete itself is assumed to consist of coarse aggregates in a mortar matrix. That is, the so-called two-phase material is as-

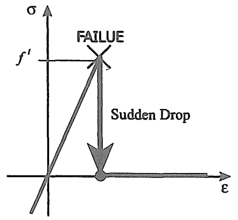

Figure 2 Constitutive model for a mortar member

sumed for the meso-scopic heterogeneity of concrete, whereas we neglect, in a more micro-scopic viewpoint, the effects of other constitutents such as fine aggregate, cement paste and air space.

Thus, an overall analysis model of reinforced concrete is composed of four types of materials; mortar, coarse aggregate, steel and bond member, each of which is a truss finite element with a simple constitutive model. In the subsequent sections, the meso-scopic constitutive model for each member is introduced in order.

(*Mortar and aggregate members*) In a meso-scopic viewpoint, concrete will exhibit elast-brittle behavior and re-distribute internal force as soon as the breaking threshold is exceeded. The brittle behavior is caused by the propagation of microcracks, and in turn the re-distributed internal force invites new microcracks. As a result, the growth of a crack is represented by the formation of macrocracks. Though this behavior is solely geometrical nonlinearity, the so-called strain softening behavior would be observed in an overall structure. In other words, the macroscopic strain softening behavior is the result of unstable response, triggered by the progress of the microcrack. This type of macroscopic material resonse is sometimes called geometrical softening; see Drescher and Vardoulakis (1982). We draw attention to such interpretation of macroscopic softening behavior to model the quasi-brittle response of concrete.

In our lattice model for concrete, the meso-scopic brittle response is assumed to be caused only by the tensile failure of mortar members. Thus, the stress-strain relation of mortar members is elast-brittle type as shown in Figure2, while aggregate members are always assumed to have a linearly elastic property and not to fail. Such material modeling for concrete is never invite a compressive and shear failure of each meso-scopic lattice member. Instead, the microcracks are activated by *tensile* force to invite shear or splitting cracks even for macropscopic compression.

(*Reinforcing steel and bond member*) A reinforcing steel bar is discretized into steel members, which is the same network of truss elements as concrete members, and whose material nonlinerity is also idealized by a 1-D constitutive model. The classical one-dimensional rate-indepedent plasticity with isotropic

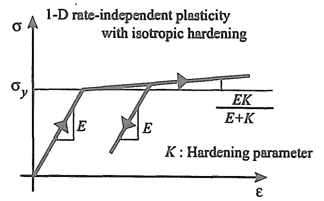

Figure 3 Constitutive model for steel and bond members

hardening model, which is illustrated in Figure 3, is employed for the constitutive model for steel members. Note here that the plastic deformation in our modeling never exhibit material softening.

The influence of bond characteristics between steel and concrete cannot be neglected to examine the failure of reinforced concrete structures. Several macroscopic bond-slip relationships are obtained from pull-out tests; see for example Salem and Maekawa(1998). However, actual bond effects would be caused by the contact and the friction between steel and the surrounding concrete, and not-reversible behavior is observed.

In this study, the bond effect is simply approximated by bond members, which is characterized by the same 1-D rate-independent plasticity as steel member. We settle the bond members between mortar and steel member, in addition, the overlapping layers of the bond and steel so as to represent craking through the steel bar, as illustrated in Figure 4.

The feature of our lattice model is summarized as follows:

1. reinforced concrete is divided into four type lattice members: mortar, coarce aggregate, steel and bond
2. material nonlinerlity for each member is provided by very simple, conventional constitututive models.

Note that simple material models would make the interaction between each members clear.

With this physical model for a reinforced concrete, the numerical solution can reflect the geometrical characteristics of aggregates as well as their positions. In order that the discritization successfully involves such meso-scopic information, the geometry models should be realistic as much as possible. Therefore, a new moleling strategy with digital images is utilized to extract the realistic morphology of concrete composition.

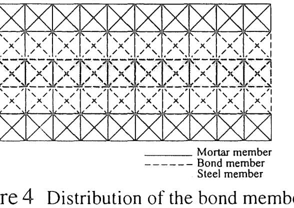

Figure 4 Distribution of the bond members

2.3 *Digital image modeling – Second modification*

In most of the existing lattice type numerical models, the heterogeneity of meso-scopic geometry of concrete is taken into account by way that random circular aggregates are disposed. Then, the circular aggrregates are mapped on a regular lattice network in one case, and, in another case, are connected to adjacent ones by a pin-joint truss at each of their centers. Unfortunately, the idealization of circular aggregates in these ways of modeling seems open to criticism. That is, actual shape of aggregates never be circular even in plane analyses and, in turn, irregular shape of aggregates would be a source of size effects and complex overall behavior. Therefore, more realistic representation of the mesoscopic geometry is dispensable in the lattice type model for reinforced concrete structures.

For the sake of simplicity, we have assumed that the concrete consists of coarse aggregates and mortar. In numerical analysis with meso-scopically, overall structural responses depend on the geometric analysis model that defines the shapes of aggregates and their position.

In this context, a novel modeling technique with digital images is developed in the area of computational homogenization to construct realistic microstructural models and is referred to as digital image-based (DIB) modeling; see Hollister and Kikuchi (1994) and Terada et al. (1997). The idea is simple and the geometry model is constructed in such a way that one pixel in the 2-D case (or one voxel in the 3-D case) is regarded as a one finite element (FE) element.

The same modeling scheme can be directly applied to the meso-scopic geometry representation of concrete, but the lattice approximation of continunous media necessitates slight modification. In this particular situation, a pixel is regarded as a square unit structure composed of several lattice members, as shown in Figure5. Then actual meso-scopic geometry of concrete is successfully captured and recognized as digitized data; that is, the images would contain the necessary information about the constituents such as an aggregate and mortar with a desirable accuracy withn image resolution.

The whole procedure can be divided into two major parts. The first step is *sampling* and the second step consists of *thresholding* and *modeling*. Figure5 shows the whole procedure of this modeling.

2.4 *Nonlinear analysis method*

When craking develops in a brittle material, stress is released on the cracked surface. The numerical analysis involving released stress is known to invite a decline of convergence rate of nonlinear solution methods. To overcome this difficulty, several algolithms have been proposed; see, for example, r_{min} method by Yamada et al. (1968) and Method of Inelastic Force(MIF) by Jirasek and Bažant (1995).

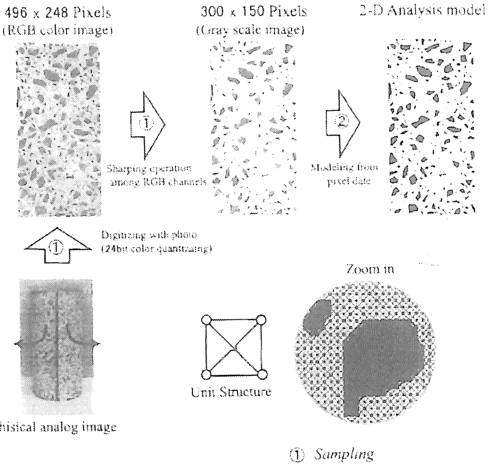

Figure 5 Whole procedure of DIB modeling for lattice type models

In order to capture the rugged responce in the load-versus-displacement curve, we shall make slight modification on the conventional MIF such that the static equilibrium state is attained by an Newton-Raphson iterative solution scheme with the arc-length control. Also, for the plastic members, the conventional return-mapping algorithm is used to attain the equilibrated state efficiently; see Simo and Hughes (1998). Each loading step in our analysis method is divided into several iteration steps for equilibrium as well as mesos-copic cracking (or equivalently macro-scopic localization) step; see Figure6. It is to be noted that the loading is not updated until the lattice system is equilibrated.

3 SIMULATION OF SPECIMENS UNDER UNIAXIAL COMPRESSION

In this section, we carry out the numerical simulation on a plane concrete specimen under uniaxial compression prior to that for reinforced concrete structures in Section 4. The purpose of this simulation is to examine the performance of our modified lattice model with the DIB modeling and, in particular, the ability to reproduce the compressive failure characteristic, which is one of the essential properties of concrete.

3.1 *Conditions of numerical simulations*

In order to simulate the effect of boundary and geometry conditions, we prepare four analysis models of specimens of different conditions as shown in Figure7. Models A-1 and A-2 are generated directly from the plane digital image of a cylinderical specimen (15 cm $\times$ 30 cm). The resolution of the digital image is adjusted into 10 pixels/cm. Models B-1 and B-2 are generated so that the height is a half of that of Models A-1 (or A-2). Models A-1 (or A-2) and Model

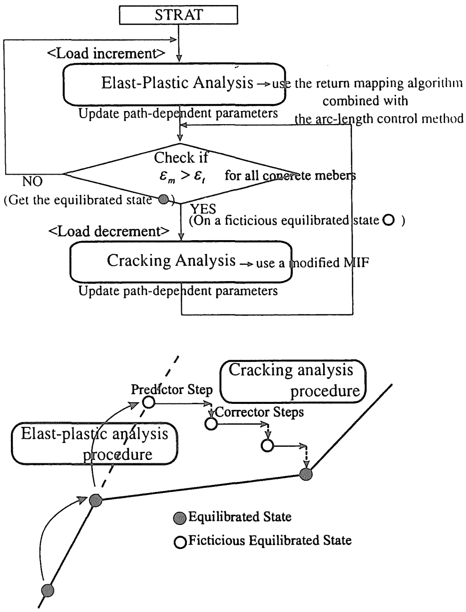

Figure 6 The whole analysis flow

B-1 (or B-2) have 150 × 300 and 150 × 150 finite elements (pixels), respectively. Here, '-1' and '-2' on each model name respectively indicate the fixed and free boundary conditions at the top and bottom surfaces. This means that Models A-1 and B-1 are subject to high friction there in the transverse direction, while Models A-2 and B-2 are not.

The material properties are chosen as follows: Young's modulus of mortar members $E_m = 20\,\mathrm{GPa}$, that of of coarse aggregate members $E_a = 60\,\mathrm{GPa}$, the breaking threshold for mortar members $f_t = 2\,\mathrm{MPa}$ (or $\varepsilon_t = f_t/E_m = 0.01\,\%$).

3.2 *Numerical result and Discussion on compressive fracture*

Figure8 shows the apparent axial stress-strain curves obtained for all the models. Here, the *apparent* means *macroscopically measured* as in actual experiments and, the stress and strain are respectively evaluated from the displacement and external force in the loading direction at the top surface. The cracking patterns at fracture are shown in Figure10. Here, the failure is understood in a numerical sense and defined as a divergence during the iteration or, equivalently, a state of unbalance between the external and the internal forces.

From these figures, the following results are obraind:

1. The slenderness makes the ultimate load decline moderately,

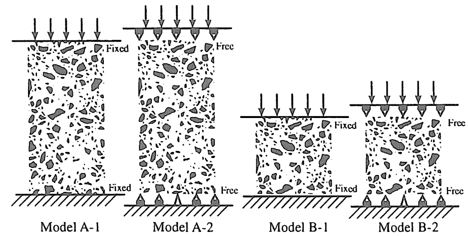

Figure 7 Analysis models for uniaxial compression

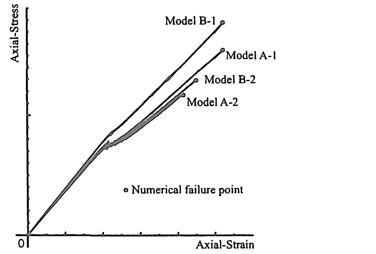

Figure 8 Axial Stress - Strain curves of compressive test

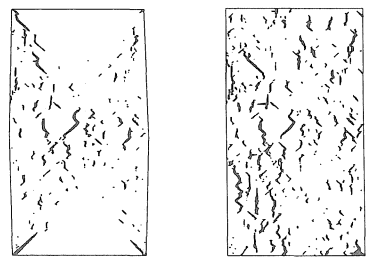

Figure 9 Cracking patterns at failure (Model A-1,-2)

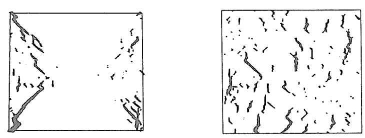

Figure 10 Cracking patterns at failure (Model B-1,-2)

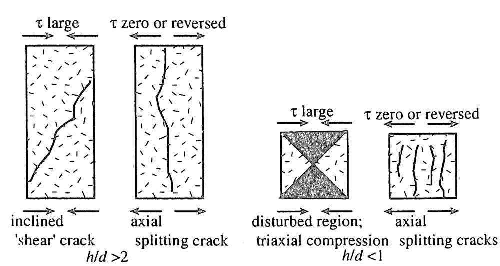

Figure 11 Different mechanisms fo concrete under uniaxial compression

2. The existence of friction at the top and bottom surface makes the ultimate load decline moderately, and

3. The two types of boundary conditions give quite diffrent failure modes,

These statements are equivalent to the claim that that effect of boundary conditions at the top and bottom surfaces agrees with the experimental tendency reported by van Mier (1998), which has been shown in Figure11.

In spite of the simplicity of our proposed model by lattice networks, the compressive failure modes are reasonably reproduced in these simulations in agreement with the well-known experimental fact. It should be noted again that our lattice model does not involves compressive or shear failure criterions and only assumes very simple material models described in Section 2.2.

To be ideal, material characterization should be essentially unique and objective. In other words, the material properties should not depend on the boundary condition as well as any other experimental conditions. Our results show that the experimental tests of specimens are very sensitive to the boundary conditions and the size of the specimens. It is emphasized here that the meso-scopic geometrical configuration in actual concrete plays an improtant role to capture such mechanical bahavior.

4 SIMULATION OF REINFORCED BEAMS

WIth the proposed analysis method, numerical analyses are conducted on a series of simply-supported reinforced concrete beams under a transverse point force at the center of the beam. Assuming that the specimens have no shear reinforcements, we expect that these beams exhibit brittle shear failure.

4.1 *Conditions of numerical simulations*

Three specimens with different values of ratio a/d and of the same effective depth d are considered. The concrete material is assumed to be the same material composition as in the previous section. The material properties used here are given in Table 1.

Table 1 Material properties

	Mortar	Aggregate	Bond	Steel
E (GPa)	20	60	20	80
A	1.0	1.0	1.0	1.0
ε_t	0.0001	-	0.0001	-
ε_s	-	-	(0.0001)	0.002
K	-	-	20	20

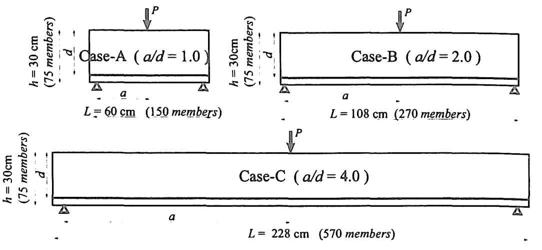

Figure 12 Analysis models of concrete beems without shear reinforcement

An analysis model of each specimen is generated from the digital images of the same cross section of a reinforced concrete which is presented in Figure12. The dimensions of Beam A, B and C are chosen as follows: 30cm $\times$ 60cm, 30cm $\times$ 108cm, 30cm $\times$ 228cm, and $a/d = 1.0$, 2.0, 4.0. The resolution of the digital images is adjusted into 2.5 pixel/cm so that the mesh size of Beam A, B and C identified with 75 $\times$ 150, 75 $\times$ 270 and 75 $\times$ 570 finite elements (pixels), respectively.

4.2 *Numerical results of reinforced beam*

Using the loading parameter and the displacement at the loading point, we plot the load-versus-vertical displacement curves in Figure13. Figure14 shows the simulated craking pattern at failure. Here, the failure states are defined in a numerical sense as in the previous section. Owing to the modified MIF, each equilibrated state is evaluated on a discrete loading step. The macro-scopic responses of the reinforced beams are characterized by the rugged lines shown in Figure13, which are actually induced by the meso-scopic shear and splitting fracture of mortar members. The calcurated ulitimate load of the reiforced beams increases as the ratio a/d becomes small. This tendency well agrees with the failure characteristics reported in other experimental studies. That is, for reinforced concrete structures, there exists a size effect of the mechanical responses on the ratio a/d.

Figure15 shows the transition of crack propagation for Case-B and its final cracking pattern is compared with a state from an experimental study at failure. As can be seen from this figure, the numerical cracking pattern provides the realistic failure mode as is reported in other experimental studies.

According to Schlangen (1995), numerical simulations with the lattice type models would provide the results dependent strongly on the employed fracture criterion and the selected element and/or mesh type. Our results, on the contrary, illustrate that a numerical concrete model with simple fracture criterion, simple structural elements and regular mesh together with accurate geometry modeling is able to reproduce the cracking patterns of macrocracks, which govern the stability of overall structure. Instead of the depen-

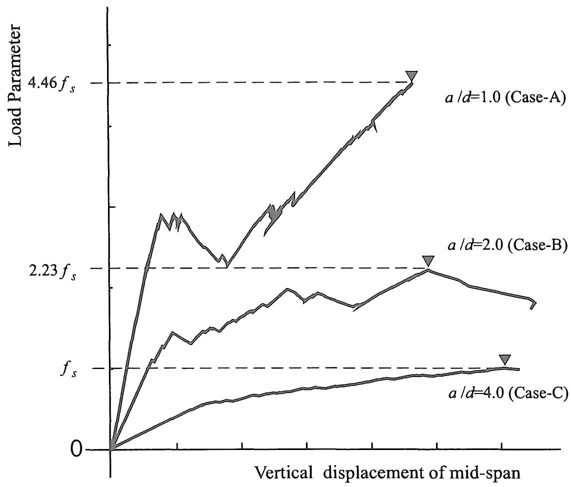

Figure 13 Load parameter - vertical displacement at the loading point

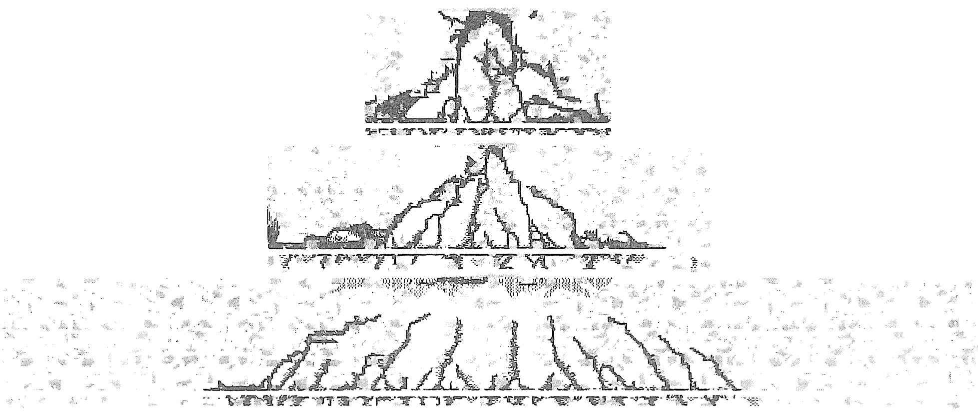

Figure 14 Simulated cracking patterns at failure

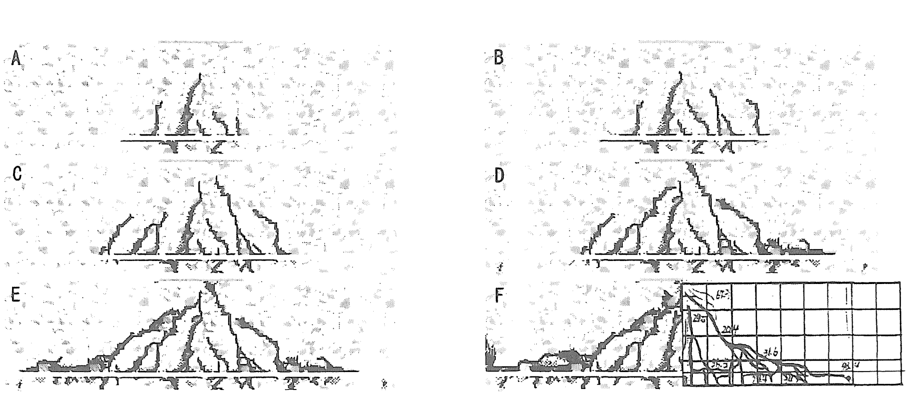

Figure 15 Propagation of Crack in Case B($a/d = 2.0$)

dency on the aforementioned conditions for particular lattice members, the mechanical behavior of reinforced concrete structures strongly depends on the boundary conditions or/and the interaction between steel and concrete. As a result of this ability to reproduce the actual phenomena, the complex failure modes of reinforced concrete structures such as shear failure modes are well simulated. Therefore, our proposed analysis method with lattice type numerical modeling is superior to other meso-scopic or macroscopic ones.

5 CONCLUSIONS

We have proposed a meso-scopic lattice type numerical model to simulate the realistic mechanical behavior of reinforced concrete structures. Numerical simulations performed in this study well illustrate its performance. This success is due to the development of computer memory and novel techniques for numerical analyses.

In conclusion, the implemented disorder must be small as possible in our physical modeling for the failure mechanism and geometry modeling for meso-scale heterogeneity. Then, this type of meso-scopic numerical concrete model could be a helpful tool to study the fracture chracteristics in concrete structures without complex mathmatical formulation.

6 REFERENCES

An, X. and Maekawa, K. (1998) Numerical simulation on shearfailure of RC beams, **Fracture Mechanics of Concrete Structures**, Proceeding of FRAMCOS-3, AEDIFICATIO Publishers, Germany, 1077–1086.

Bažant, Z. P. and Oh, B. H. (1983) Crack band theory for fracture of concrete, **Material and Structures**, 16(93), 155–177.

Bažant, Z. P., Tabbara, M. R., Kazami, M. T. and Cobat, G. R. (1990) Random particle model for fracture of aggregate or fiber composites, **J. Engrg. Mech.**, ASCE, 116(8), 1686–1705.

Bhatt, P. and Adbel-Kader, M. (1998) Prediction of shear strength of reinforced concrete beams by nonlinear finite element analysis, **Computers and Structures**, 68, 139–155.

Cundel, P. A. and Strack, O. D. L. (1979) A discrete numerical model for granular assemblies, **Geotechnique**, 29, 47–65.

Drescher, A. and Vardoulakis, I. (1982) Geometric softeing in triaxial tests on granular material, **Geotechnique**, 32(4), 291–303.

Hollister, S. J. and Kikuchi, N. (1994) Homegenization theory and digital imaging: a basis for the mechanics and disign principles of bone tissue, **Biotechnology and Bioengineering**, 32(1), 27–62.

Jirasek, M., and Bažant, Z. P. (1995) Macroscopic fracture characteristic of random particle systems, **Int. J. Frac.**, 69, 201–228.

Kwan, A. K. H., Wang, Z. M. and Chan, H. C. (1999) Mesoscopic study of concrete U: nonlinear finite element analysis, **Computers and Structures**, 70, 545–556.

Nagai, G., Yamada, T. and Wada, A. (1998) Stress analysis of concrete material based on geometrically accurate finite element modeling, **Fracture Mechanics of Concrete Structures**, Proceeding of FRAMCOS-3, AEDIFICATIO Publishers, Germany, 1077–1086.

Salem, H. M. and Maekawa, K. (1998) Coupled bond and bridging stress transfer in cracked reinforced concrete, **Fracture Mechanics of Concrete Structures**, Proceeding of FRAMCOS-3, AEDIFICATIO Publishers, Germany, 1353–1362.

Simo, J. C., and Hughes, T. J. R. (1998) *Computational Inelasticity*, Springer-Verlag New York, Inc.

Schlangen, J. M. and van Mier, J. G. M. (1992) Simple lattice model for numerical simulation of fracture of concrete materials and structures, **Materials and Structures**, 25, 534–542.

Schlangen, E. (1995) Computational aspects of fracture simulations with lattice models, **Fracture Mechanics of Concrete Structures**, Proceeding of FRAMCOS-2, AEDIFICATIO Publishers, Freiburg, 913–927.

Terada, K., Miura, T. and Kikuchi, N. (1997) Digital image-based modeling applied to the homegenization analysis of composite materials, **Computational Mechanics**, 20, 331–346.

van Mier, J. G. M. (1998) Failure of concrete under uniaxial compression: an overview, **Fracture Mechanics of Concrete Structures**, Proceeding of FRAMCOS-3, AEDIFICATIO Publishers, Germany, 1169–1182.

Yamada, Y., Yoshimura, N. and Sakurai, T. (1968) Plastic stress-strain matrix and its application for the solution of elast-plastic problems by the finite element method, **Int. J. Mechanical Science**, 10, 343–354.

Zaitsev, Y. B. and Wittmann, F. H. (1981) Simulation of crack propagation and failure of concrete, **Mater. Const.**, 14, 357–365.

Zubelewicz, A. (1980) Contact element method, Thesis presented to The Technical University of Warsaw, Poland, in fulfillment of the requirements for the degree of Doctor of Philosophy, 1980., P. A. and Strack, O. D. L.

Zubelewicz, A. and Mróz, Z. (1983) Numerical simulation of rockburst processes treated as problems of dinamic instability, **Rock Mech. and Engrg.**, 16, 253–274.