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Modelling of crack propagation in large structures with a two scales approach

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ABSTRACT: We present in this paper a simplified computational technique based on a refined global-local method, due to Noor et al. (1986) and Mao et al. (1991) among others, which separates the scale of the structure modelled with large size finite elements from the scale of the fracture process zone in which crack propagation and progressive cracking occurs, modelled with a dense finite element grid. The finite element solution is split into two parts: one for the coarse mesh in which the material behaves elastically, and another one for the fine grid where the material, in the examples, follows a non local damage model. This two levels method of analysis, also called "zoom technique" has been implemented in the finite element package CASTEM 2000 and numerical examples showed that the proposed method yields accurate solution with significant savings in computing time compared with a dense finite element mesh. Further this method is suitable for parallel computation.

1 INTRODUCTION

The prediction of the carrying capacity and the evaluation of the long term safety of large structures requires usually a sound and robust computational method of assessment. On top of the difficulties involved in (i) the description of the inception and propagation of cracks at the material level and (ii) the implementation of adequate numerical algorithms, another issue is to be able to deal with finite element models which contain a large number of degrees of freedom. In the case of large structures such as cooling towers or containment vessels, there is a balance to achieve between two extremes: practical numerical models are made of finite elements with a relatively large size (e.g. quadrilaterals of size 1m x 1m) while constitutive relations based on plasticity or damage which describe progressive cracking require finite elements which are at least one order of magnitude smaller.

In the literature, there are several ways to achieve this balance. The easiest solution might be to model the displacement discontinuity due to the presence of the crack directly. It requires mesh adaptation, crack propagation criteria, and a crack initiation criterion. The crack geometry might be directly modelled or it can be embedded within a finite element. A second possibility, which combines initiation and propagation of cracking into the same set of constitutive relations, is to implement some automatic remeshing techniques and models which deal with strain localisation due to cracking. Adaptive meshing may be required in both solutions because an accurate description of crack initiation and crack propagation is quite demanding in term of finite element fineness. Furthermore, discrete crack models hardly handle crack initiation and distributed cracking in a straightforward fashion.

Based on the global-local method (Noor et al. 1986, Mao et al. 1991), we propose in this paper a simplified technique for large structural analysis. In this method, the scale of the structure modelled with large size finite elements, in which the material behaves elastically is separated from the scale of the fracture process zone in which crack propagation and progressive cracking occurs, which is captured with a dense finite element grid, where the material, in the examples, follows a non local damage model.

This two levels method of analysis, also called "zoom technique" has been implemented in the finite element package CASTEM 2000 (Verpeaux et al. 1988). It is illustrated first in the simple case of mode I cracking and compared with experimental data and with a non-linear calculation performed with a dense finite element mesh (density of the small scale mesh in the fracture process zone). It is shown that a similar level of accuracy is achieved with a reduction of computing time for the zoom technique. The difference between the two numerical solutions is a few percent on the load-CMOD curves, and the computing time is at least divided by two. A second example (DEN beam) is presented in order to show that the method can handle curved crack propagation

2 GLOBAL-LOCAL ANALYSIS STRATEGY

The motivation for developing the proposed method is to improve the efficiency of structural analysis by reducing a large structural problem into a number of small problems.

2.1 *Principle*

For the sake of illustration, consider the case of mode I crack propagation in plate (Fig. 1), the entire analysis is performed in four steps as follow :

- In the first step, a relatively coarse mesh (large scale), in which the material behaves elastically, is generated over the whole domain of the structure, to calculate an approximate solution of the displacements and stress field, using the regular finite element method.

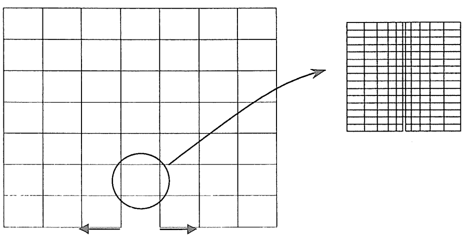

Figure 1. Global coarse mesh and the local refined region of interest

- In the second step, finite element which contain the crack tip is taken for consideration. In this local region a refined mesh is used, where the material follows a nonlocal damage model (Mazars 1984), in order to capture damage propagation properly. Along the boundary of this mesh, the displacement from the first step of computation (the global analysis) are used to enforce displacement boundary condition. This calculation yields the distribution of damage and the resulting forces at the external boundaries of the finite element grid. The reason for using the displacement field of the global analysis as a boundary condition for the local region is as follow. Consider a finite element in the coarse mesh used in the global analysis. If a linear displacement function is used for the element, the only variation in displacement distribution within this element is through the variation of the nodal displacements owing to the fine mesh used in the local analysis, the resulting displacement especially its shape, over the same region agrees much better with the exact displacement, (Fig. 2).

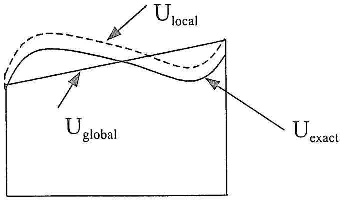

Figure 2. Displacement field of various solutions in an element

- In the third step, analysis is performed over the global region by projection at the larger scale the force distribution obtained in the second step. The same global coarse mesh as in the first step is adopted. Because cracking and damage have propagated, there is a difference between the nodal forces of the element which contains the crack tip in the large scale solution and the forces which results form the small scale solution. This difference is applied as external forces in the large scale solution.
If necessary, step 2 and step 3 can be repeated to produce successively improved solution. Convergence occurs when the norm of the nodal forces difference at the local region is small. Usually few iterations are sufficient to achieve convergence.
- Once the convergence is achieved, we passed to the fourth step which consist to verify the location of the crack in the local region. When the considered finite element is entirely crossed by the crack, it can be either removed or its stiffness is set close to zero, then the same procedure (step 1 to 4) can be applied with the next element on the crack path at the large scale. In order to avoid boundary effects which may appear when the crack tip is very close to the boundaries of the structure at the small scale, the structure at this scale is preferably made of a patch of two elements: one which contains the crack tip and the element which is next to the crack tip in the crack direction.

2.2 *Formulation*

Consider an infinite two-dimensional solid subjected to remote uniform boundary traction producing a uniform stress field σ_∞. The solid Ω is made of a linear elastic material Ω_G meshed with a relatively coarse mesh and a non-linear elastic material Ω_L meshed with a fine mesh (it represents the zone which contains the crack tip), the two zones are separated by the imaginary local boundary Γ_L.
The stress and displacement fields for this problem can be solved by superposing the solutions of two simpler problems (Fig. 3).
From the virtual work principle, the equilibrium equation of the complete system Ω is expressed as :

$$\delta\{u\}^T \int_{\Omega} [B]^T \sigma . d\Omega = \delta\{u\}^T \{F\} \tag{1}$$

Where $\delta\{u\}^T$ = variation of nodal displacement vector; and $\{F\}$ = nodal force vector.

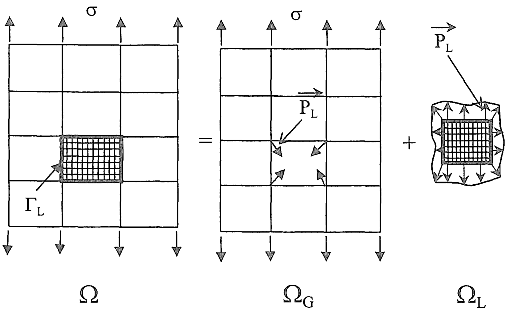

Figure 3. Superposition Scheme

$\Omega = \Omega_G \cup \Omega_L$; $\Omega_G \cap \Omega_L = \Gamma_L$.
By superposition, the first part of Equation 1 may be written as :

$$\{\delta u\}^T \int_{\Omega} [B]^T \sigma \cdot d\Omega = \delta\{u\}^T \int_{\Omega_G} [B]^T DB .\{u\} \cdot d\Omega + \delta\{u\}^T \int_{\Omega_L} [B]^T \sigma_L \cdot d\Omega \tag{2}$$

$$\text{with } \sigma_L = [D] \cdot [B_L]\{u_L\} \tag{3}$$

σ_L = stress vector field of the local fine grid; $\{u_L\}$ = resulting displacement vector from the local fine analysis; $\{u_G\}$= resulting displacement vector from the global coarse analysis; and $[D]$= constituve matrix of the material.
At the local boundary Γ_L, we suppose that the average displacement field of both global and local analysis are equal, we can then calculate the displacement field $\{u_L\}$ knowing $\{u_G\}$ via the shape functions, (Zienkiewicz 1997):

$$\{u_L\} = [N_L]\{\delta_G\} \tag{4}$$

$[N_L]$ = interpolation function; and $\{\delta_G\}$=the generalised displacements at element nodes at the large scale. Substituting into equation 2 we have :

$$\delta\{u\}^T \left[\int_{\Omega_G} [B]^T \sigma_G \cdot d\Omega \right] = \delta\{u\}^T \left[\{F\} - \int_{\Omega_L} [B]^T \sigma_L \cdot d\Omega \right] \tag{5}$$

$$\text{where : } \{P_L\} = \int_{\Omega_L} [B]^T \sigma_L \cdot d\Omega \tag{6}$$

represent projection of the interface traction obtained from local analysis, on the boundary Γ_L of the global system to equilibrate stress tensor σ_L. Finally substituting into equation 5, we have:

$$\delta\{u\}^T \left[\int_{\Omega_G} [B]^T \sigma_G \cdot d\Omega \right] = \delta\{u\}^T [\{F\} - \{P_L\}] \tag{7}$$

1. first we calculate σ_G and $\{u_G\}$by applying on Ω_G the boundary condition of system Ω from equation 1, then we calculate the displacement field $\{u_L\}$according to equation 4, and σ_L according to equation 3. Equation 6 yields the value of $\{P_L\}$.
2. The new pressure $\{P_L\}$is applied as external forces on the local region for the new global analysis, within that local region, the original boundary conditions is applied. We recalculate σ_G and $\{u_G\}$according to equation 1, σ_L from equation 3 and $\{P_L\}$according to equation 6.
3. Step 2 is iterated until the change $\{P_L\}^{l+1} - \{P_L\}^l$ of the interface traction from iteration becomes small enough. The convergence criterion is that:

$$\frac{\left\| \{P_L\}^{l+1} - \{P_L\}^l \right\|}{\left\| \{P_L\}^l \right\|} \le e \tag{8}$$

in which e is a given small tolerance; e = 0.01 was used in computation and usually few iterations were needed. For small enough e, this iterative procedure should approximate the exact solution as closely as desired.

3 EXAMPLES

The "zoom technique" algorithm presented in section 2.2 has been implemented in the finite element package CASTEM2000. Two examples are treated here, the first one deals with the wedge splitting test in the case of mode I cracking. The second example is the double-edge notched concrete specimen, in the case of mode (I+II) cracking. Results of this method are compared with those performed with a global fine mesh and with experimental data if available.

3.1 *Wedge splitting test*

The chosen example is a plane stress model of the wedge splitting test, where displacements at nodes 1 and 2 were controlled (Fig.4). First a Global fine mesh analysis is performed, figure 4 shows the finite element mesh used for this analysis. As shown in this figure, a very dense mesh is generated in the fracture process zone, where crack propagation and progressive cracking occurs. The material in this zone follows a non-local damage model (Pijaudier-Cabot et al. 1991), with the following parameters, κ_0 = 1.0×10^{-4} E = 32000 MPa; ν = 0.2; A_t = 1.0; B_t = 10000; A_c = 1.4; B_c = 1500; β = 1.06; lc = 20 cm.

For the rest of structure, coarse mesh are used and the material behaves elastically.

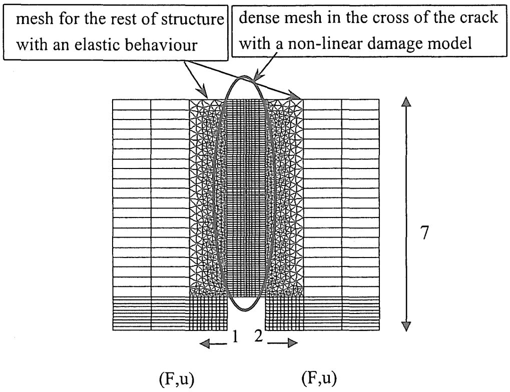

Figure 4. Wedge splitting test, finite element mesh (dimension in m)

Concerning the "zoom technique", the global analysis (the first step) employed a global coarse mesh as shown in figure 5a. The size of the finite element compared to the internal length lc is h = 5*lc. The local analysis (the second step) was performed on a local region covered by 2 coarse elements and the refined mesh consist of 165 quadrilaterals in every coarse element (Fig. 5b) (size of the finite element = 0.5*lc). The global mesh, should be as fine as possible to represent the initial elastic stiffness. In order, to avoid such initial errors we refined the mesh near the applied load zone.

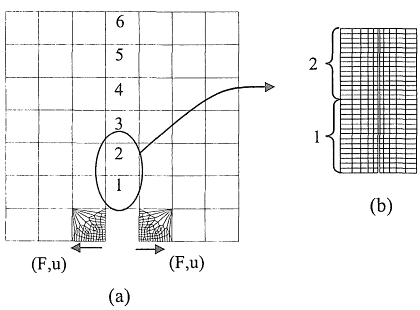

Figure 5. zoom technique : a – global coarse mesh ; b – local fine mesh

Figure 6 shows the plot of the applied load F vs. the crack mouth opening displacement (CMOD), obtained from the global fine mesh. We had plotted in the same figure, the result obtained from the zoom technique analysis, results are quite satisfactory.

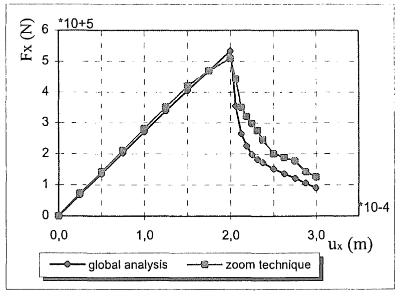

Figure 6. wedge splitting test; load vs. CMOD response

Figure 7a and figure 7b show the distribution of damage at different steps of calculation. We can see that the evolution of crack in the structure is similar.

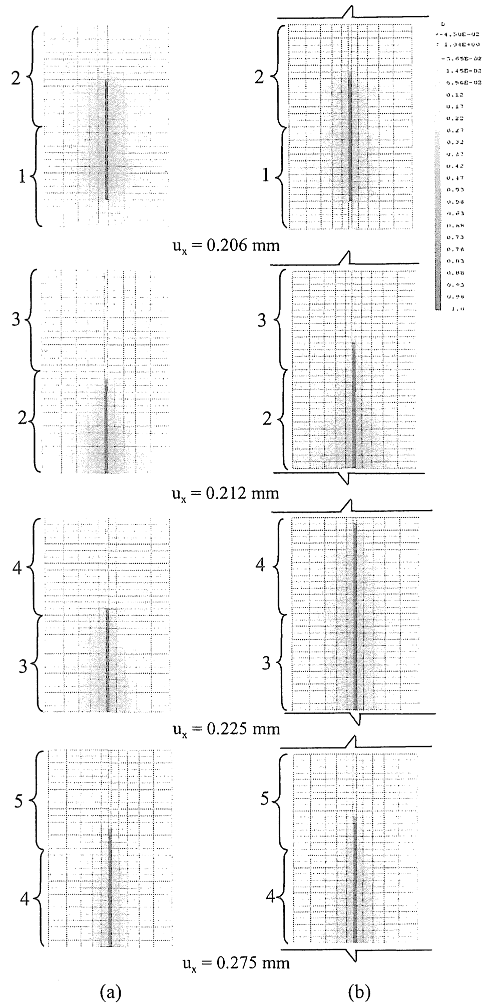

Figure 7. distribution of damage : a- zoom technique , b- global analysis

Concerning the CPU time, the zoom technique yields a decrease in computing time compared to the global fine analysis, it is approximately divided by two.
CPU time for global fine mesh analysis ≈ 46h
CPU time for zoom technique ≈ 26h.
on DEC-alpha ev5 machine.

3.2 *Double-edge notched specimen*

The second example deals with the double edge notched concrete beam. The geometry of the specimen and loading apparatus are schematized in figure 8a. The notch depth in the center from both side is 16 cm and the notch width is 5 cm. First, the panels are loaded by a shear force denoted as Ps. This load is kept constant while uniaxial tension is applied to the specimen. The tensile force P is controlled by the relative tensile displacement δ measured between two points D and D' as shown in figure 8a.

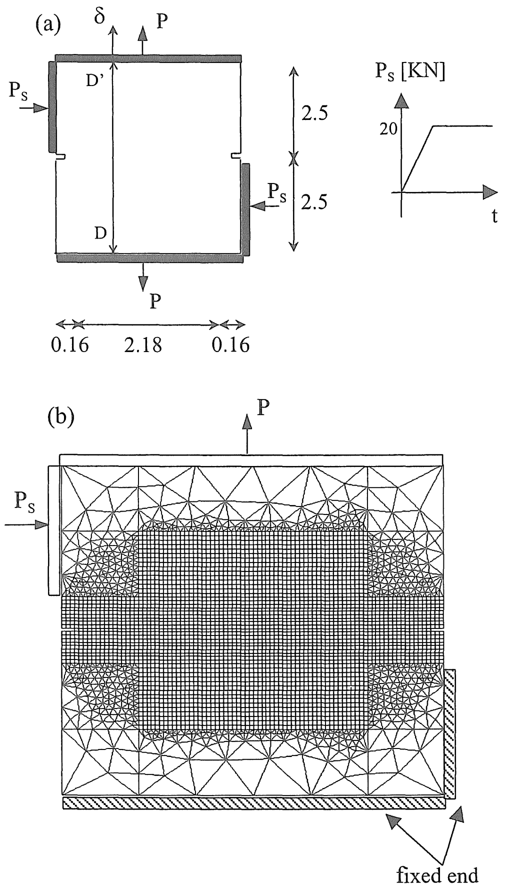

Figure 8. Double-edge notched specimen : (a) geometry and loads (dimensions in m); (b) global finite element mesh

Figure 8b shows the finite element discretization used for the global fine analysis. Same for the previous calculation, the fine region in the cracked zone follows a damage model with the following parameters : $\kappa_0 = 1.0 \times 10^{-4}$; E = 32000 MPa; $\nu = 0.2$; $A_t = 1.0$; $B_t = 10000$; $A_c = 1.4$; $B_c = 1500$; $\beta = 1.06$; lc = 0.0 cm; (the local version of the non-linear model was used in this section, because the computing time needed to achieve the total calculation with non local version was too large). For the rest of structure, a coarse mesh is used and the material behaves elastically.
For the zoom technique, the global coarse mesh (first step) is presented in figure 9a and the local analysis (second step) is performed on a local region covered by 2 coarse elements as shown in figure 9b, the refined mesh consist of 342 quadrilaterals in every coarse element. For the global fine analysis, the local version of non-linear model was employed in the zoom calculation, in order to compare the two results. For simplicity, in the zoom technique calculation, we suppose a symmetry regarding to the axe connecting the two notches, and thus we study propagation of cracks only from one side of the specimen as shown in figure 9.

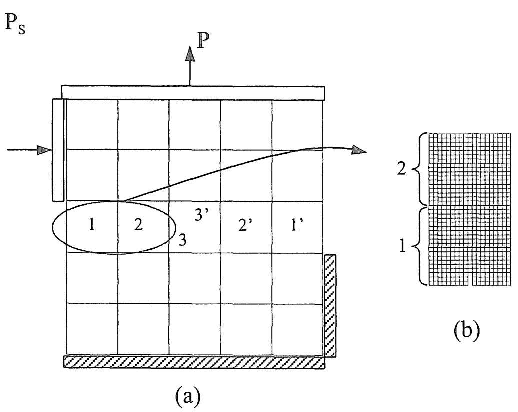

Figure 9. zoom technique : a – global coarse mesh; b – local fine mesh

3.2.1 *Crack distribution at peak load*

The maximum tensile load in the global fine analysis is obtained for $\delta = 1.5575$ mm, and its value is Pmax = $7.020 * 10^{+5}$ N. Figure 10a shows the crack distribution at the peak load. An unstable crack propagation occurs once the maximum load is reached. This is the reason why we will compare in the present paper, just the damage distribution at the peak load only. The present zoom technique cannot accommodate large crack propagation within a single load step because the crack tip may jump outside the plementation of the zoom technique. In Figure 10b, we present the damage distribution obtained for the zoom technique. Maximum load is obtained for δ = 1.751 mm, with the value of Pmax = 8.10 * 10^{+5} N. We can see, that zoom technique gives a good representation of crack distribution in the structure compared to the global fine analysis as shown in this figure.

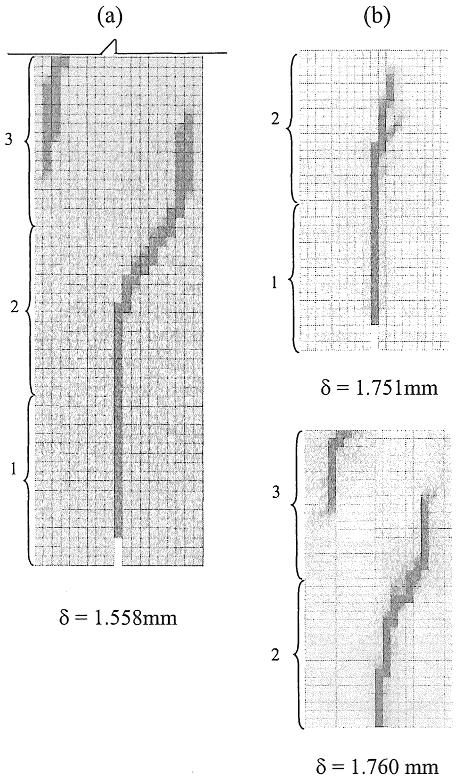

Figure 10. Crack distribution at peak load : a – global fine analysis ; b-zoom technique

4 CONCLUDING REMARKS

An efficient zooming method for finite element analysis is presented in this paper. A global solution is obtained using a coarse grid, then the detailed stress and damage distribution near the crack (local region) are obtained by zooming on that area, refining the model and using the displacements from the coarser model as an input for refined mesh. This method has been implemented in the finite element code CASTEM2000. Numerical calculations demonstrate the efficiency of the method. This kind of zooming technique can be quite useful in large structural calculations in order to predict inception and propagation of cracks by refining and analyzing only the local region containing crack tip. Application to mode I cracking and comparisons with non-linear calculations performed with a dense finite element mesh indicated that the present technique yielded substantial time savings in computations.

A number of questions, in connection with this technique, remain open issues, namely:

(1) criteria for selecting the local region for refining and analyzing.

(2) behaviour of elements totally cracked, in the case of crack closure and shear.

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