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Cyclic and Seismic Nonlinear Modelling of Concrete Structures Using Damage Model and Multilayered Beam Elements

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ABSTRACT: A complete method to predict the behavior of reinforced concrete structures with beams and columns controlled by flexure is proposed. The concrete is modelled with a damage mechanics approach. The parameters of the model are adjusted on material tests and various procedures that account for confinement and cyclic response. Steel is modelled with a simple cyclic model. A simplified finite element program allows the prediction of the global and the local behavior from the constitutive laws of the material. This program uses multilayer beam elements. Localization of damage for softening structures is taken into consideration by a meshing procedure. The methodology is used to predict the behavior of three different kinds of structures subjected to three different types of loading. Predictions are in very good agreement with experimental results.

1 INTRODUCTION

In a certain number of civil engineering problems, it is necessary to predict the behavior of reinforced concrete structures under cyclic or dynamic loading. For example, to assess the safety of structures subjected to seismic loading, it may be necessary to use a time-history analysis. This is a common practice in Japan or in America. The analysis should account for the main phenomena influencing the behavior of reinforced concrete elements for a wide range of concrete strength and yield strength of confining steel, namely: (i) Confinement of concrete through passive action of transverse steel; (ii) Cyclic behavior of materials; and (iii) Localization of stress and strain that are experimentally observed. In such analysis, it is of prime interest to use simple and cost-effective methods that would require less experimental data.

In engineering practice, generally, only the uniaxial stress-strain law of material are known, or can be estimated with confidence. The structural behavior of elements can only be estimated from these laws and generally structural effect should be predicted without experimental validation. For this purpose, finite element analysis is very practical, but often very long, complex and expensive. Usually, finite element programs are general purpose softwares which are difficult to use even in simple cases. But finite element modelling is not solely a software problem. Methodology is of prime interest. The same methodology should give good results on a wide range of problems of interest. This methodology should insure that structural effects and constitutive laws of material result from an objective choice with reproducible operations.

A methodology applicable to beams or columns elements with behavior dominated by flexure is presented. In the first part, constitutive laws of materials are described. Then, numerical tools are presented. It is a simplified finite element analysis based on beam elements with superimposed layers. Finally, the discretization process is described. This procedure is used to predict available experimental data on three different structures subjected to three different types of loading.

2 CONSTITUTIVE LAWS

2.1 *Concrete*

To adequately model the behavior of a material under cyclic loading, it is necessary to account for the history of loading. It can be simple variables, as maximum experienced strain. For concrete, the problem is more complex since the behavior is very different in tension and in compression. It has been shown that the damage theory is well adapted to model the behavior of concrete (Mazars 1986). The damage theory is based on the thermomechanics of continuum where the stress-strain laws are derived from the free energy, for instance the Gibbs energy. It is practical for cyclic analysis since the damage variables are in fact a trace of the loading history.

In this study, the Laborderie (1991) model is used. The model can be written in its most general 3-dimensional form. In this paper, only its unilateral formulation is presented. The behavior of concrete is controlled by two damage variables, i.e. D_1 for damage in tension and D_2 for damage in compression. The concrete strain is defined as a function of concrete stress as (LaBorderie 1991):

$$\varepsilon_c = \frac{\sigma^+}{E_c(1-D_1)} + \frac{\sigma^-}{E_c(1-D_2)} + \frac{\beta_1 D_1}{E_c(1-D_1)} f'(\sigma) + \frac{\beta_2 D_2}{E_c(1-D_2)} \tag{1}$$

where E_c is the initial Young modulus, σ^+ and σ^- are the positive and negative stresses respectively (for tension stress, $\sigma^+ = \sigma$ and $\sigma^- = 0$; for compression $\sigma^+ = 0$ and $\sigma^- = \sigma$). Both D_1 and D_2 are damage variables in traction and compression, respectively; β_1 and β_2 are parameters defining the inelastic behavior; f is a function that accounts for the closure mechanism of cracks. Six parameters define completely the monotonic behavior of concrete: Y_{01}, A_1 and B_1 for tension, and Y_{02}, A_2 and B_2 for compression. The evolution of damage variables is controlled by energy restitution rates defined as:

$$Y_1 = \frac{\sigma^{+2} + 2\beta_1 f(\sigma)}{2E_0(1-D_1)^2} \tag{2}$$

$$Y_2 = \frac{\sigma^{-2} + 2\beta_2 \sigma}{2E_0(1-D_2)^2} \tag{3}$$

For unconfined concrete in compression, the parameters are identified on complete stress-strain curves obtained on cylinder tests or on estimated stress-strain law. While adjusting the various parameters, it is possible to fit the experimental curves. For confined concrete, the parameters are adjusted in order to fit the confined concrete compressive curve chosen. Here we use the Cusson & Paultre (1995) model, modified by Légeron & Paultre (1997a). The stress-strain curve for confined concrete (Fig. 1) is totally defined by the knowledge of the unconfined concrete stress-strain curve and the *effective confining stress*, $f_{\ell e}$. $f_{\ell e}$ is the unifom stress applied on the whole surface of the concrete core that would induce the same effect on the strength and ductility enhancement as the lateral steel when concrete reaches its maximum stress. This effective confining stress is computed on the bases of compatibility of deformation and static equilibrium in the transverse plane. Légeron & Paultre (1997a) proposed a simple method to determine this confining stress from the geometry of the member and from the characteristic of the transverse reinforcement steel.

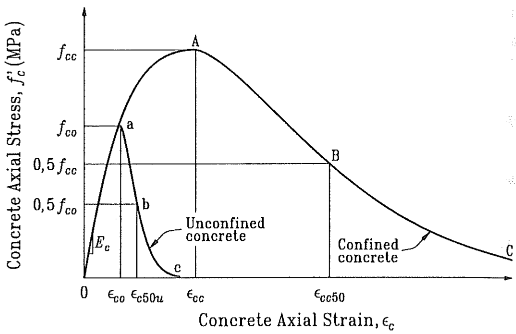

Figure 1: Stress-strain curve for confined concrete

The tensile strength is determined by conventional tests. However, it is difficult to obtain the complete tensile stress-strain curve with a stable post-peak behavior. Hence, before cracking, the behavior is considered elastic with the tensile elastic modulus equal to the compression elastic modulus and the parameters defining the post-peak softening are fitted to agree with a tension stiffening behavior of reinforced concrete as described by Collins and Mitchell (1991).

In compression, the cyclic behavior of concrete is defined by the parameter β_2. For this purpose, it is necessary to know the evolution of plastic strain during cyclic tests. Generally, those tests are not known at the project level. Hence, the approach proposed by Dodd & Cooke (1994) is used. This approach relates the permanent strain (strain at zero stress) to the maximum reached strain and corresponding stress as well as the strain at peak stress. This permanent strain is used to identify parameter β_2 with a procedure defined by Légeron (1998).

The parameter β_1, which controls the cyclic behavior in tension, is taken as a function of tensile strength, as proposed by Légeron (1998). In fact, the cyclic behavior of concrete is difficult to determine by tests. Therefore, it is preferable for practical applications, to use predetermined values for β_1.

2.2 *Steel*

A bilinear envelop curve is used for steel. For the cyclic effects, the model proposed by Dodd & Cooke (1994) is simplified. Under hysteretic loading, the degradation of the reloading stiffness, depends on the maximum experienced strain in each direction (Fig. 2). A power-function describes the response from zero stress to the maximum strain attained in the opposite direction. This model was validated on steel coupons tested by Dodd & Cooke (1994) (Fig. 3).

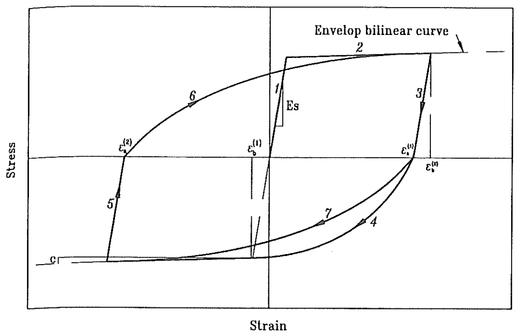

Figure 2: Model for cyclic behavior of steel

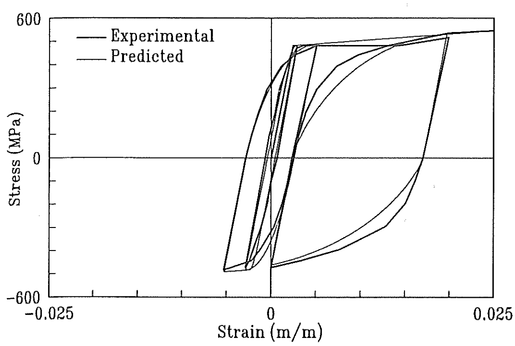

Figure 3: Prediction of cyclic test on steel coupons

3 NUMERICAL TOOL

It is now common practice to use the finite element method to compute the nonlinear behavior of complete structures subjected to various loadings such as earthquakes, blasting, etc. However, this particular method requires highly sophisticated software and high computing costs. The LMT (*Laboratoire de Mécanique et Technologie* in Cachan, France) developed a simplified approach implemented in the computer program EFiCoS. This program uses multilayer beam elements (Fig. 4). Each element is constituted of superimposed layers. Each layer is made of either plain concrete or homogenized steel-concrete composite. The kinematics is simplified as plane sections remain plane (Bernouilli hypothesis), which is applicable to a large number of practical situations. This limits the number of degrees of freedom in a problem but, in the other hand, the program enables to account for realistic material behavior. In each layer, a damage model is used to describe the behavior of concrete (LaBorderie 1991). The interpolation polynomials are the same as those used for conventional beam elements. Each layer has its two damage variables, which are modified at each equilibrium state. The point of evaluation of the damage is at the middle of the layer and at the center of the element. The mesh is therefore very important since the precision of the prediction will depend on the number of points where the damage variables are evaluated.

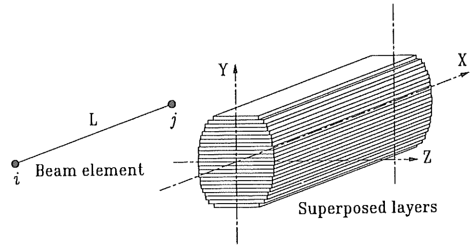

Figure 4: Beam element in EFiCoS

4 MESHING

In columns subjected to high compressive stress, the global behavior may be softening. Hence, in the finite element method, the solution is not unique and depends on the size of the elements. Figure 5 shows the response of a column subjected to axial load and combined flexure. The response was computed with three different lengths of elements: 100, 250 and 500 mm. We observe that the response is highly related to the size of the elements.

It is believed that post-peak softening is not only related to material response but also to the structure in which it is used. To avoid localization problem two possibilities are possible: fix the size of the elements or alter the stress-strain law of materials with the size of elements. The fist approach is more practical and is adopted here. A stability approach (Bažant 1977, Legeron 1998) was developed for this purpose to evaluate whether the post-peak behavior of the beams was stable or not. For each specific experimental set-up, a stability function is

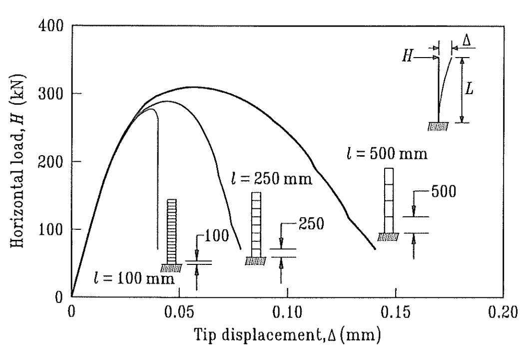

Figure 5: Response of columns with different size elements

determined for structures that can be softening. From this procedure, it appears that a minimum length of the more damaged element is required to assure stability of the process. This length is used as the minimum length of the element. It has been shown by L egeron (1998) that it is possible to use the length of the equivalent plastic hinge which gives comparable results in practical situations and is a lot easier than a stability approach. An appropriate expression is given by Priestley, Seible & Calvi (1996) as:

$$\ell_p = 0.08L + 0.022 d_b f_y \qquad (4)$$

where L is the length between the end of the member and the point of contraflexure, d_b is the diameter of the longitudinal bars and f_y is the steel yield strength. This formula predicts quite well the equivalent length of plastic hinge on various experimental results (fig 6) considering the large experimental error.

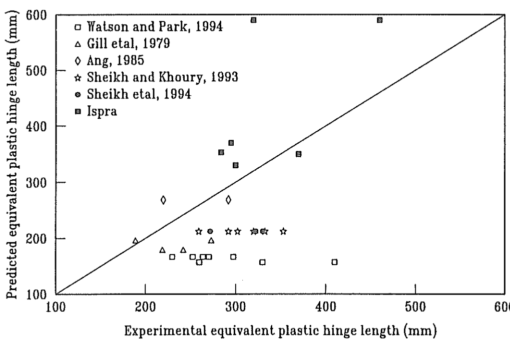

Figure 6: Prediction of experiment equivalent plastic hinge for columns

For structures with hardening behavior, localization is not importance on the global behavior. However, a certain strain localization occurs. It is also necessary in this case to use an element size not too small because strain limits could be reached earlier than it is really experienced. Therefore, the ultimate strength is much affected by this kind of localization which implies that available ductility depends on the size of elements. Numerical tricks are often used to solve this problem such as alteration of hardening properties of steel. Here we have chosen to fix the size of the elements to the length of the equivalent plastic hinge. This approach seems to correlate well with experimental results.

5 COMPARISON TO TEST DATA

In this research, three types of data are used: the response of over-reinforced beams with normal strength and high strength concrete (Van Mier and Ulfjaer, 2000), the cyclic quasi-static tests on high-strength concrete (HSC) columns (L egeron and Paultre 1997b), and the pseudo-dynamic tests on bridge piers (Pinto et al. 1996). The methodology proposed above is used in the predictions.

5.1 *Flexural Behavior of Over-Reinforced Beam*

Van Mier and Ulfkjaear (2000) tested 12 beams with four different configurations with three repetitions. The beams were tested under monotonic four point loading. Three large beams were made with normal strength concrete (NSC) and with the same configuration. The nine other beams were small beams broken into three beams constructed with normal strength concrete (NSC), three with high-strength concrete (HSC) and three with fiber high-strength concrete (FHSC). In these beams, all the characteristics were similar. In 1997, a benchmark was organized by Van Mier and Ulfkjaear (2000) on the prediction of over-reinforced beams. They reported that the experimental results were known by the authors of the present paper at the moment of prediction. However, the predictions were submitted about one year before the seminar organized during FRAMCOS3 where the results were unveiled. Hence, it is underlined that the predictions reported in the benchmark during FRAMCOS3 by the authors were made without knowing the results. Confinement and strain localization were not an issue since no confinement steel was provided and the maximum moment was equal in all the central part of the beams between the applied loads. The experimental stress-strain law of materials and the complete geometry were provided for the benchmark. The parameters of the model were adjusted on this experimental concrete behavior. The predicted stress-strain curve for the different concretes are compared to experimental ones in Figure 7. For HSC, the model was not able to predict the unstable post-peak behavior. It is believed that the testing machine was not stiff

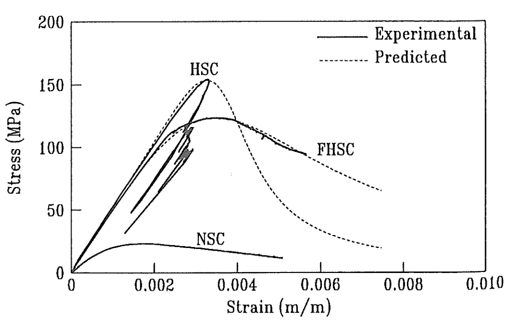

Figure 7: Prediction of compressive stress-strain curve of concrete of the over-reinforced beams

enough or the control of the test was not totally appropriate for testing very high strength concrete specimens. A typical value of 0.004 for the strain at 50% of maximum stress was used for this concrete.

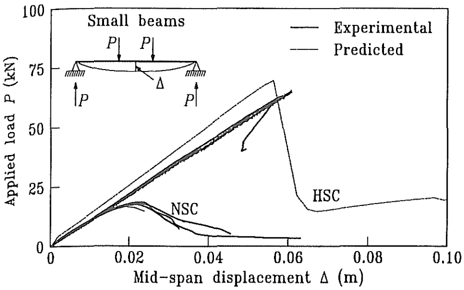

Figure 8: Prediction of the HSC and NSC over-reinforced concrete small beams

The predictions of mid-span deflection as a function of the applied load is performed with the EFiCoS program. They are shown in Figures 8 and 9 for the small beams and in Figures 10 for the large beams. For the small beams with NSC, the prediction is very good (Fig. 8) as the strength and the post-peak behavior is well captured. For the HSC beams, the stiffness is slightly overestimated but the predicted strength is very close to the experimental value. The experienced post-peak behavior is unstable which is quite well predicted since a rather steep numerical post-peak results experimentally in an unstable behavior except if very good control and stiff experimental frame is provided. For FHSC (fig.9), the stiffness and maximum load is well predicted while the displacement at peak is somewhat overestimated. However, the post-peak behavior is very well predicted. For the large beams (fig.10), the behavior is very well predicted from the initial stiffness, the strength as well as the post- peak response.

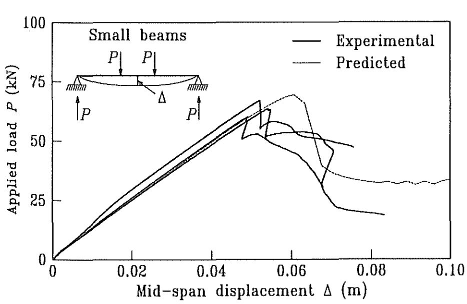

Figure 9: Prediction of the FHSC over-reinforced concrete small beams

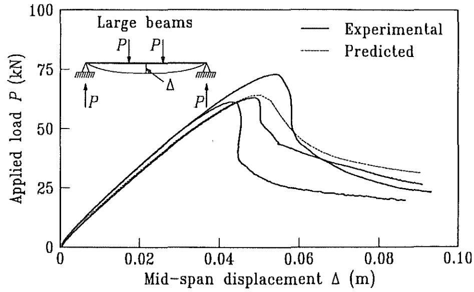

Figure 10: Prediction of the NSC over-reinforced concrete large beams

5.2 *Cyclic Tests on HSC Columns*

L egeron & Paultre (1997b) tested 12 HSC columns subjected to constant axial load and reversed flexure (fig 11. The columns were heavily confined. The volumetric ratio of transverse steel reached 4.26% for certain columns. Other columns were confined with high-yield strength steel (HYSS). Longitudinal reinforcements were made of Grade 400 MPa steel, and reinforcement ratio was constant at 2.15%. The specimens tested represent a ground-floor HSC column in a typical building, 4-m high, with a 305 × 305 mm cross section. The column is connected to a massive stub simulating a rigid member of a foundation. A transverse load is applied at the tip of the specimen, two meters from the base of the column. The parameters of the damage model were identified on monotonic material tests, as described in the Constitutive Law section. The length of the elements was taken as 400 mm, which was the length of the equivalent plastic hinge. This resulted in five elements on the full length of the column. This type of columns were very sensitive to the size of elements.

Examples of predictions made with EFiCoS are shown in Figures 12 to 14. The overall behavior is very well predicted: loading and unloading branches as well as maximum capacity and even the pinching effects. This means that confinement effects and crack closure are taken into consideration in a proper manner. What is also very interesting for seismic analysis, and specifically for performance-based design, is that local behavior seems also to be well predicted, i.e., onset of spalling of the concrete cover at peak and yielding of steel. This also translates into a very good estimate of the behavior of the columns. Cyclic behavior of material is also well modelled. The problem here is very comparable to practical engineering problem where the behavior has to be predicted with simplified analysis, knowing only the geometry of the structure and uniaxial stress-strain law of materials. With this approach, engi-

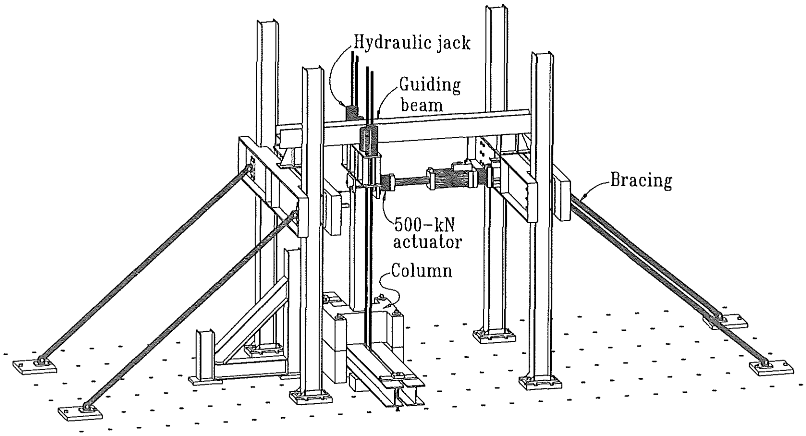

Figure 11: Tests of HSC columns under axial load and reversed flexure

neers are able to model structures with more complex behavior as it is going to be shown hereafter.

5.3 *Tests on Bridge Piers*

The *European Laboratory for Structural Assessment* (ELSA), at Ispra in Italy tested piers of model bridges under seismic loading with a sub-

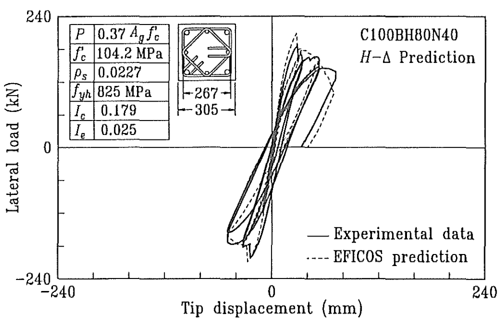

Figure 12: Prediction of the behavior of column C100BH80N40

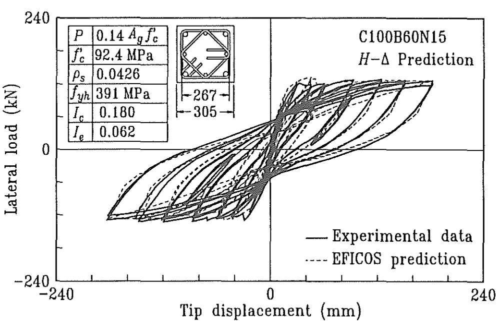

Figure 13: Prediction of the behavior of column C100B60N15

structure pseudo-dynamic method (Pinto et al. 1996). Only the results of bridge B232 (Fig. 15) are presented here. This bridge is a 4 span continuous prestressed concrete bridge with three piers: (i) two 14 m high lateral piers (ii) a central pier with a height of 21 m. The bridge is modelled with a 1/2.5 scale factor. In fact, in this case, the deck is not tested because its behavior is supposed to be linear elastic and computed using a sub-structuring technique (Pinto et al. 1996). The damage model parameters were identified based on test results of the materials. EFiCos is a 2D analysis program. Since the bridge was loaded in a transverse direction a 3D discretization was necessary. However, the 2D elements are still used since the response of the piers is in one direction.

The displacement at the top of the high pier and one of the medium pier were recorded during the test and predicted with EFiCos. The results shown in Figures 16 and 17 are obtained for the accelerogram that represents a generated earthquake for two times the nominal acceleration chosen. The piers were designed for the nominal acceleration

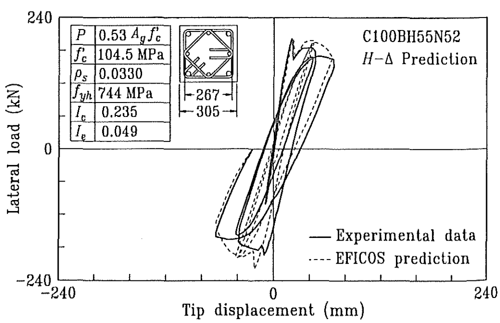

Figure 14: Prediction of the behavior of column C100BH55N52

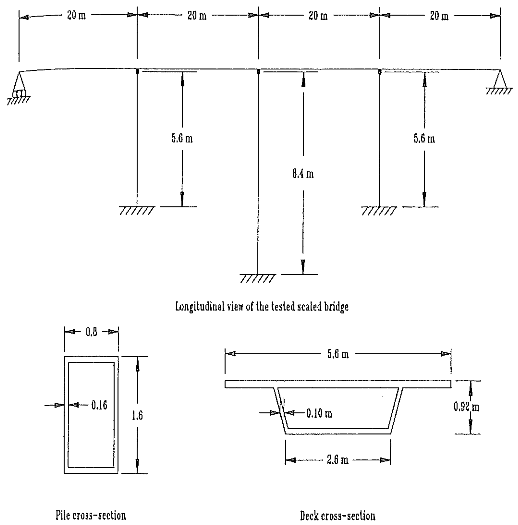

Figure 15: Bridge tested at Ispra

and twice this level means a high ductility demand is placed on the piers with expected a highly non-linear response. The predicted responses are in good agreement with the test data. At some places, small differences are recorded. It may be explained by various factors other than imperfections in the modelling. During the tests, a special algorithm was developed at ISPRA for the non linear analysis and the sub-structuring technique which was not used in EFiCoS. Of particular importance to the results, the ISPRA's integration method has a certain numerical damping which was not reproducible by the traditional Rayleigh damping. However some variation of the Rayleigh coefficients altered the response, but it was not the purpose here to fit the experimental results. With the data comparable to what is available to an engineer at the project level, it was interesting to see that it is possible to predict quite well the response of the bridge even subjected to high non linear demand.

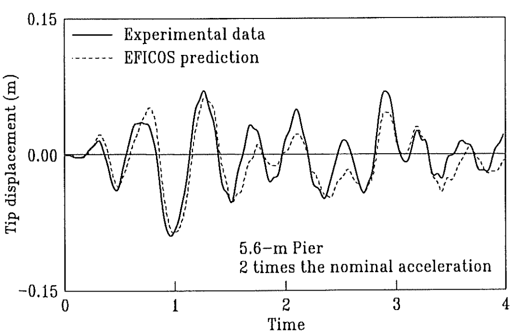

Figure 16: Prediction of the top displacement of 5.6 m high pier

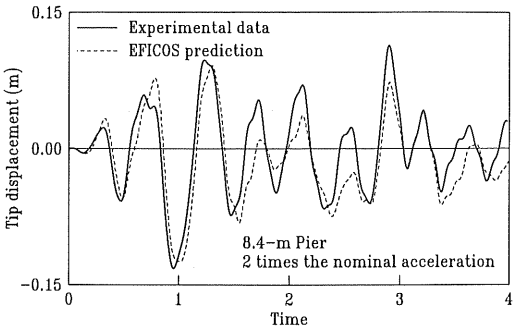

Figure 17: Prediction of the top displacement of 8.4 m high pier

6 CONCLUSION

In this paper, a complete approach to the prediction of concrete structures subjected to cyclic and earthquake-type loadings is presented. This method is based on damage mechanics and on a simplified approach of confinement and cyclic behavior of materials. From stress-strain curves which are comparable to the data available to an engineer confronted to a practical application, it was possible to predict the response of HSC columns subjected to cyclic loading, bridge piers subjected to seismic input and over-reinforced beams. The global response is very well predicted and the local behavior is also available as spalling of concrete cover, cracking, unloading stiffness, yielding of bars, etc. It is a powerful tool for seismic analysis and other engineering problems where non linear analysis of structures is dominated by flexure. Moreover, the damage mechanics provide a good picture of the level of damage that a structures has accumulated during an earthquake, which is of prime importance for all "Performance-based design" approach.

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