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Statistical Property of Internal Cracks in Concrete

M.Saichi & H.Shinohe
Tohoku Institute of Technology, Sendai, Miyagi, Japan

H.Mihashi
Tohoku University, Sendai, Miyagi, Japan

ABSTRACT: This paper presents a study on distributing feature of internal cracks, which is concerned about the statistical law of internal cracking. The fitness of Poisson distribution on the number of internal cracks and the fitness of log-normal distribution on the length of those cracks were discussed. Concrete specimens were observed for internal cracking by means of digital microscope after they were subjected to several types of cyclic loading. The agreement of shapes between expected distribution and observed histogram was verified. The expedience of applying probability distribution to the number as well as length of cracks was substantiated. This paper is limited to work on internal cracks observed on the surface cut perpendicular to the direction of the compressive loading.

1 INTRODUCTION

In our study, the focus of interest is originally put on the field of damage assessment of concrete within existing RC structures. This paper presents a study conducted as a part of the series of investigation on distributing feature of internal cracks.

1.1 *Relation between damages and internal cracks*

Structures are subjected to a large number of loads applied during their design lives, and concrete is known to deteriorate both in strength and in stiffness. It has long been recognized that concrete is damaged by application of stress lower than the ultimate stress of the material. To date, it has been shown that micro cracks have an important influence upon the behavior of concrete and are largely responsible for the failure process. On the other hand, damage has a close connection with internal cracking in concrete (Jover, 1991). Many investigators have implied that the intensity of damage is related to irreversible cracking. These and similar studies underscore the important role of internal cracking for the fatigue behavior and/or life of the concrete.

1.2 *Microscopic observation of internal cracks*

While progressive cracking has mainly been observed by means of indirect examinations, including ultrasonic test (Robinson, 1965), AE technique (Maji & Shah, 1986), etc., direct observation is preferable because this would provide the precise information pertinent to the mechanism of cracking, for example, location or the exact size of internal cracks. The use of powerful microscopes has been able to study the formation of internal cracks, and some insight has been obtained on the change in surface feature of concrete. A direct observation method by means of microscope for studying microstructure and crack growth within concrete was introduced by Hsu (1963) and Shah & Sanker (1987). As demonstrated by them, a meso-level microscopic observation such as crack map could be useful tool for describing distributed internal cracking.

In addition, generally, for studying the internal cracking properties of concrete, it is required to get the microscopic information from the sliced specimens. On the other hand, their preliminary work and subsequent reading in microscopic technique consume much of time. Hence, a lot of time, patience and experience have been required to make a reliable observation.

1.3 *Quantitative parameters of internal cracks*

The most apparent properties of internal cracks are number and size including length, width and depth. Some investigators (e.g. Shah & Chandra, 1970) obtained total crack area by multiplying the total length of cracks with the observed average width of the cracks in a given slice. In practice, the characteristics, which should be studied, and possibly observed on the surface subjected to the damage loading, are the number and the length of cracks developed during or before the damage loading.

In this paper, variability of number and length of internal cracks are studied.

1.4 *Significance of statistical investigation into internal cracking*

The internal cracks in the damaged concrete have been considered randomly generated, and their characteristics, number and length, were not necessarily considered to follow the certain natural law. Since the meso-crack system observed through the microscopic examination could be considered involving a variable governed by an accidental cause, those characteristics may also be considered as a random variable. If those characteristics considered being a random variable, and probability distributions are known with sufficient accuracy, statistical analyses could be applicable to the subjects of the internal cracking. For example, if the information involving probability distribution were available for number and length of the cracks, it would be possible to estimate the extent of internal cracking in concrete subjected to external forces. It would also make it possible to deduce the development of the internal cracking, and opens up the possibility of studying complicated problems, e.g. crack quantification, with specified statistical information.

Moreover, the application of such techniques could be helpful to study the natural laws governing the inherent variability of the internal cracking and how this behavior would change when damage is applied to the concrete. However, little is known about the probability distribution, which governs a law for the crack development.

This study intended to clarify the statistical law governing the occurrence of internal cracks and thereby facilitates the development of probability model, which should make it possible to establish the substantial aspect of the population for internal cracks. Consequent findings might provide useful information for assessing damage feature of concrete.

2 EXPERIMENTAL WORK

2.1 *Damage simulation on external forces*

While cracks at the interface between coarse aggregate and mortar exist in plain concrete before it is subject to any load, additional external loading was introduced in this study for damage simulation. In practice, the damage can be a result of an external loading, environmental condition or built-imperfections. The external loading is one of the major factors, which weakens materials.

It has long been recognized that concrete is damaged by application of cyclic stress lower than the strength of the concrete. There have been numerous proposals to define the damage by means of various cyclic loadings, (e.g. Chrisp, 1993, Shah & Winter, 1966).

In this study, specimens were subjected to a series of cycles of loading and unloading for assuming damage. The effects of these cyclic loads could be divided into a primary simulation for the mechanical damage. Although these cyclic loadings may considered to be unrealistic compared with load histories that may actually be experienced, such loadings are essential for an understanding of low cycle fatigue behavior or damage (Grzybowski & Meyer, 1993).

In order to establish the damage index stage, the term "stress level" is introduced. The stress level is defined as the ratio of the applied stress divided by the ultimate stress of respective specimens. In this paper, the damage value is defined as the ratio of the applied stress to the strength of respective specimen, and the form of percentage will be used for the representation.

2.2 *Outline of experimental work*

2.2.1 *Specimen preparation*

All the experimental tests described in this paper were conducted on concrete prisms whose dimensions were $100 \times 100 \times 200\text{mm}^3$. The nominal value of concrete strength used was 30MPa and 60MPa. Ordinary Portland cement was used in concrete preparation. The coarse aggregate was crushed gravel with the maximum size of 20 mm and the fine aggregate was river sand. No chemical admixture and no mineral addition were applied to the concrete mix. Approximately 48 hours after casting, specimens were removed from the steel molds and transferred to a standard moist curing room where curing was continued until the day before testing at approximately 20℃.

2.2.2 *Damage loading*

Specimens were tested at an age of 28 days. The damage loadings were conducted under controlled load with intensities, which were selected in reference to the cylinder strength measured under monotonous loading; the intensity were loaded according to the 21-grade system of stress level from 0% to 100% with increments of 5%. Three specimens from the batch were first tested to determine the average static strength, on which basis the stress levels were fixed for the damage loading. This made the stress levels of each set of specimens cover an extent of 0.0 to 1.0 (0% to 100%). Each cylinder was manually loaded with the rate at about 0.2 to 0.3 MPa/sec until it reached the stress described above, and then the load was released at the same rate. After unloading, the load was applied again from a bottom stage of unloading. The loading was repeated five times, and then the load was applied until the ultimate stress was reached without any interruptions in loading. The damage index is shown as a function of the stress level $\sigma / \sigma c$, where σ is the intensity of load and σc is the ultimate stress. Hereafter, in this pa-

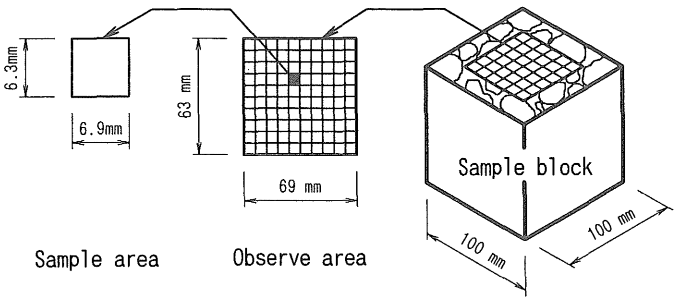

Figure 1. Sample areas used for microscopy.

per, above cyclic loading would be sometimes designated as the damage loading.

2.2.3 *Preparation for microscopy*

In total 42 concrete prisms were used to observe the cross section under a microscope. For observing internal cracks, it should be necessary to cut the specimen in a given size. The 200mm-long prisms were sawed into 100 mm cubes for microscopic observations. A horizontal section (perpendicular to the direction of loading) was cut directly through the center of the specimen. Thus, the block of specimen was 100 mm square in section. Each section would be observed for internal cracking by means of optical microscope, which equipped CCD digital camera. As this device did not require the slicing, it was suitable for estimating the cracking area within a cross-section of prismatic specimens. Cross sectional area was ground carefully by precision grind machine until no trace of saw could be seen. A red liquid for penetration was sprayed over the area for the staining. This was done to distinguish more clearly between cracks at the interface and those through the matrix.

After several times, mortar surface became a color of light pink, and deep red lines representing internal cracks could be clearly distinct in the view. The stained surface was then ground again on a grinding wheel. The sections were then slowly dried in room where atmosphere was kept at 20℃ and approximately 65 percent relative humidity. Neither the sawing nor the drying on specimens was considered to significantly affect the measured characteristics of cracking because their effects were verified negligible through the preparatory tests.

2.2.4 *Microscopic observation*

All the quantitative work was done for the sections with an optical system using 35-power magnification. The system consists of a high-resolution microscope connected to a CCD camera. In order to mark the location and size of crack, each crack was plotted on the image, and then, they were recorded on a MO-media as a digital image. The microscopic images extracted were processed by the software, which enables to draw a cracking map, and then the number and the length of cracks were quantified. The length of cracks was measured along the curves of the cracks. The observed area used in the microscope observation was 6070mm^2 per section in total so that a sufficiently wide range of sizes could be covered. The observed data presented herein were obtained from the sample areas determined by dividing the observe area into 10×10 segments. The data consist of the number of cracks and the crack length observed over the view of each segment (see, Figure 1).

One of the problems in the observation is the instability of discrimination between real crack and line like aperture. Cracks smaller than 500 μ m were not recorded because it was difficult to decide whether they were true cracks or just very porous mortar adjacent to the aggregate. Cracks, throughout this paper, refer only to those observed in examination of transverse sections. Although their length may increase longitudinally with increasing strain, no observations of this phenomenon were made.

3 EXPERIMENTAL RESULTS AND DISCUSSION

3.1 *Type of cracks*

The failure of concrete under uni-axial compressive stress would be caused by progressive internal micro cracking. This cracking starts at approximately 10 to 30 percent of the ultimate stress at the interface between coarse aggregate and mortar. Cracks can be divided into three types, viz., cracks at the interface between aggregate and mortar (hereafter called a bond cracks), cracks through the mortar (mortar cracks), and cracks through the aggregate. Mortar cracks begin to increase noticeably and form continuous crack patterns at about 70～90 percent of the ultimate load (Hsu et al. 1963, Carrasquillo & Slate 1983).

Figure 2 make a comparison on share between bond and mortar cracks. Although plots are not

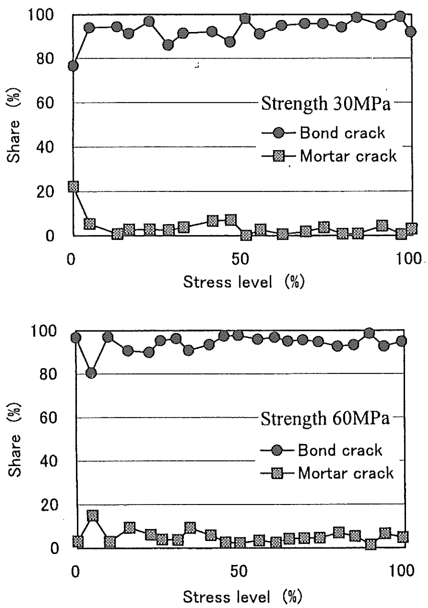

Figure 2. Comparison on share between bond and mortar cracks.

showing a positive tendency for mortar cracks to follow the feature mentioned above, it can be seen that bond cracks predominate over mortar cracks at all stages of stress level. It suggests that major crack would develop at the interface between mortar and aggregate irrespective of the extent of damage loading.

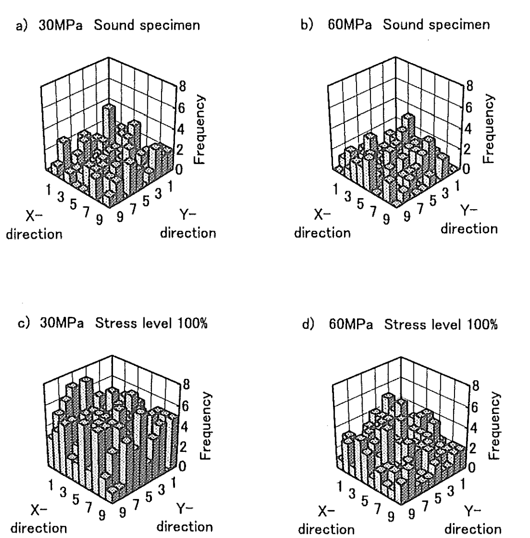

Figure 3. Comparison of extent of internal cracks within each sample area for sound and damaged specimens in strength 30MPa and 60MPa.

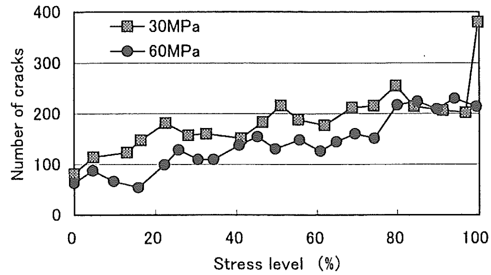

Figure 4. Increasing of total numbers of crack within observed area as a function of stress level.

3.2 *Relation between damage intensity and cracks*

Figure 3 shows the comparison of extent of internal cracks within each sample area for sound and damaged specimen. For clarity, results relating only to four concrete specimens are presented. Compared to Figures 3-a), b), with Figures 3-c), d), it is clear that the damaged specimens produce a higher extent of internal cracking than the sound those. Figures 3-a) and b) also showed that cracks at the interface between coarse aggregate and mortar are widely spreaded even in unloaded (sound) concrete. There are varieties of phenomena that may cause bond cracking in plain, nonleaded concrete (Sriravindrarajah & Swamy, 1989). These include settlement of fresh concrete, hydration of cement paste, drying shrinkage, and carbonation shrinkage. This is a manifestation of the well-known phenomenon of existing cracks observed experimentally and/or microscopically by many investigators.

Figures 4 and 5 represent the increasing feature for total numbers and lengths of cracks as the stress levels increase. In 30MPa concrete, both value were in a trend to be somewhat higher than the 60MPa concrete. Number and length of cracks in damaged concrete were larger than those in sound concrete; the difference was estimated to be about 100 cracks for numbers and 100 mm for lengths.

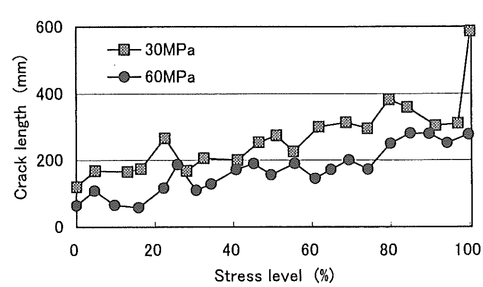

Figure 5. Increasing of total crack lengths within observed area as a function of stress level.

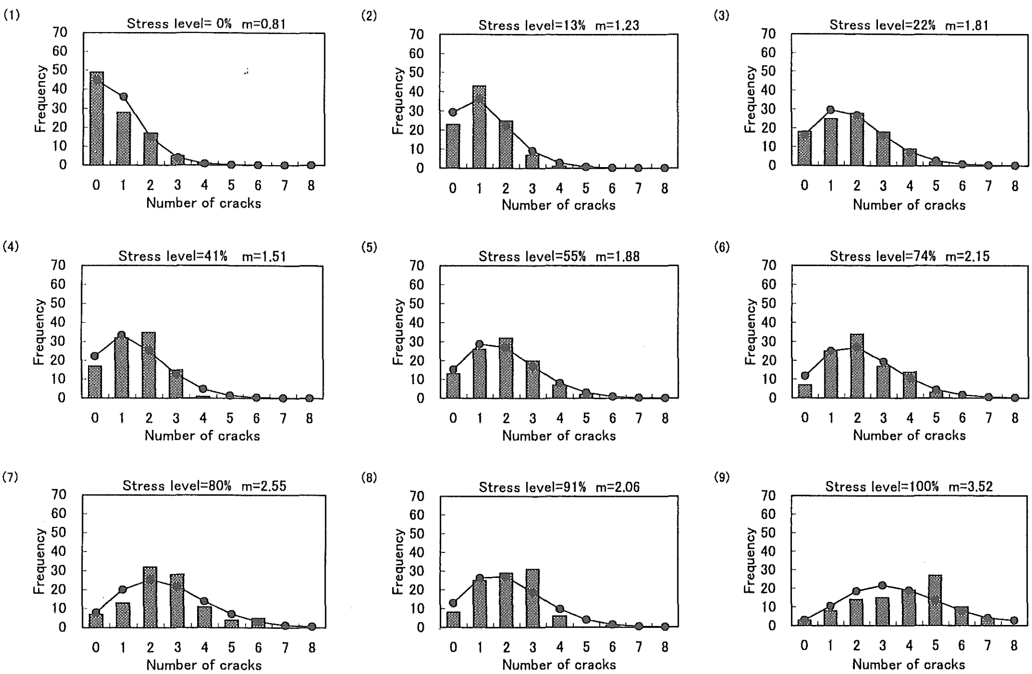

Figure 6. Comparison of experimental value of cracks and theoretical Poisson distribution patterns in each stage of stress level for Strength 30MPa.

3.3 *Relation between number of cracks and Poisson distribution*

Typical results of the number of cracks are illustrated in Figure 6 for several different damage loadings. Although other diagrams were omitted to save space, they were essentially the same as those presented for the companion specimens subjected the similar damage loading. The bar represents the number of the sample areas counted for the given ranks of crack, and the line is plotted according to the Poisson distribution law. There is an interesting agreement between bars and line shown in each diagram.

Poisson distribution has been used to denote the occurrence of the defective events, and in a case of this study, the probability function $P(x)$ would be defined by the following equation:

$$P(x)=e^{-m}\frac{m^{x}}{x!} \quad (x=0,1,2\cdots) \qquad (1)$$

where $P(x)$ = probability of the event occurring; e = base of the natural logarithm, 2.71828; m = experimentally determined average number and also the variance of cracks.

The parameter m, which could be considered denote the defects (cracks) in population, is equivalent of the mean of the Poisson distribution, and governs the overall characteristics of the distribution.

A Chi-square test carried out on each grouped data of the diagrams in Figure 6 did not reject the agreement at the usual 5 percent significance level except diagrams 8 and 9. These results suggest that the distribution of observed number of cracks can be approximated by a Poisson distribution.

In other words, it suggests that the Poisson distribution may be used with reasonable accuracy to estimate the crack development within the concrete. When a certain specimen can be calibrated in terms of parameter m, the prescribed Poisson distribution might be a good predictor of the internal cracking.

3.4 *Relation between crack length and log-normal distribution*

The histogram on crack length for the strength 30 MPa is illustrated in Figure 7. There is a consistent pattern of results for frequency in each diagram, their balances are positively skewed.

Since the log-normal distribution should be clearly asymmetric, the curve given by its probability density function is also superimposed on each diagram. As shown in each diagram, some departure from log-normality could be finding at the vicinity of the peak. Therefore, according to the Chi-square test, the agreement has all rejected. Nevertheless, considering the agreement in the sections more than

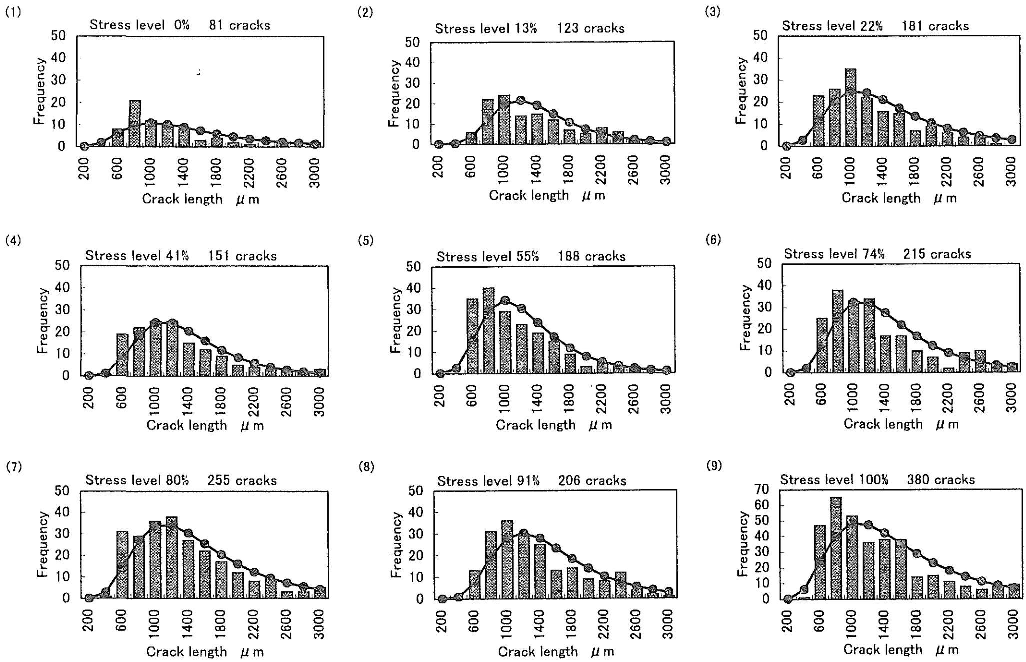

Figure 7. Comparison of experimental value of crack length and theoretical lognormal distribution patterns in each stage of stress level for Strength 30MPa. .

1000 μ m, it would not be unreasonable to assume a log-normality for the development of crack lengths.

When the distribution curve was assumed to be log-normality, the probability function would be defined as follows:

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma_y \cdot x} \exp\left[-\frac{1}{2} \cdot \frac{\left(y-\bar{y}\right)^2}{\sigma_y^{\,2}} \right] \tag{2}$$

where $f(x)$= probability density function; x = crack length; $y = \ln(x)$; $\bar{y}$ = average of y; σ_y = standard deviation of y.

4 DEDUCTION FOR DEVELOPMENT OF CRACKS WITHIN DAMAGED CONCRETE

4.1 *Variation of experimental Poisson distribution parameter with damage intensity*

Plots of all experimental data for the average number of cracks (m) versus stress levels are shown in Figure 8 for the two strength groups. Despite the differences in the strength value an approximately linear relationship exists between two parameters, and the average number of cracks estimated from that relationship was 1.08 to 2.06 for low strength

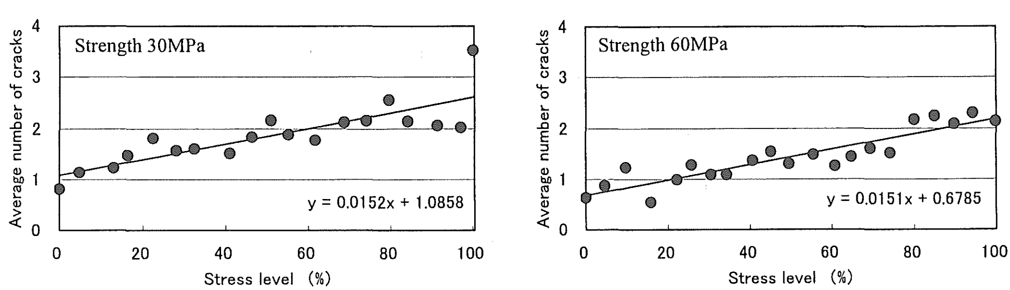

Figure 8. Correlation between Poisson parameters (average crack number, m) and stress levels.

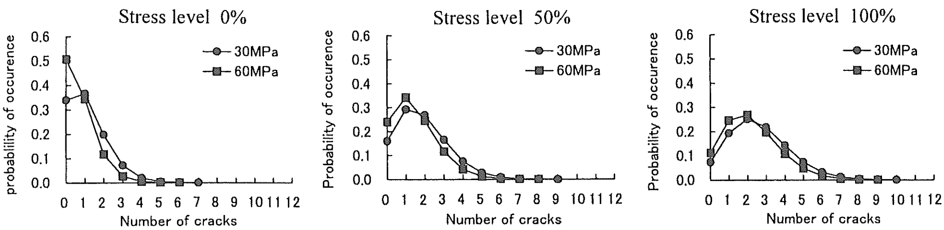

Figure 9. Comparison of expected distribution patterns between strengths 30MPa and 60MPa for the three stages of stress level.

(30MPa), and 0.68 to 2.19 for high strength (60MPa); their lower value is in case of sound concretes, and higher one is in damaged those.

Those data are reworked in the following section to obtain probability of crack occurrence inferred from Poisson distribution by using the estimated average crack number m.

4.2 *Occurrence of cracks with damage*

Calculations were made for three stages of stress level and for the low strength (30MPa) as well as the high strength (60MPa); the results are shown in Figure 9. On the basis of the average number of cracks (m) estimated previous section, it appears that the probability distribution curve on cracks could be deduced for stress levels at 0, 50, and 100 percent as shown in each diagram. Despite the differences in the extent of the stress level, the appearance probability of the number of cracks is very similar to the both strength levels. As shown in those diagrams, Poisson distribution could be a useful tool for predicting the number of cracks developed by damage loading. At present, however, estimated values of m would be significant only on a relative basis since the average number of cracks could be considered an arbitrarily gotten variable.

5 CONCLUSION

In this paper, variability of number and length of internal cracks were reviewed and demonstrated the validity of the statistical approach in investigating the internal cracking by means of probability distributions.

Based on the results of this study, the following conclusions can be drawn:

Two statistical distributions, Poisson distribution and log-normal distribution, were proposed for crack-development in damaged concrete. With respect to the internal cracking, it appeared that the numbers and the total lengths of the observed cracks were closed to the expected those derived from probability distributions. Their agreements were statistically verified through the Chi-square test, and in case of the number of cracks excellent correlations were found between assumed statistical distributions and extensive experimental data.

The frequency of internal cracks in certain area could be deduced by the single parameter m of the Poisson distribution. Experimental results indicated the existence of a correlation between the experimental parameter m and the stress level of cyclic loading. Such loading, referred to as the damage loading in this paper, led the value of the parameter m to the condition of 1.08 to 2.60 in proportion to the magnitude of the stress levels.

On the other hand, the appearance pattern on the length of the internal cracks appears to be indicated consistently by the log-normal distribution despite the magnitude of the damage loading. In case of the crack length, however, a few discrepancies are discovered between expected statistical distributions and experimental values.

Through above experimental process the expedience of applying probability distribution to the number of cracks as well as crack length was substantiated.

The research also suggested that the consideration for the number is more important than for the length of cracks. Similar results were obtained from concrete specimens of different strength tested under similar conditions.

This paper is limited to work on internal cracks observed on the surface cut perpendicular to the direction of the compressive loading.

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