Over-nonlocal microplane model M4: mode I fracture simulations

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ABSTRACT: The paper analyses the effectiveness of the regularization technique known as nonlocal model of integral type when applied to the microplane model M4. An "over-nonlocal" generalization of the type proposed by Vermeer and Brinkgreve (1994) is adopted. Moreover, the symmetric nonlocal formulation proposed by Borino et al. (2003), and the "visibility criterion" is introduced in the over-nonlocal formulation of the microplane model M4. The model has been applied to the simulation of a recent experimental investigation on mode I fracture.

Keywords: microplane model, nonlocal continuum, concrete, damage.

1 INTRODUCTION

The finite element solutions for strain softening materials are afflicted by serious problems, which arise from the differential equations which govern the problem. These equations lose the hyperbolicity (for dynamic loading) or ellipticity (for static loading) so that the boundary value problem becomes ill-posed. This means that the numerical calculations cease to be objective, exhibiting pathological spurious mesh sensitivity and excessive damage localization as the mesh is refined. To recover a well-posed problem and to prevent the localization of damage into a zone of zero volume, many solutions have been proposed, based on the introduction of the characteristic length of the material (Bažant 1976, Bažant & Oh 1983, Lasry & Belytschko 1988). One of the most successful techniques, which have a justification from the micro-cracks interactions (Bažant 1994), is based on the concept of *nonlocal continuum*. The nonlocal concept was introduced in the 1960s (Eringen 1966, Kröner 1968, etc.) for elastic deformations and later expanded to hardening plasticity. In a nonlocal continuum, the stress at a certain point depends not only on the strain at that point but also on the strain field in the neighborhood of that point. Bažant (1984) and Bažant et al. (1984) introduced the nonlocal concept as a localization limiter for a strain-softening material. This formulation was later improved in the form of the nonlocal damage theory (Pijaudier-Cabot & Bažant 1987) and was applied in real problems. To refine the nonlocal formulation, Vermeer & Brinkgreve (1994), proposed a novel "over-nonlocal" formulation, in which the nonlocal variable is enlarged by a factor m larger than 1, while the corresponding local variable is reduced by the factor (1-m). Borino et al. (2003) recently proposed a symmetric nonlocal damage theory in which they introduced a symmetric nonlocal integral operator which is self adjoint at every point of the solid, including the zones located near the solid boundary. This formulation is able to preserve the uniform fields and to reproduce a physically correct nonlocal quantity at ever point of the domain, including the zones near the body boundary. Moreover, observing some difficulties for the correct calculation of the nonlocal integral in the vicinity of a sharp notch, the concept of the visibility criterion, taken from the Meshless Methods (Belytschko et al. 1996), has been introduced in the symmetric nonlocal formulation. According to the visibility criterion, a line connecting a point to another point in the nonlocal averaging is imagined to be a ray of light and the boundary of the body to be an opaque surface. If the ray encounters an opaque surface, it is terminated and the point is not included in the nonlocal average.

This symmetric nonlocal formulation with the visibility criterion has been applied for the simulation of a recent experimental investigation on mode I fracture based on three-point-bending tests and Brazilian tests.

2 THE SYMMETRIC OVER-NONLOCAL FORMULATIONS

The nonlocal model, in general, consists in replacing a certain local variable f(**x**), characterizing the softening damage of material, by its nonlocal counterpart f(**x**). The nonlocal variable is defined as

$$f(\mathbf{x}) = \int_V \hat{\alpha}(\mathbf{x},\xi) f(\xi) dV(\xi) \tag{1}$$

where V is the volume of the structure, **x** e ξ are the coordinates vectors, and $\hat{\alpha}(\mathbf{x},\xi)$ is a weight function. The weight functions in Equation 1 has the following characteristics: 1) it is a positive function; 2) it has its maximum value for **x**=ξ; 3) it is a monotonic decreasing function to zero of the distance r=|**x**−ξ| (the nonlocal average in a fixed point have a certain finite influence volume).

The basic weight function α(**x**−ξ) is often taken as a bell-shaped function; its analytical expression is

$$\alpha(\mathbf{x},\xi) = \begin{cases} \left[1 - \left(\dfrac{|\mathbf{x}-\xi|}{R}\right)^2\right]^2 & \text{if} \quad 0 \le |\mathbf{x}-\xi| \le R \\ 0 & \text{if} \quad R \le |\mathbf{x}-\xi| \end{cases} \tag{2}$$

where R, called the interaction radius, is proportional to the material characteristic length *l*, R=ρ_0*l*. The coefficient ρ_0 is determined so that the volume under function α(**x**−ξ) be equal to the volume of the uniform distribution. The bell-shaped function is often used in the numerical applications since it has a limited support R.

One of the requirements which we expect from the nonlocal average is that a uniform field is not influenced by the nonlocal formulation (Equation 1): if the local field is uniform also the nonlocal field has to be uniform. In order to satisfy this condition, Pijaudier-Cabot & Bažant (1987) proposed the following normalized nonlocal formulation

$$\hat{\alpha}(\mathbf{x},\xi) = \frac{\alpha(\mathbf{x},\xi)}{V_R(\mathbf{x})}; \quad V_R(\mathbf{x}) = \int_V \alpha(\mathbf{x},\xi) dV(\xi) \tag{3}$$

in which $\alpha(\mathbf{x},\xi)$ is the basic nonlocal weight function for an unbounded medium (Equation 2); $V_R(\mathbf{x})$ is called the representative volume and it is a constant if the unrestricted averaging domain does not tend to protrude outside the boundaries. The nonlocal formulation in Equation 3 ensures the normalization condition

$$\int_V \hat{\alpha}(\mathbf{x},\xi) dV(\xi) = 1 \tag{4}$$

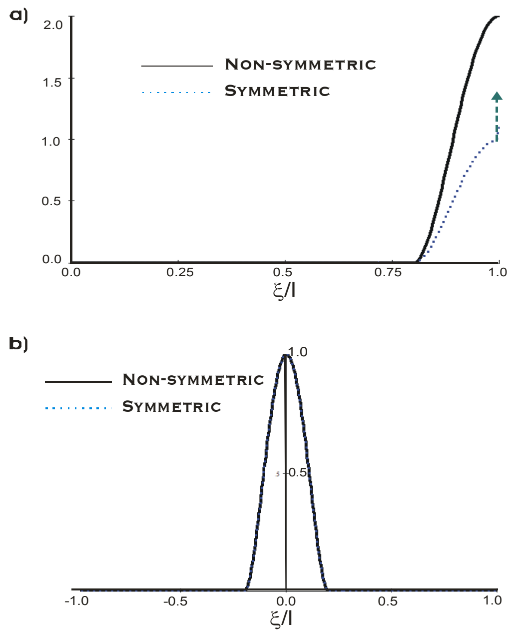

Figure 1. Comparison of one-dimensional weight functions for symmetric and non-symmetric nonlocal formulations (with R = 0.2l): a) weight functions for a point in the right boundary; b) weight functions for a point far from the boundary.

It is worth noting that the nonlocal formulation in Equation 3 is not symmetric, i.e. $\hat{\alpha}(\mathbf{x},\xi) \neq \hat{\alpha}(\xi,\mathbf{x})$ in general. This lack of symmetry makes the tangent operators non symmetric. To overcome this problem, Borino et al. (2003) proposed a new nonlocal formulation in which they introduced the following weight function

$$\hat{\alpha}(\mathbf{x},\xi) = \left(1 - \frac{V_R(\mathbf{x})}{V_\infty}\right)\delta(\mathbf{x},\xi) + \frac{\alpha(\mathbf{x},\xi)}{V_\infty} \tag{5}$$

where δ(**x**−ξ) is the Dirac delta function, α(**x**−ξ) is the basic weight function (Equation 2), $V_R(\mathbf{x})$ is the representative volume and V_∞ is the value of the representative volume far from the boundaries where it has a constant value. The first term in Equation 5 is a local term which is activated only for points near the boundaries, while for points far from the boundaries this first term tends to vanish ($V_R(\mathbf{x}) \to V_\infty$). As we can see from Equation 5, this formulation is symmetric everywhere and for an unbounded solid or for points of a bounded solid sufficiently far form the boundaries this formulation coincides with the classical nonlocal formulation, (Equation 3). In Figure 1b for the one-dimensional case using the bell-shaped weight functions, Equation 3 and Equation 5 are plotted together for a point far from the boundary where the two formulation coincide. In Figure 1a, instead, Equation 3 and Equation 5 are plotted for a point located at the boundary where the two formulations differ.

Originally Vermeer & Brinkgreve (1994) and later Planas et al. (1996), and Strömberg & Ristinmaa (1996) (see also Bažant & Planas, 1998), introduced a refinement of the standard nonlocal formulation, called *over-nonlocal* formulation because m>1, in which a combination of the local and the nonlocal variable is defined as follows

$$f^*(\mathbf{x}) = m\bar{f}(\mathbf{x}) + (1-m)f(\mathbf{x}) \tag{6}$$

where $f^*(\mathbf{x})$ is the over-nonlocal average of the variable $f(\mathbf{x})$, $\bar{f}(\mathbf{x})$ is the nonlocal variable obtained from Equation 1, and *m* is an empirical coefficient (over-nonlocal parameter). The previous works on this formulation, applied to simple softening plastic models, confirmed the avoidance of spurious localization if *m*>1. Planas et al. (1996) rigorously proved, for a uniaxial stress field, that the localization zone is finite if and only if *m*>1. Later Bažant & Di Luzio (2003) showed the necessity that *m* be larger than 1 for the nonlocal generalization of the microplane model M4.

The same refinement can also be obtained rewriting the weight function in Equation 1 as a function of the over-nonlocal parameter *m*. For the non-symmetric over-nonlocal formulation one obtains

$$\hat{\alpha}_m(\mathbf{x},\xi) = (1-m)\delta(\mathbf{x},\xi) + m\frac{\alpha(\mathbf{x},\xi)}{V_R(\mathbf{x})} \tag{7}$$

where $\delta(\mathbf{x},\xi)$ denotes the Dirac delta function and *m* is the over-nonlocal parameter (for *m*=1 the standard nonlocal formulation is recovered). For the symmetric over-nonlocal formulation we have

$$\hat{\alpha}_m(\mathbf{x},\xi) = (1-m)\delta(\mathbf{x},\xi) + m\left[\left(1-\frac{V_R(\mathbf{x})}{V_\infty}\right)\delta(\mathbf{x},\xi) + \frac{\alpha(\mathbf{x},\xi)}{V_R(\mathbf{x})}\right] \tag{8}$$

3 REVIEW OF THE NONLOCAL MICROPLANE MODEL M4

The microplane constitutive model has the potential to capture the complex inelastic behavior of concrete by using simple constitutive relations between stresses and strains acting on a plane in the material called the microplane, which has an arbitrary orientation. In the microplane model M4 based on the kinematic constraint the static equivalence (or equilibrium) of stresses between the macro and micro levels is expressed by the principle of virtual work (Bažant 1984). This numerical integration is done according to an optimal Gaussian integration formula for a spherical surface (Stroud 1971, Bažant & Oh 1986). An efficient formula which involves 21 microplanes (Bažant & Oh 1986) and yields acceptable accuracy has been used in this work. Other formulas with 28 (the Stroud's formula), 37 and 61 can be used to achieve better accuracy.

The most general explicit constitutive relation on the microplane level give σ_N, σ_L and σ_M as functionals of the histories of ε_N, ε_L and ε_M, possibly supplemented by a yield condition in terms of σ_N, σ_L and σ_M. But, in general, it is sufficient to assume that each of σ_N, σ_L and σ_M depends only on its corresponding strain ε_N, ε_L and ε_M because cross dependence on the macro level, such as shear dilatancy, is automatically captured by interaction among microplanes of various orientations. An exception is the frictional yield condition relating the normal and the shear components on the microplane with no strain dependence.

In the microplane model M4 (Bažant et al., 2000), the constitutive relation in each microplane is defined by 1) incremental elastic relation and 2) stress-strain boundaries (softening yield limits) that cannot be exceeded. For unloading and reloading, the elastic microplane moduli are assumed to be functions of the current strain and the maximum strain reached so far. The stress-strain boundaries, which may be regarded as strain dependent yield limits, consist of the following conditions:

$$\begin{gathered} \sigma_N \le F_N(\varepsilon_N) \quad \sigma_V \ge F_V^-(\varepsilon_V) \\ F_D^-(\varepsilon_D) \le \sigma_D \le F_D^-(\varepsilon_D) \\ |\sigma_L| \le F_T(\sigma_N, \sigma_V, \varepsilon_I) \quad |\sigma_M| \le F_T(\sigma_N, \sigma_V, \varepsilon_I) \end{gathered} \tag{9}$$

Except for the last two conditions, which model friction, interactions among various components need not be considered, since the cross effects are adequately captured by interactions among various microplane due to the kinematic constraint. The unloading conditions are formulated separately for each microplane component. The constitutive model, formulated and tested in Bažant et al. (2000) and Caner & Bažant (2000), is completely defined on the microplane level. However, some modifications on the constitutive law have been recently introduced (Merlo 2003).

As already mentioned, the microplane model differs from the classical tensorial model of plasticity and continuum damage because the stress-strain boundaries, which define the inelastic strain, depend on the total strain only. This suggests a nonlocal generalization in which the stress-strain boundaries are evaluated from the nonlocal total strains (instead of being evaluated from the local total strain, with the nonlocal averaging postponed until after the inelastic strains have been evaluated). Based on these considerations, Bažant & Di Luzio (2003) proposed a new kind of nonlocal formulation in which the elastic stress increments are local and the boundaries in Equation 9 are modified as follows:

$$\sigma_N \le F_N\left(\varepsilon_N^*\right) \quad F_D^-\left(\varepsilon_D^*\right) \le \sigma_D \le F_D^-\left(\varepsilon_D^*\right)$$
$$\left|\sigma_L\right| \le F_T\left(\sigma_N, \sigma_V, \varepsilon_I^*\right) \quad \left|\sigma_M\right| \le F_T\left(\sigma_N, \sigma_V, \varepsilon_I^*\right) \tag{10}$$

For the sake of generality, the strains in these conditions are considered as over-nonlocal;

$$\varepsilon_V^* = \varepsilon_{kk}^* / 3 \quad \varepsilon_N^* = N_{ij}\varepsilon_{ij}^* \quad \varepsilon_D^* = \varepsilon_N^* - \varepsilon_V^*$$
$$\varepsilon_L^* = L_{ij}\varepsilon_{ij}^* \qquad \varepsilon_M^* = M_{ij}\varepsilon_{ij}^* \tag{11}$$

where $\varepsilon_{ij}^* = m\bar{\varepsilon}_{ij} - (1-m)\varepsilon_{ij}$, ε_{ij} are the Cartesian components of ε, and the over-bar denotes the nonlocal counterpart of the variable as defined in Equation 1. The standard nonlocality is the special case for *m*=1. It is crucial to recognize that the elastic strains on the microplane (as well as any hardening inelastic strains) must depend only on the local strain, or else one would engender zero energy instability modes (such modes plagued the original nonlocal strain-softening continuum model, the so called imbricate model, and had to be suppressed by parallel elastic coupling, which precluded the strain-softening to terminate with zero stress). This means that, in every constitutive law in which the softening depends on the strains, objectivity can be reached by making the softening function dependent on the nonlocal strains. As shown by Bažant & Di Luzio (2003), only using *m*>1 a realistic description of the fracturing process is achieved. They showed that the fracturing strain is localized into a finite length, independently of the number of elements, only if *m* is larger than 1. On the other hand, if the classical nonlocal model (*m*=1) is adopted, the fracturing strain tends to localize into one element even if the global response is correct (i.e., objective) in terms of the stress-displacement curve.

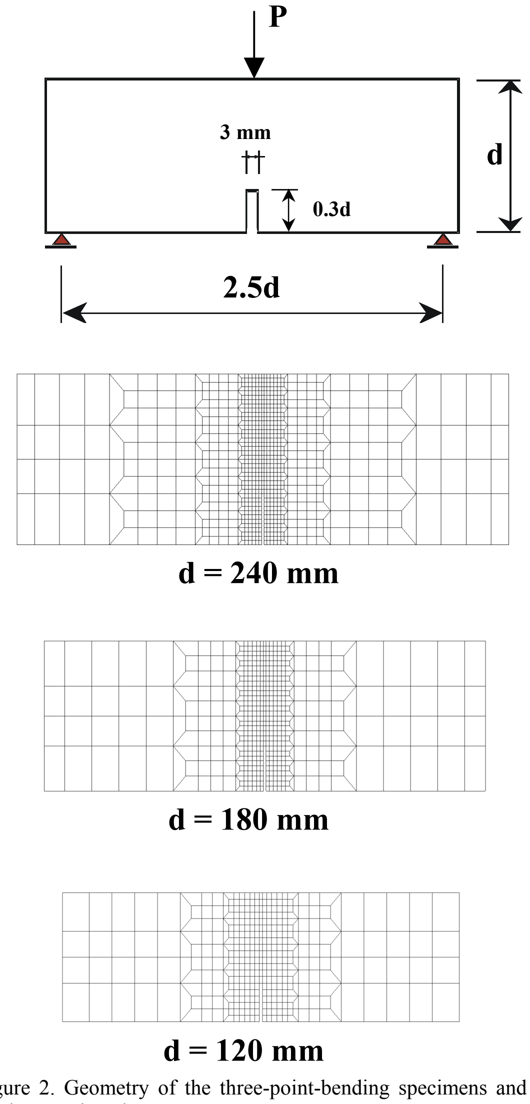

Figure 2. Geometry of the three-point-bending specimens and meshes employed.

4 MODE-I FRACTURE ANALYSES

One important consequence of nonlocality is the size effect. To demonstrate this, the results of a recent experimental investigation on three-point-bending and Brazilian specimens are considered (Taini 2003).

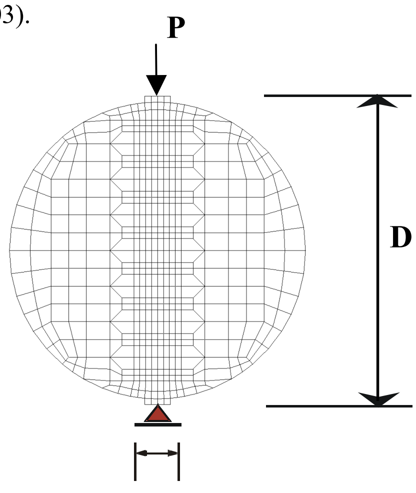

Figure 3. Mesh of the Brazilian Test.

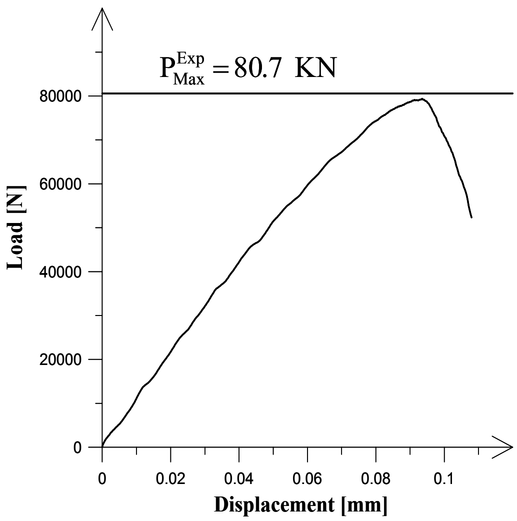

Figure 4. Numerical load-displacement curve for the Brazilian test and experimental maximum load.

Three-point-bending specimens of the same geometry and of three different sizes, with ratio 1:1.5:2, have been considered (Fig. 2). The smallest specimen depth is d=120 mm. The thickness is b=80 mm for each size. Taking into account the weight of the specimens, the average measured maximum loads for the three specimen sizes are 5000, 6450 and 7920 N, respectively.

For the Brazilian test a cylinder with a diameter D=100 mm and a length l=200 mm has been considered (Fig. 3). This test gave a mean maximum load of 80.7 kN, which leads to a tensile strength of 2.53 MPa ($f'_t = P_{MAX}/(\pi\ d\ l)$).

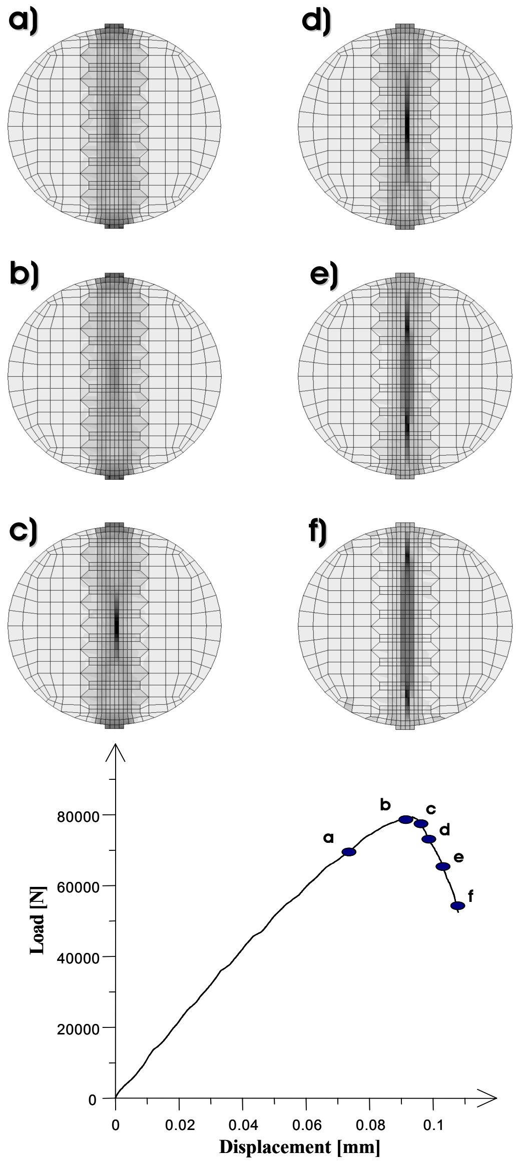

Figure 5. Evolution of the maximum inelastic principal strain for different values of the applied displacement.

The mechanical properties of the concrete obtained from independent tests are given by compressive strength f'_c=28.5 MPa and a Young modulus E_c=24.2 GPa. The tensile strength and the nonlocal parameters, i.e. the characteristic length (the over-nonlocal parameter *m* is assumed equal to 1.04), which control the fracture energy of the model, have been calibrated by fitting the experimental data.

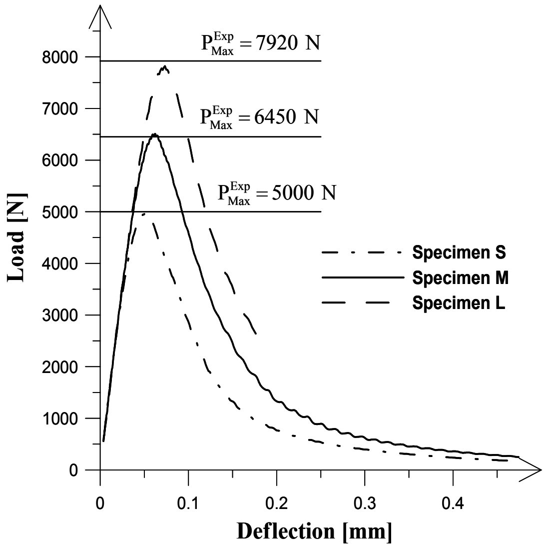

Figure 6. Load-deflection curves for the simu-lation of the three-point-bending tests compared to the experimental maximum loads.

Using the symmetric over-nonlocal microplane model calibrated as described above, the experimental tests have been simulated. The maximum loads obtained from the numerical simulations are: for the Brazilian test 79.4 kN (80.4 kN the experimental value with an error of 1.25%), for the three-point-bending tests 5050 N, 6550 N and 7800 N for the three sizes (5000 N, 6450 N and 7920 N are the corre-sponding measured values with an error of 1%, 1,55% and 1,52%). In Figure 4 the plot of the load-displacement curve obtained for the Brazilian test shows a good agreement with the observed maximum load. Figure 6 shows that the load-deflection curves of the three three-point-bending specimens agree very well with the observed ultimate loads. A tensile strength of 2.22 MPa and a characteristic length of 55.2 mm are the material parameter which lead to the above numerical results. The tensile strength has been reduced from the value of the Brazilian test (2.53 MPa) of about 12.25% which is in the range generally accepted. Analyzing the experimental and the numerical results through the Bažant's size effect law, we obtained a G_f equal to 35.4 N/m and 31.7 N/m, respectively. The fracture energy calculated as the integral of load-deflection curve divided by the fracture area is 99.5 N/m, with a ratio between this value and G_f of 3.14. Figure 5 and Figure 7 show the evolution of the maximum inelastic principal strain during the loading of the Brazilian specimen and the small three-point-bending specimen, respectively.

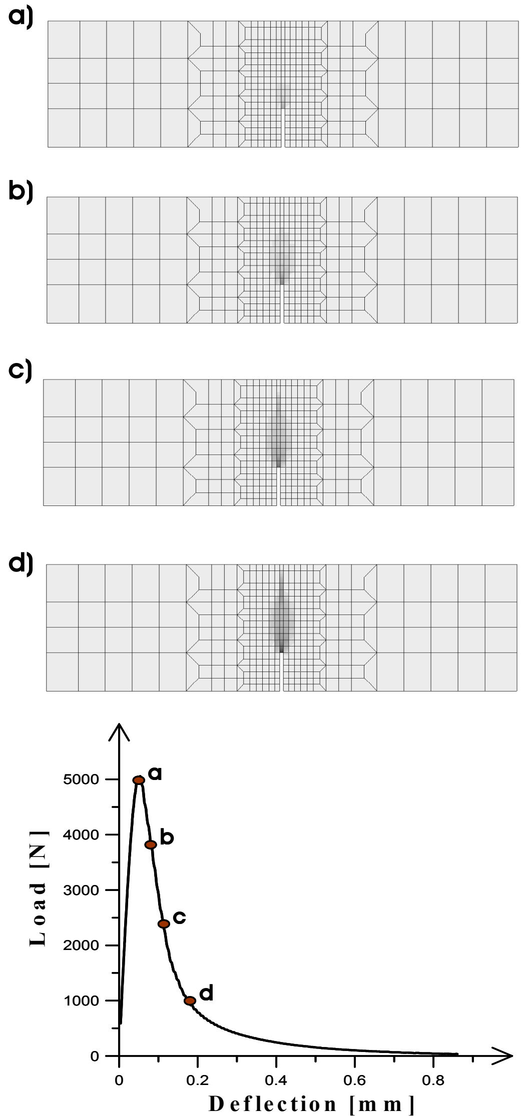

Figure 7. Evolution of the maximum inelastic principal strain for different values of the applied displacement for the small three-point-bending specimen.

5 CONCLUDING REMARKS

The symmetric over-nonlocal microplane model M4 is a constitutive law capable of predicting the fracturing behavior of the concrete in a realistic way. It is able to avoid some of the typical problems of this kind of constitutive laws, such as the localization into a band of zero thickness. Moreover, it is able to give a physical meaningful reproduction of the non-local variable in the vicinity of a boundary, which is a typical problem of the classical non-local formulation. This model has been applied for the simulation of different types of mode I fracture tests (Brazilian test and three-point-bending test for three sizes) with a good agreement with the experimental data (maximum error of 1.55% on the ultimate loads).

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