Fracture Studies on Notched Concrete Disc Specimens at Various Sizes

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ABSTRACT: The main objective of this work is to study the size effect on the nominal strength of concrete disc specimens in Mode I loading condition. As the size of the specimens increases, the nominal strength decreases for all notched specimens. The decrease is more noticeable in the specimens with small notches. The results are also compared with the findings of the approaches based on two types of fractal based models. The test results obtained from the notched disc specimens confirm the existence of the Size Effect Law, the Multifractal Scaling Law, and the Modified Fractal Method proposed in this work.

Keywords: size effect, notched disc specimen, splitting strength, fractal.

1 INTRODUCTION

Since concrete is a widely used material in structural engineering, it is essential to have sufficient information about crack initiation and propagation in concrete. The information is needed especially for the finite element analysis of concrete behavior. The improvements in the modern computer aided design and the wide usage of concrete in special structures such as reactor containment vessels, dams, high rise concrete buildings, and missile storage silos have led to a growing interest in the cracking behavior of concrete (Shah and Tasdemir, 1994).

In practice, concrete mix design is based on the compressive strength criterion. Such an approach, however, ignores the importance of the brittleness of concrete. In the last three decades, some fracture mechanics based models have been proposed for the determination of the fracture parameters and the brittleness of concrete. These are: i) the fictitious crack model (Hillerborg et al., 1976), ii) the Size Effect Law (Bazant and Kazemi, 1990), iii) the two parameter fracture model (Jenq and Shah, 1985), and iv) the effective crack models (Nallathambi and Karihaloo, 1986; Swartz and Refai, 1988). The first three models are known as the RILEM recommendations. Each of these models provides a way to measure the brittleness of concrete. In order to determine the ductility of the mix, the characteristic length in the fictitious crack model is successfully used in the numerical analysis of concrete structures, and also in the design of new cementitious composites, e.g. by controlling the microstructural brittleness to prevent microcracking (Karihaloo, 1995). Characteristic length (l_{ch}) is defined, in terms of modulus of elasticity (E), fracture energy (G_F), and direct tensile strength (f_t'), by the equation $l_{ch}=EG_F/(f_t')^2$ (Hillerborg, 1985). According to the fictitious crack model, G_F is defined as the area under the load versus displacement at mid-span curve (RILEM FMC-50, 1985). The Size Effect Law requires a minimum of three different sizes of beams to be tested, but only the maximum load needs to be measured (RILEM TC-FMT, 1990a). According to the two parameter model, at least two parameters are needed to describe the fracture process; these parameters are K_{IC}^S (the critical value of stress intensity factor) and $CTOD_C$ (the critical value of crack tip opening displacement) (RILEM TC-FMT, 1990b). Recently, a simple test using only peak load measurements was proposed to determine these parameters (Tang et al., 1992 and 1996, Yang et al., 1997, Eser et al., 2002, and Eser, 2002). The use of centre notched disc specimens has an important potential in testing core specimens taken from an existing structure to evaluate the fracture parameters. Although the Size Effect Law has been proposed for the three-point bending of beams as one of the RILEM recommendations, the peak load measurements on notched disc specimens were used in this study to determine fracture parameters of concrete.

2 TEST SPECIMENS

Disc specimens, shown in Figure 1, were produced using the same batch of concrete. Mixture proportions of concrete were as follows; cement : water : sand (0-4 mm) : limestone powder (0-4 mm) : crushed stone I (4-10 mm) : crushed stone II (8-20 mm): admixture = 1:0.522: 1.679 : 0.685 : 1.713 : 1.596 : 0.01. The cement content was 325 kg/m^3 and a superplasticizer was used. The specimens were cast in a specially designed steel mold, and a steel blade was inserted vertically through the specimen before casting. Various sizes of steel blades were used to maintain different 2a/d ratios for each disc specimen. The steel blade was removed about 4 hours after casting to enable easy removal. Thereafter, the molds were removed. All specimens were kept under wet burlap for 7 days. Then, they were exposed to the laboratory environment until 56 days of age. At last three specimens of each size were tested at constant loading rate. All specimens were tested using the same machine by changing the loading capacities. The post-peak response was not measured, and a low scatter was observed during the tests.

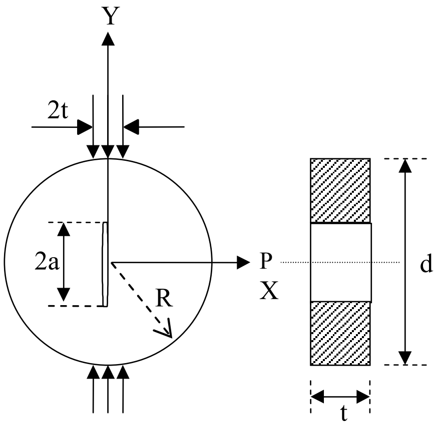

Figure 1. Loading setup condition and specimen geometry used in splitting-tensile tests.

The compressive strength and modulus of elasticity of concrete were determined according to standard procedures using cylinders 150 mm in diameter and 300 mm in length. The static modulus of elasticity was calculated using the ascending part of the stress-strain curves in compression up to a stress level that approximately equals to the 30 percent of the strength. The 56th day cylinder compressive strength and modulus of elasticity of concrete were 27.5 MPa and 24.2 GPa, respectively. The disc specimens were 100 mm, 150 mm, 225 mm, 300 mm, 450 mm, and 600 mm in diameter and 50 mm, 60 mm, 112,5 mm, 150 mm, 225 mm, and 300 mm in thickness, respectively. The ratio of 2a/d was chosen to be 0.1, 0.3, and 0.5, where 2a is the length of the notch and d (i.e. 2R) is the diameter of the disc specimens.

3 RESULTS AND DISCUSSION

Experimental results obtained indicate that the splitting tensile strength of notched concrete disc specimens decreases with increasing size of disc, and then nearly remains stable for disc specimens greater than 450 mm in diameter, as shown in Figure 2.

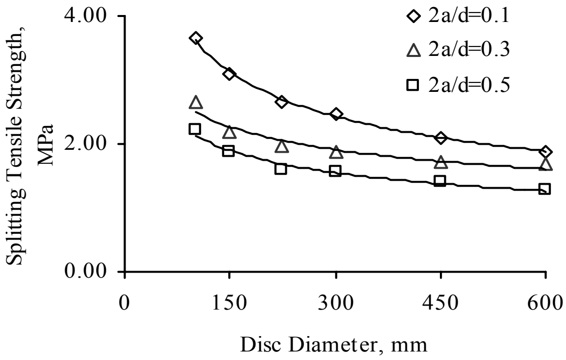

Figure 2. Relationship between the disc diameter and splitting tensile strength of concretes with notch.

The disc size and nominal strength were found to be inversely proportional. The decrease in strength is more noticeable in the case of notched specimens of smaller size. However, the splitting tensile strength of concrete stays nearly constant when the sizes of concrete become very large. It is clear that the size effect on the splitting tensile strength may be mainly explained by the fracture process zone.

3.1 *Size effect law*

The size effect in the failure of geometrically similar specimens can be expressed in terms of the nominal stress at failure. Size Effect Law proposed by Bazant (1984) is defined as follows:

$$\sigma_N = c_n \frac{P_u}{bd} \qquad (1)$$

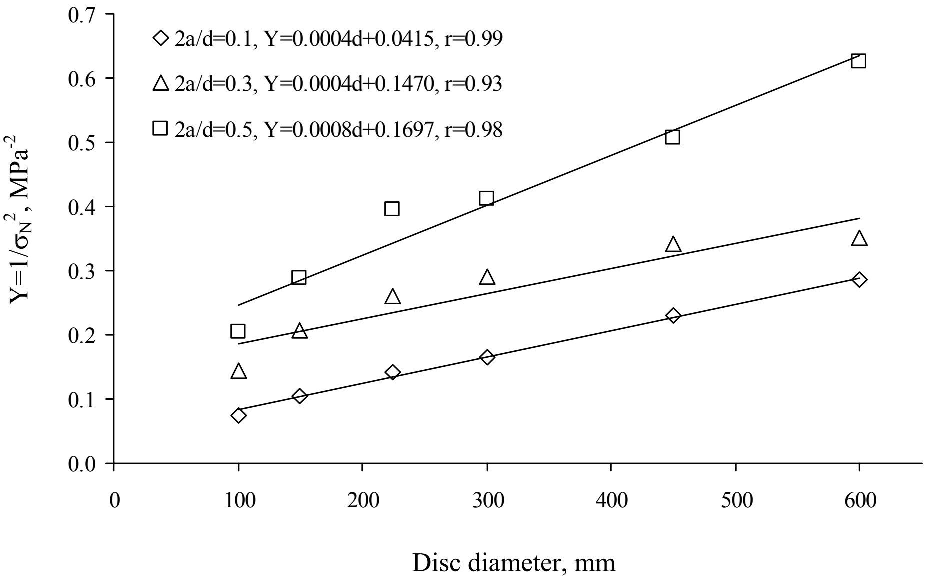

Figure 3. Linear regression results for notched concrete disc specimens obtained by Eser, 2002.

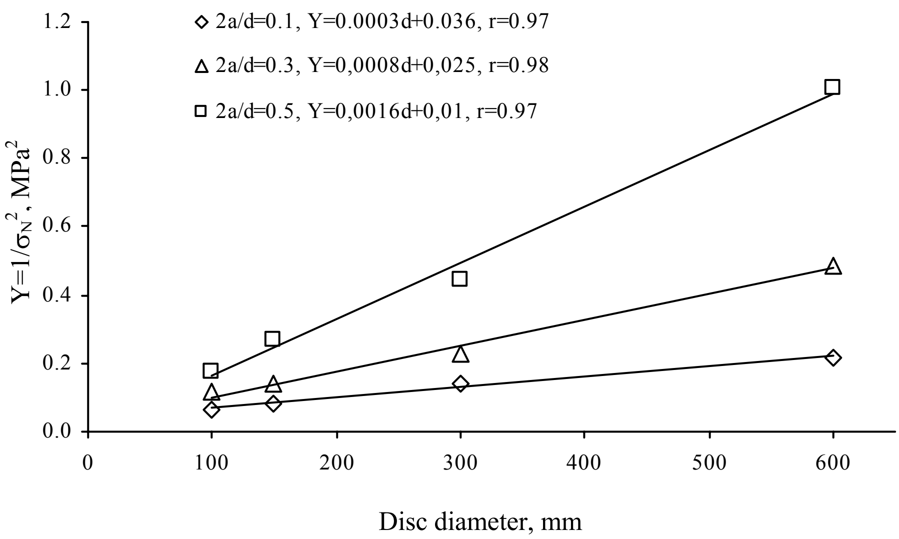

Figure 4. Linear regression results for notched concrete disc specimens obtained by Atahan, 1996.

where, P : the maximum load, b: specimen thickness, d: characteristic dimension of the specimen, and c_n: a coefficient introduced for convenience (Bazant & Kazemi, 1990). The size effect based on fracture mechanics approach can be approximately described as:

$$\sigma_N = Bf_t^{'} / \sqrt{1+\beta} \tag{2}$$

As shown in Figure 3, a line with an equation of of Y=AX+C was fitted to the experimental data for a range of disc diameters from 100mm to 600mm. In each linear regression, the correlation coefficient was found to be greater than 0.93. As seen in Figures 3 and 4, the slope of the line where 2a/d=0.5 is significantly greater than the slopes of the other lines.

The size effect curve in $\log(\sigma_N/Bf_t')$ versus $\log(d/d_0)$ graph obtained in this study is illustrated

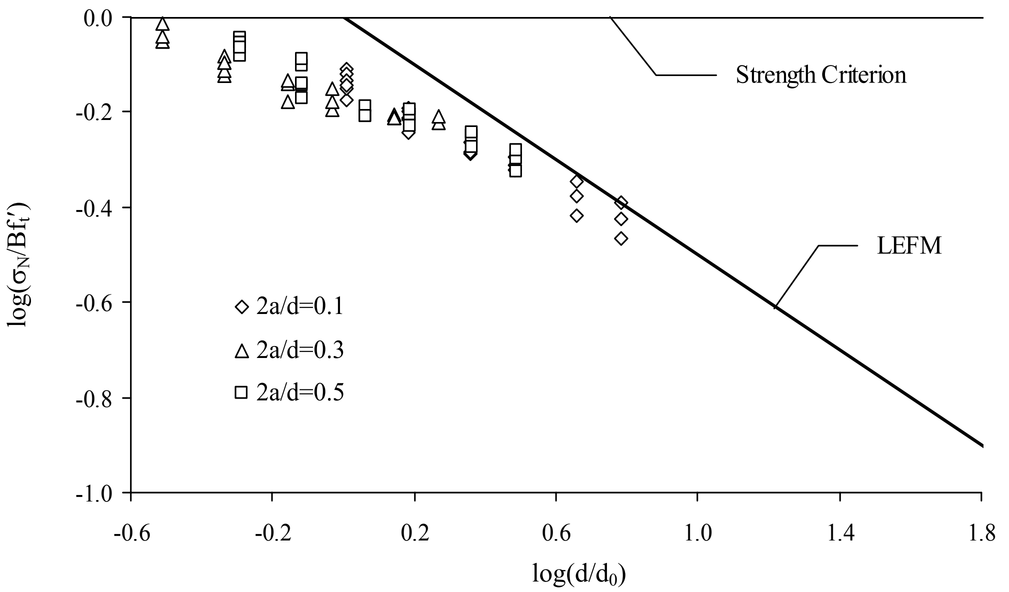

Figure 5. Test results of disc specimens obtained in this study displaying the size effect.

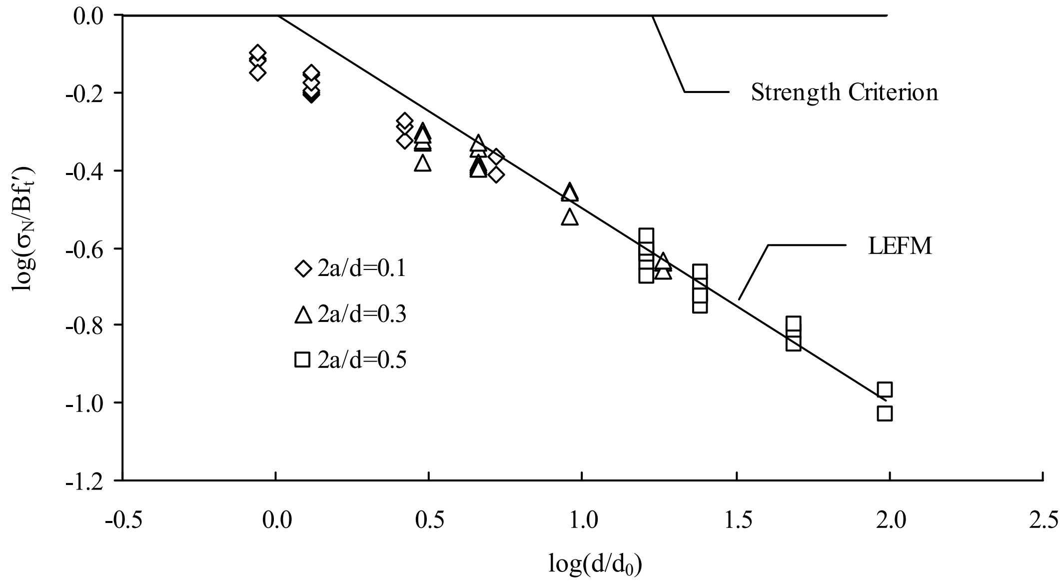

Figure 6. Test results of disc specimens obtained by Atahan, 1996 displaying the size effect.

in Figure 5. Individual test results for all sizes are shown in the figure. Although the current design criterion ignores the size effect, the results obtained show the existence of a strong size effect for the disc specimens tested. For small sizes of specimens, these curves approach a horizontal asymptote, and for large sizes the experimental points approach an inclined asymptote of slope -0.5, which corresponds to LEFM.

As seen in Figure 5, the Size Effect Law which appears to be compatible with the experimental results obtained, represents a certain transition between the case of plasticity (where the sizes are small i.e. β approaches zero, at which there is no size effect) and the case of LEFM (where sizes are large i.e. β goes to infinite, at which the size effect is strongest). As seen in Figures 4 and 6, the similar results were obtained by Atahan (1996). Thus, the results obtained confirm the existence of the Size Effect Law proposed by Bazant (1984).

3.2 *Fractal approach*

In regular systems, such as long wires, large thin plates, or large solid cubes, the dimension f characterizes how the mass M(L) changes with the linear size L of the system. Let us consider a small part of the system of linear size bL (b<1), where M(bL) is decreased by a factor of b^f. Thus, the following equation can be written:

$$M(bL)=b^f M(L) \quad (3)$$

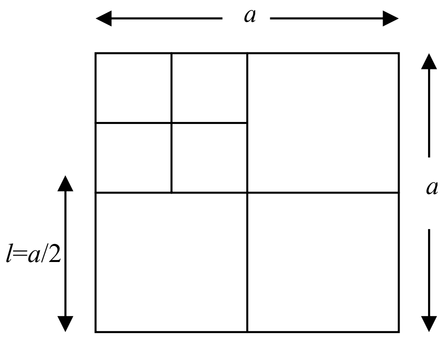

Figure 7. Division of square into small squares

In Figure 7, the fracture surface area of concrete at failure is accepted to be analogous to the division of a square of *a* cm by *a* cm in size into smaller squares with sides of *l* cm.

Based on the geometric similarity, Equation 3 can be written as $M(L)=AL^f$. Thus, defining u=*a*/*l* as a dimensionless quantity, and if A=1, the following equation can be obtained (Peitgen & Saupe, 1988; Bunde & Havlin, 1994; Akyuz & Tasdemir, 1997).

$$M(u)=u^2k=u^f \quad \text{or} \quad u^{2-f}=1/k \quad (4)$$

Thus, the fractal size of the fracture surface area of concrete takes the form of *f=2+(logk/logu)* where it is suitable to take u=5. For all disc specimens, from the smallest disc to the largest one, the calculated pairs of (f;k) are (1.877;0.82), (1.760;0.68),(1.672;0.59),(1.594;0.52),(1.461;0.42) and (1.348;0.35), respectively (Eser et al., 2002).

If the tensile forces were applied to concrete cubes (geometrically similar specimens) with different sizes, the following equation can be written:

$$T_1 = \frac{F_1}{A_1} \neq T_2 = \frac{F_2}{A_2} \neq ... \neq T_n = \frac{F_n}{A_n} \quad (5)$$

where F_1, F_2, F_3, ...,F_n are failure loads and A_1, A_2, A_3, ..., A_n are crossections of cubes tested.
The normalized tensile strength, T^*, which represents the strength of the material with no defect, can be calculated as follows:

$$\frac{T^*}{k^*} = \frac{T_1}{k_1} = \frac{T_2}{k_2} = ... = \frac{T_n}{k_n} \quad (6)$$

In Equation 6, k_1, k_2, k_3, ..., k_n satisfy the following equations:

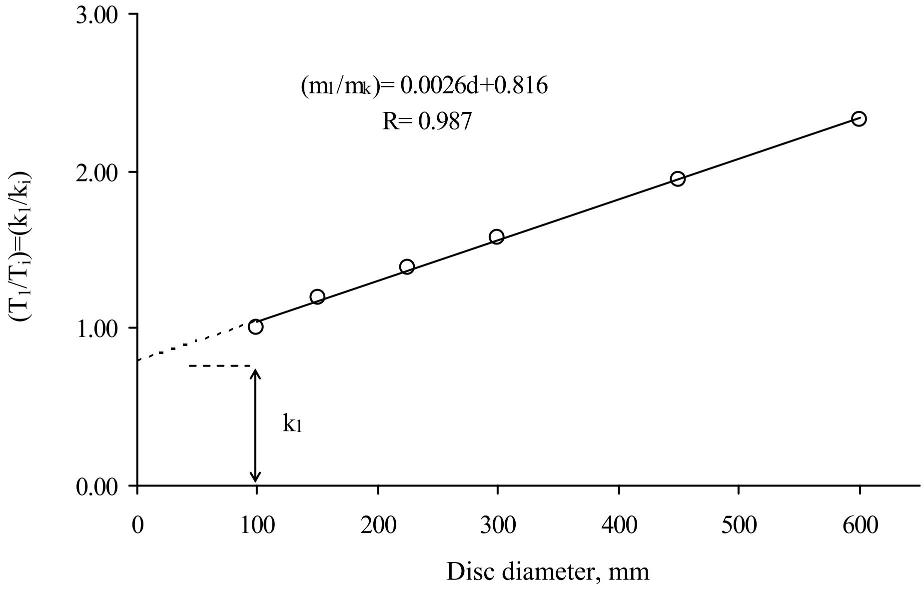

Figure 8. Linear relation between $(T^1/T_i)=(k_1/k_i)$ and disc diameter.

$$\frac{T_1}{T_2}=\frac{k_1}{k_2},\frac{T_1}{T_3}=\frac{k_1}{k_3},......\frac{T_1}{T_n}=\frac{k_1}{k_n} \quad (7)$$

In this work, it is shown that the relation between $(T_1/T_i)=(k_1/k_i)$ and d_i (diameter of the disc) is a straight line, where i=1, 2, ..., n and n=6. As shown in Figure 8, as d approaches zero ($d\rightarrow0$), k_i becomes 1 (perfect microstructure). Thus, k_1 takes the value where the regression line intersects the vertical axis. From Figure 8, the ratios of k_1/k_2, k_1/k_3, k_1/k_4, k_1/k_5, and k_1/k_6 are obtained as 1.20, 1.38, 1.57, 1.95, and 2.32, respectively (Eser, 2002 ; Eser et al. 2002).

3.2.1 *Mathematical model of size effect*

The relationship between $(T_1/T_i)=(k_1/k_i)$ and d_i mentioned in Paragraph 5 can be written as:

$$\frac{k_1}{k_i}-1=m(d-d_1) \quad (8)$$

where m is the slope of the line. Equation 9 can be obtained by expressing Equation 8 in a different form:

$$k_i=\frac{k_1}{1+m(d-d_1)} \quad (9)$$

If Equation 9 is inserted in the equation below:

$$T_i=T^*k_i \quad (10)$$

(which can be obtained from Equation 6), the following equation can be obtained:

$$T_i=\frac{T^*k_1}{1+m(d-d_1)}=\frac{T^*k_1}{1-md_1+md} \quad (11)$$

and finally, by taking $T^*k_1/m=A$ and $(1-md_1)/m=B$, Equation 11 can be expressed as:

$$T_i=\frac{A}{B+d} \quad (12)$$

In the two limiting cases of the characteristic size of the specimen, tensile strengths of the specimen attain the following values:

$d\rightarrow\infty\Rightarrow T=0$ and,

$d\rightarrow0\Rightarrow T=T^*$.

3.2.2 *Modified fractal method (MFM)*

Akyuz & Tasdemir (1997) proposed a Modified Fractal Method (MFM) to represent the nominal tensile strength (σ_N) as follows:

$$\sigma_N=\frac{A}{B+d} \quad (13)$$

where d is the size of the specimen, and coefficients A and B are constants to be determined from experimental data. To determine the coefficients A and B, Equation 13 can be written in the following form:

$$\frac{1}{\sigma_N}=\frac{1}{A}d+\frac{B}{A} \quad (14)$$

1/A represents the slope of the regression line of the experimental $1/\sigma_N$ values versus d graph, and B/A is the Y-intercept of the fitted line.

3.3 *Multifractal scaling law (MFSL)*

According to Carpinteri et al. (1995), the effect of microstructural disorder on the mechanical behavior becomes progressively less important at larger scales, whereas it represents the fundamental fracture at small scales. The expression of MFSL is :

$$\sigma_N=\left(A+\frac{B}{d}\right)^{0.5} \quad (15)$$

where σ_N is the nominal tensile strength, d is the characteristic structural size, and A and B are constants with the same units as the square of a stress and SIF, respectively. For determining MFSL parameters, the following linearization of Equation 15 can be more useful:

$$\sigma_N^2=A+B\frac{1}{d} \quad (16)$$

in which X=1/d and $Y=\sigma_N^2$. Hence, the MFSL is described by a linear function Y=A+BX in the X-Y plane, where A and B are the same parameters expressed in Equation 15.

3.4 *A comparative study on MFSL, MFM, and SEL*

Typical comparison of SEL, MFSL, and MFM are shown in Figures 9 and 10. For the different models used in this study, the correlation coefficients obtained on log σ_N versus log d curves are given in Table 1.

As shown in this table, MFLS gives the same or a slightly higher correlation coefficient than SEL and MFM. Based on Eser's test results, it can be concluded that the curve of MFSL is concave up on the interval considered. However, the curves of both SEL and MFM are concave down on the same

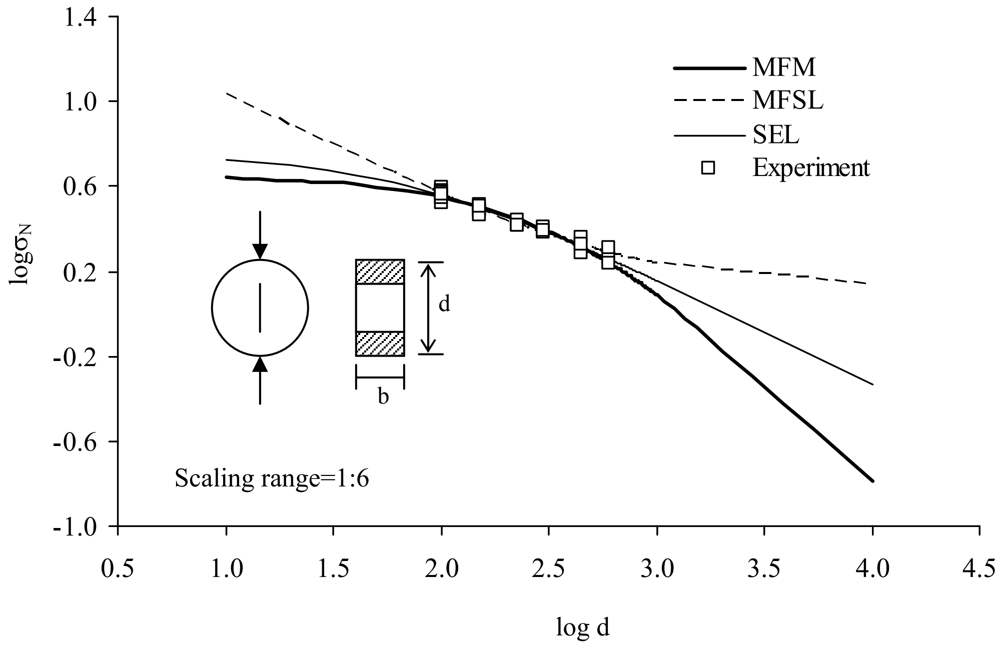

Figure 9. A typical comparison of MFM, MFSL, and SEL for 2a/d=0.1

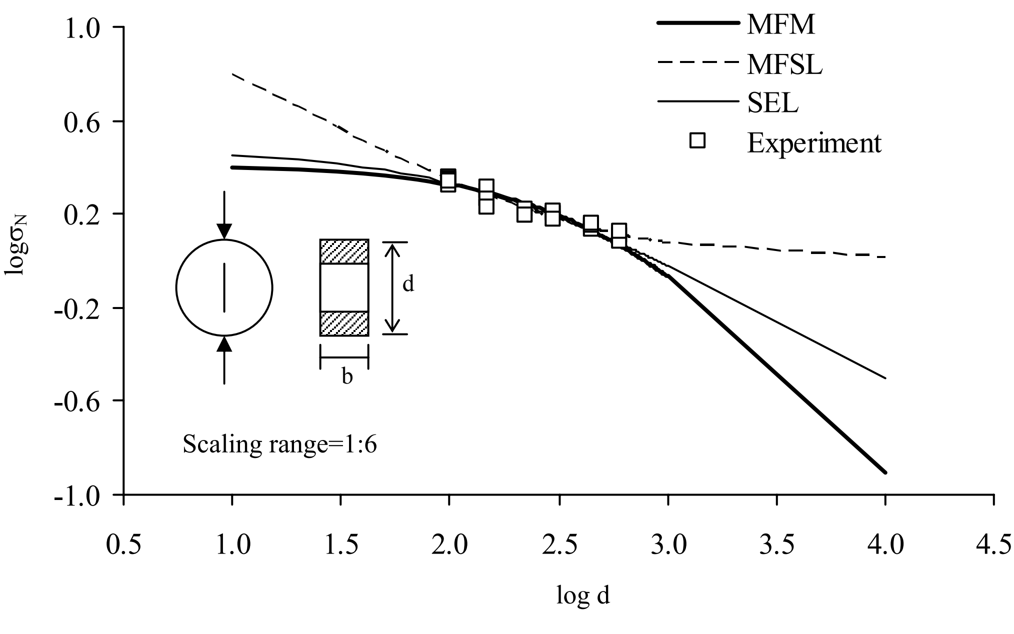

Figure 10. A typical comparison of MFM, MFSL, and SEL for 2a/d=0.5

interval (Eser, 2002). It can be concluded that these approaches give very high correlations for the dimensions of the laboratory size specimens.

As shown in Table 1, in one of the series experimentally obtained by Eser, the correlation coefficients in MFSL was slightly greater than those in the other models. In contrary, in another series obtained by Atahan, the correlation coefficient in MFSL was determined to be slightly less than the coefficients in the other models.

Table 1. Comparison of correlation coefficients for three different models

	Correlation Coefficient, (Eser, 2002)		
2a/d	0.1	0.3	0.5
SEL	0.97	0.91	0.95
MFM	0.96	0.88	0.93
MFSL	0.98	0.95	0.97
	(Atahan, 1996)		
SEL	0.97	0.98	0.97
MFM	0.96	0.98	0.97
MFSL	0.96	0.92	0.94

4 CONCLUSIONS

The following conclusions can be drawn from the investigation of the size effect on the fracture of centre notched concrete disc specimens in Mode I loading condition:

1) The splitting tensile test results of the notched disc specimens show a strong size effect during the failure. As the diameter of the specimen increases, the nominal strength decreases; the decrease is more dramatic on the disc specimens with small notch.

2) SEL, MFSL, and MFM represent a transition between the strength criterion and LEFM.

3) The splitting tensile strength of concrete is expressed by a force acting on a surface having fractal dimensions between 1 and 2; the fractal dimensions decrease, as the diameter of the disc increases. There is a linear relationship between the calculated ratios of the reciprocal of nominal tensile strength and the size of specimen.

4) The test results confirm the existence of the SEL, MFSL, and MFM.

5) A notched disc can be used as a valid fracture mechanics specimen with many advantages and future investigations of cracking in concrete may be carried out on this specimen with a strong potential for standardization.

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