Composite Damage Mechanics And Applications to Modeling of Degradation of Concrete

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ABSTRACT: A new theory of composite damage mechanics at the mesoscale level is developed. The mechanical behavior of a distressed composite is described by the combination of two different phases (matrix and inclusion), both of them are linear elastic isotropic materials. The matrix represents the original material without damage, and the inclusions represent the material with ultimate damage. The inclusion volume fraction is used as the variable to characterize the extent of the damage, instead of the conventional scalar damage parameter. The major difference from the scalar damage theory is that the elastic modulus of the inclusion is not zero, which allows various combinations of the two constituent phases, representing different forms of damage evolutions. Specifically, two models based on parallel and serial configurations are introduced. For example, one can simultaneously combine the parallel model representing stiffness of the composite and the serial model representing stress on the composite. Linear and exponential softening stress-strain curves are used to construct the damage models. The stress of distressed composite can be expressed as a function of the level of damage, and as a result, the upper and lower bounds for the stress can be obtained for a given level of damage.

Keyword: damage mechanics, composite mechanics, composite damage mechanics, concrete.

1. INTRODUCTION

The concept of continuum damage mechanics is based on the theory initially introduced by Kachanov in 1958 [Lemaitre, 1992 and Chaboche, 1999]. Kachanov's theory is described by *one scalar variable, d*, where $0 \le d \le 1$ (0 for undamaged state and 1 for failure). d is frequently termed the scalar damage variable (parameter). Degradation of the elastic properties can be described by this parameter.

After Kachanov, many researchers have used his initial concepts and developed many theories based on his one-scalar theory. Lubliner, et al. [1989] proposed a constitutive model based on an internal variable-formulation of plasticity theory. This model, which is frequently used for non-linear description of concrete, is now termed the Barcelona model [Lee & Fenves, 1999]. For cyclic loading applied to concrete, using the concept of fracture-energy-based damage similar to the Barcelona model and plastic-damage concepts [Simo & Ju, 1987], Lee & Fenves [1999] developed a damage model for concrete subjected to cyclic loading histories. Within the framework of scalar damage theory, Ozbolt & Ananiev [2003] considered different degradation mechanisms of concrete, such as linear and exponential degradations in the softening part of stress-strain responses.

In this paper, we deal with the damage process using composite mechanics rather than the traditional scalar damage variable, d. Using composite mechanics to deal with the effect of damage on various properties of distressed materials is called *composite damage theory* [Xi 2002]. The composite damage theory was first used to handle the effect of damage on transport properties of composite materials, such as diffusivity of concrete [Xi, 2002; Xi & Nakhi, 2003]. In this paper, we will use the composite damage theory to characterize the effect of damage on mechanical properties of distressed concrete. We call the theory as *composite damage mechanics*.

A distressed material can be considered as a two-phase composite material comprising of a linear elastic isotropic matrix and linear elastic isotropic inclusions. The matrix is considered as the original material without damage (i.e. the effective material of a composite), and the inclusion is the distressed material at the final stage (with ultimate damage). Both phases are linear elastic, but with different stiffnesses. The damage process is characterized by the variation of the volume fraction of the inclusion [Xi, 2002; Xi & Nakhi, 2003]. The major difference between the conventional scalar damage mechanics and the composite damage mechanics is the stiffness of the inclusion. In scalar damage mechanics, the stiffness of the inclusion is zero (cannot hold any load), while in the composite damage mechanics, the stiffness of the inclusion is not zero, and furthermore, the properties of the inclusion are linear elastic. According to the scalar damage mechanics, when damage takes place, the stiffness of some of original material changes to zero, while in the composite damage mechanics, some of the matrix (the original material) changes to the inclusion with a reduced stiffness (non-zero). In this sense, the scalar damage mechanics can be considered as a special case of the composite damage mechanics.

There are several advantages of using the composite damage mechanics:

(1) Application of composite mechanics. All available elastic composite theories can now be readily used to deal with damage process in materials.

(2) Multi-phase theory. The two-phase composite damage mechanics can be further generalized into multi-phase theory [Xi 2002]. A distressed material can be described by a multi-phase composite with one phase as the original material and the other phases as damaged materials of different levels of damage. Note that, the damaged phases remain linear elastic and their elastic properties remain constants. As a result, there will be more than one volume fraction of the damaged phases that can be used to characterize the damage process.

(3) Softening and hardening. When the stiffness of the inclusions is considered to be lower than the stiffness of the matrix, we characterize the process of softening. On the other hand, we can also consider hardening process by using inclusions with stiffness higher than that of the matrix.

(4) Plasticity. When we consider that the matrix and the inclusion have the same strength but different moduli of elasticity, the composite damage mechanics can be used to describe elastoplastic behavior of materials (see Section 5 for more explanations).

(5) Morphology of damage. The constituent phases with damage can be distributed in many different ways to characterize the morphology of the damage distribution, while in the scalar damage theory, the original phase and the damaged phase can only be arranged parallel to the loading direction (the so-called parallel coupling or iso-strain model). Serial model (or iso-stress model) is not valid, because of the zero-stiffness of the damaged phase.

In order to show some of the advantages of the composite damage mechanics, both parallel and serial models will be used in the present paper. Since the overall stress and, for example, stiffness of the constituent phases are not mathematically related, we can use either the parallel or serial model for stress, and the other for stiffness.

In this paper, we consider two different degradation mechanisms, i.e. the linear and exponential degradations of hardening/softening of stress-strain response. As a special case, our models reduces to the scalar damage models obtained by Ozbolt & Ananiev [2003], if we reduce the stiffness of the inclusions from non-zero values to zero.

2. VOLUME FRACTION AND SURFACE FRACTION OF EACH PHASE

The entire region of solid with volume V contains volume V_1 for Phase 1 and volume V_2 for Phase 2. $V_1 + V_2 = V$. The volume fraction V_i / V, (i=1, 2) is defined as c_i and is termed the matrix volume fraction if i=1, and the inclusion volume fraction, if i=2.

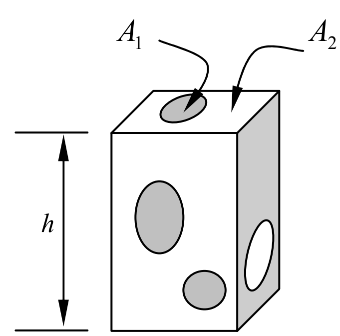

Fig. 1 Definition of inclusion surface fraction

It is assumed that at any cross section of the entire solid both phases are present (Fig. 1). If the area of the cross section is A, the area related to Phase 1 is denoted by A_1 and the area related to Phase 2 by A_2. The surface fraction A_i / A, (i=1, 2) is denoted as a_i and termed the matrix surface fraction, if i=1, and inclusion surface fraction, if i=2. It is assumed that at any cross section normal to h (see Fig. 1) a_i is constant, which means that the surface fraction of each phase is constant. In view of these considerations, it cannot be inferred that the position of each phase is fixed in the cross section. In addition, it does not mean that the distributions of the phases are constant. By this assumption, based on Fig. 1, one can write:

$$A_1 h + A_2 h = V = Ah \tag{1}$$

where V is the volume of the representative volume element. From this equation and the definitions given above, we infer

$$a_1 + a_2 = 1, \quad a_1 = c_1, \quad a_2 = c_2 \tag{2}$$

3. EQUILIBRIUM

Based on the definition of the surface fractions of the phases, the Cauchy equation of surface traction has the following form

$$\boldsymbol{Tn} = \boldsymbol{t} \tag{3}$$

Where

$$T_{ij} = (1-c_2) T_{ij}^{(1)} + c_2 T_{ij}^{(2)} \tag{4a}$$

$$t_i = (1-c_2) t_i^{(1)} + c_2 t_i^{(2)} \tag{4b}$$

In these equations, $\boldsymbol{T}^{(1)}$ and $\boldsymbol{T}^{(2)}$ are the symmetric stress tensors belonging to Phase 1 and Phase 2, respectively, whose components are $T_{ij}^{(1)}$ and $T_{ij}^{(2)}$, respectively. $\boldsymbol{n}$ is the unit tensor of the surface area with stress vector $\boldsymbol{t}$, whose components are t_i. The indices i and j vary from 1 through 3. Eqs. (4a) and (4b) show the parallel combination of the stress tensors and stress vectors. One can use any other combination of the tensors and vectors. In this paper, we also define the serial combination of the tensors and vectors as follows

$$T_{ij} = \left[\frac{(1-c_2)}{T_{ij}^{(1)}} + \frac{c_2}{T_{ij}^{(2)}}\right]^{-1} \tag{5a}$$

$$t_i = \left[\frac{(1-c_2)}{t_i^{(1)}} + \frac{c_2}{t_i^{(2)}}\right]^{-1} \tag{5b}$$

To support Eqs. (5a) and (5b) to be physically acceptable, consider, for example, a simple case of tensile stress due to a tensile force, say f, applying on the surface of a composite. In Section 2, we denote the current area of the surface by A and the area of its constituents by A_1 and A_2, and we denote the fraction of force f applied on Phases 1 and 2 by f_1 and f_2

$$f_1 = (1-c_2) f \tag{6a}$$

$$f_2 = c_2 f \tag{6b}$$

Then the total tensile stress, T, is $T = f/A$, which implies

$$1/T = A_1/f + A_2/f \tag{7}$$

Substituting Eqs. (6a) and (6b) into Eq. (7), we have

$$1/T = (1-c_2)(A_1/f_1) + c_2 (A_2/f_2) \tag{8}$$

in which, by defining $T_1 = f_1/A_1$ and $T_2 = f_2/A_2$, one can obtain Eqs. (5a) and (5b).

In the same manner, the equation of equilibrium at a material point is written as

$$T_{ij,j} + \rho b_i = \rho \ddot{u}_i \tag{9}$$

Where ρ is mass density; $\boldsymbol{u}$ is the displacement vector of the material point. The symbol "," in the subscript denotes the derivative with respect to the spatial coordinate, while the superscript "." is used for the time derivative. In the parallel combination

$$\rho = (1-c_2)\rho^{(1)} + c_2 \rho^{(2)} \tag{10a}$$

$$b_i = (1-c_2) b_i^{(1)} + c_2 b_i^{(2)} \tag{10b}$$

and in the serial system

$$\rho = [(1-c_2)\rho^{(1)^{-1}} + c_2 \rho^{(2)^{-1}}]^{-1} \tag{11a}$$

$$b_i = [(1-c_2) b_i^{(1)^{-1}} + c_2 b_i^{(2)^{-1}}]^{-1} \tag{11b}$$

The same argument as given in the paragraph right after Eqs. (5a) and (5b) is valid here for the mass density if we keep in mind that the inclusions originally are a part of the matrix with the same density. If M_1 and M_2 are the mass of Phase 1 and Phase 2, then they equal $(1-c_2)M$ and $c_2 M$, where M is the total mass. Equation (11b) is a direct consequent of (11a).

4. CONSTITUTIVE LAW

The basic assumptions used to develop the response function of the material at the mesoscale level are

(1) The response of the material in each phase depends only on the current configuration of the phases, so does the entire material at the mesoscale level.

(2) We assume that the behavior of each phase follows Green-elasticity, which means there exists a strain energy function for the material of each phase, so does the entire material.

Assumption (1) means that the behavior considered here is limited to a material without memory. It may happen that one phase of the entire body contains matter in such a way that its elasticity tensor is zero. In this case, the strain energy function for that material is always zero.

Based on these assumptions, by changing time, the properties of one phase may change to another. When damages are present in the region, we characterize the damaged region by Phase 2, and in this case, Phase 2 has a degraded elasticity tensor. Thus, based on assumption (2), the strain energy function in Phase 2 is identically zero. The damage can be characterized by changing Phase 1 (the material with original elasticity tensor) to

Phase 2 (with degraded elasticity tensor). When the entire Phase 1 changes to Phase 2, we have a state of complete damage.

It follows from assumption (2) that, there exists a free energy potential for the material point at the mesoscale level as a function of the strain level and composite variable c_2

$$\psi = \psi(\varepsilon_{ij}, c_2) \tag{12}$$

Based on Coleman & Gurtin [1967], c_2 is an internal variable. Following Coleman & Noll [1964] and Coleman & Gurtin [1967], and based on Clausius-Duhem's inequality (the Second Law of Thermodynamics) we have

$$T_{ij} = \frac{\partial \psi}{\partial \varepsilon_{ij}} \tag{13}$$

and

$$-\frac{\partial \psi}{\partial c_2}\dot{c}_2 \geq 0 \tag{14}$$

Using Taylor series expansion of the free energy about $\varepsilon_{ij} = 0$ and a linear variation with respect to c_2, and knowing that the energy in the natural state is zero, one can write (12) as

$$\psi(\varepsilon_{ij}, c_2) = \frac{1}{2}(1 - c_2)C^{(1)}_{ijkl}\varepsilon_{ij}\varepsilon_{kl} + \frac{1}{2}c_2 C^{(2)}_{ijkl}\varepsilon_{ij}\varepsilon_{kl} \tag{15}$$

Where the higher terms have been neglected. $C^{(1)}_{ijkl}$ and $C^{(2)}_{ijkl}$ are fourth-order elasticity tensors of Phase 1 and Phase 2, respectively. When the materials in Phase 1 and Phase 2 are isotropic, $C^{(1)}_{ijkl}$ and $C^{(2)}_{ijkl}$ are

$$C^{(m)}_{ijkl} = \lambda^{(m)}\delta_{ij}\delta_{kl} + \frac{\mu^{(m)}}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \tag{16}$$

Where m varies from 1 to 2, and $\lambda^{(m)}$ and $\mu^{(m)}$ are Lame's constants.

In the following sections, we consider different hardening and softening situations. The hardening and softening rules are taken from the uniaxial test data [Ozbolt & Ananiev, 2003]. The hardening and softening rules are understood here as dependence on tensile/compressive stress and inclusion (or matrix) volume fraction. Following Ozbolt & Ananiev [2003], we introduce different degradation mechanisms for the tensile and compressive part of the stress-strain response curves under uniaxial loading.

5. ONE DIMENSIONAL LINEAR DEGRADATION

As a first approximation, we consider a case for one dimensional linear degradation. As shown in Fig. 2, if $f_t^{(1)}$ and $f_t^{(2)}$ are the tensile strength of the materials in Phases 1 and 2, respectively, then a linear degradation is selected to use for prediction of the tensile strength of the entire material. With replacement of $f_t^{(1)}$ and $f_t^{(2)}$ by $f_c^{(1)}$ and $f_c^{(2)}$, we will find the linear degradation for compressive part. $\varepsilon_E^{(1)}$ and $\varepsilon_E^{(2)}$ are the strains in Phases 1 and 2 at the tensile strength limit in such a way that $0 < \varepsilon_E^{(1)} < \varepsilon_E^{(2)}$. For the compressive case we have $\varepsilon_E^{(2)} < \varepsilon_E^{(1)} < 0$. By these definitions, if $f_t^{(1)}/\varepsilon_E^{(1)} \succ f_t^{(2)}/\varepsilon_E^{(2)}$, we have softening, and if $f_t^{(1)}/\varepsilon_E^{(1)} \prec f_t^{(2)}/\varepsilon_E^{(2)}$, we have hardening. In the case of $f_t^{(1)}/\varepsilon_E^{(1)} = f_t^{(2)}/\varepsilon_E^{(2)}$, the two phases contain materials with the same properties.

By defining fracture energy G_F^* as

$$G_F^* = \frac{f_t^{(1)} + f_t^{(2)}}{2}(\varepsilon_E^{(2)} - \varepsilon_E^{(1)}) \tag{17}$$

we can find

$$\frac{f_t(c_2)}{E(c_2)} = \frac{2G_F^*}{{f_t^{(1)}}^2 - {f_t^{(2)}}^2}\left[f - f_t(c_2)\right] \tag{18}$$

where

$$f = f_t^{(2)} + \frac{{f_t^{(1)}}^2 - {f_t^{(2)}}^2}{f_t^{(1)} + f_t^{(2)}} + \frac{{f_t^{(1)}}^2 - {f_t^{(2)}}^2}{2G_F^*}\varepsilon_E^{(1)} \tag{19}$$

If Phase 2 has zero stiffness, then $f_t^{(2)} = 0$, and c_2 is equal to the damage parameter d. In this case, we have the scalar damage theory, and thus Eq. (18) coincides with Eq. (2) of Ozbolt & Ananiev [2003].

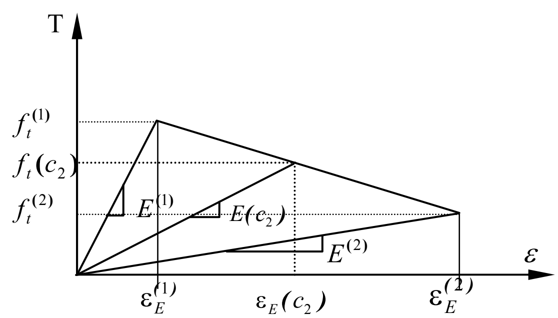

Fig. 2 Linear degradation of tensile stress.

By rearranging (18), we find the explicit formulation for $f_t(c_2)$ in terms of other parameters

$$f_t(c_2) = \frac{2G_F^* E(c_2)}{{f_t^{(1)}}^2 - {f_t^{(2)}}^2 + 2G_F^* E(c_2)} f \tag{20}$$

If we use a parallel model to characterize the stiffness modulus in uniaxial loading, then the Young's modulus for the entire volume representative element is

$$E(c_2) = (1-c_2)E^{(1)} + c_2 E^{(2)} \tag{21}$$

which shows a linear variation of the uniaxial stiffness modulus in terms of the inclusion volume fraction. By substituting (21) into (20), we have

$$f_t(c_2) = \frac{2G_F^*[(1-c_2)E^{(1)} + c_2 E^{(2)}]}{f_t^{(1)^2} - f_t^{(2)^2} + 2G_F^*[(1-c_2)E^{(1)} + c_2 E^{(2)}]} f \tag{22}$$

One can see that, the tensile strength of the composite, $f_t(c_2)$, is a nonlinear function of c_2. In the case of the serial model for the stiffness modulus in uniaxial loading, the Young's modulus for the entire volume representative element is

$$E(c_2) = \left[\frac{(1-c_2)}{E^{(1)}} + \frac{c_2}{E^{(2)}} \right]^{-1} \tag{23}$$

In this case, $E(c_2)$ is not a linear function of c_2. Substituting (23) into (20), gives

$$f_t(c_2) = \frac{2G_F^* E^{(1)} E^{(2)}}{[(1-c_2)E^{(2)} + c_2 E^{(1)}][f_t^{(1)^2} - f_t^{(2)^2}] + 2G_F^* E^{(1)} E^{(2)}} f \tag{24}$$

It is very clear from Eq. 24 that if we reduce the stiffness of any one of the two phases to zero, then the strength of the entire composite is identically equal to zero, which means that the serial model cannot be used in scalar damage theory. Physically, this should be evident to readers.

In Eq. (22), if $f_t^{(1)} = f_t^{(2)}$, then f_t (c_2) = a constant ($f_t^{(1)}$ or $f_t^{(2)}$), which means the softening part in Fig. 2 is a horizontal line. In this way, the model developed here can be used for elastoplastic degradation.

Fig. 3 shows the two different tensile strengths of the composite, $f_t(c_2)$, when parallel and serial models are used for the stiffness. The parameters used in Fig. 3 are $E^{(1)} = 10000$, $f_t^{(1)} = 10$, $E^{(2)} = 100$, and $f_t^{(2)} = 1$. For the same volume fraction of damage, if the serial model is used for the stiffness of the composite, then the tensile strength of the composite degrades much faster than the case of the parallel model. On the other hand, for the same level of stress, the inclusion volume fraction (the damaged phase) is much higher for the parallel model than that for the serial model.

In fact, since the parallel model is the upper bound and the serial model is the lower bound for the stiffness of the composite, the two curves in Fig. 3 define the upper bound and lower bounds for the volume fraction of the damaged phase when a level of stress is given. On the other hand, when the volume fraction of the damaged phase is given, the two curves define the upper and lower bounds for the load carrying capacity of the distressed composite.

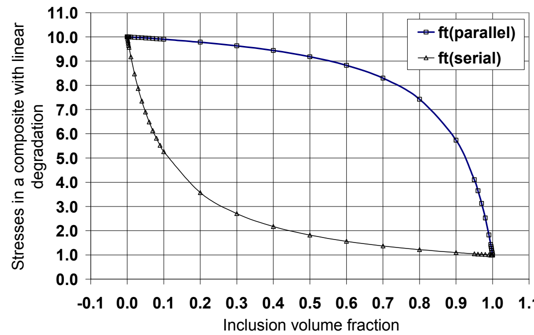

Fig. 3 Variation of the tensile stress of the composite in terms of inclusion volume fraction, when the parallel and serial models are used for the stiffness of the composite.

6. ONE DIMENSIONAL EXPONENTIAL DEGRADATION

Next, we consider an exponential degradation (Fig. 4) of the tensile/compressive stress, which is frequently used in the literature. The same notation described in the previous section is used here. We consider the case, which only softening is involved. Thus, $f_t^{(1)} / \varepsilon_E^{(1)} \succ f_t^{(2)} / \varepsilon_E^{(2)}$. The exponential function connecting points $(\varepsilon_E^{(1)}, f_t^{(1)})$ and $(\varepsilon_E^{(2)}, f_t^{(2)})$ in a one dimensional strain-stress space is

$$f_t(c_2) = f_t^{(1)} \exp[\alpha(\varepsilon_E(c_2) - \varepsilon_E^{(1)})] \tag{25}$$

where

$$\alpha = \frac{Ln(f_t^{(2)}) - Ln(f_t^{(1)})}{\varepsilon_E^{(2)} - \varepsilon_E^{(1)}} \tag{26}$$

By obtaining the area under the softening stress-strain response curve, which is the area under stress-strain response curve between $\varepsilon_E^{(1)}$ and $\varepsilon_E^{(2)}$, we find G_F^* as

$$G_F^* = \frac{f_t^{(1)}}{\alpha} \left(\frac{f_t^{(2)}}{f_t^{(1)}} - 1 \right) \tag{27}$$

In the case of presence of zero stiffness of Phase 2, $f_t^{(2)}$ is zero and G_F^* is related to the fracture energy. In this case, finding α from (26) and substituting it into (25), the strength at any level of strain is

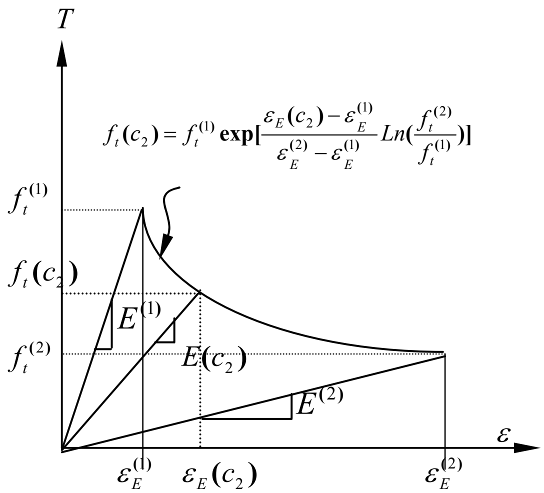

Fig. 4 Exponential degradation of tensile stress

$$f_t(c_2) = f_t^{(1)} \exp\left(-\frac{f_t^{(1)}}{G_F^*}(\varepsilon_E(c_2) - \varepsilon_E^{(1)}) \right) \qquad (28)$$

By defining $E(c_2) = f_t(c_2)/\varepsilon_E(c_2)$, and substituting $\varepsilon_E(c_2)$ from this definition into (25) and (26), an implicit function is obtained for predicting f_t in terms of c_2

$$\Theta(f_t, c_2) = f_t - f_t^{(1)} \exp[\frac{f_t^{(1)}}{G_F^*}\left(\frac{f_t^{(2)}}{f_t^{(1)}} - 1\right)(\frac{f_t}{E(c_2)} - \varepsilon_E^{(1)})] = 0 \qquad (29)$$

Depending on the parallel or serial model for the stiffness in uniaxial loading (equation (21) or (23)), equation (29) has a different solution for f_t. In each case, this solution can be evaluated numerically using a standard Newton-Raphson technique.

7. CONCLUSIONS

Based on composite mechanics, a new theory of composite damage mechanics has been developed. The composite is considered to be made of two different phases (called matrix and inclusion). Each phase is assumed to be a linear elastic isotropic material. The matrix is considered as the intact material, and the inclusion is the damaged material. The inclusion remains linear elastic during the damage process, therefore, the inclusion volume fraction is the one that characterizes the damage evolution, playing the same role as the conventional scalar damage parameter.

Since the stiffness of the inclusion is not zero, the effective stiffness of the distressed composite can be described by different models, i.e. parallel and serial models. This enable us to describe a variety of properties of the composite. For example, we can use parallel combination of the body force and serial combination of tractions at the same time. Any other combinations (models) that satisfy the limiting conditions can also be used.

Two different degradations, i.e. the linear and exponential degradations of hardening/softening of stress-strain response curve have been introduced. The stress of distressed composite is expressed as a function of the volume fraction of the damaged phase (the inclusion).

Based on the two degradation mechanisms (i.e. the linear and exponential degradations) and the two models used for stiffness of the composite (i.e. parallel and serial models), the upper and lower bounds of the stress of distressed composite are obtained, for a given level of damage (the volume fraction of the inclusion). Furthermore, the upper and lower bounds for the damage level are also obtained for a given level of stress.

8. ACKNOWLEDGEMENT

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