Rate dependent interface model formulation for quasi-brittle materials

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ABSTRACT: a rate-dependent constitutive formulation for interface is presented. The model will be considered for the analysis of concrete failure processes at the mesomechanic level of observation. The interface elastoviscoplatic formulation is based on a continuum Perzyna-type extension of the rate independent interface model by Carol and Lopez (1999). The generalized viscoplastic consistent condition for the interface is obtained which leads to the explicit formulation of the continuum tangent operator and, subsequently, to the algorithmic tangent tensor when the algebraic problem is considered.

Keywords: interface, rate dependent, viscoplasticity, brittle failure.

1 CONTINUOUS PERZYNA RATE DEPENDENT FORMULATION

Similar to the flow theory of plasticity, the constitutive relations of Perzyna (1963, 1966) type elasto - viscoplastic material formulations may be written

$$\dot{\boldsymbol{\sigma}} = \dot{\boldsymbol{\sigma}}_e - \dot{\boldsymbol{\sigma}}_{vp} = \mathbf{E} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_{vp}) \qquad (1)$$

$$\boldsymbol{\varepsilon}_{vp} = \mathbf{g}(\psi, F, \boldsymbol{\sigma}) = \frac{1}{\eta} \langle \psi(F) \rangle \mathbf{m} \qquad (2)$$

$$\mathbf{m} = \mathbf{A} : \mathbf{n} = \mathbf{A} : \frac{\partial F}{\partial \boldsymbol{\sigma}} \qquad (3)$$

$$\psi(F) = \left[\frac{F(\boldsymbol{\sigma}, \mathbf{q})}{F_0} \right]^N \qquad (4)$$

$$\dot{\mathbf{q}} = \frac{1}{\eta} \langle \psi(F) \rangle \mathbf{H} : \mathbf{m} \qquad (5)$$

where $\boldsymbol{\varepsilon}_{vp}$ represents the viscoplastic portion of the total strain tensor $\boldsymbol{\varepsilon}$, η the viscosity and **q** the set of hardening/softening variables defined as a tensor of arbitrary order. The relation 1 follows the additive decomposition of the total strain rate into an elastic and a viscoplastic part $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_e + \boldsymbol{\varepsilon}_{vp}$, quite similar to the Prandtl-Reuss equations in case of inviscid elasto-plastic constitutive relations. Equations 2 and 3 describe a general non-associated flow rule, whereby the direction of the viscoplastic strains **m**, is obtained by a modification of the gradient tensor **n** of the yield surface F by means of the fourth order transformation tensor **A**. Moreover, $\psi(F)$ is a dimensionless monotonically increasing over-stress function whereby F_0 represents a normalizing factor. The power N in equation 4 defines the order of the Perzyna viscoplaticity while the McCauley brackets in equation 2 defines the features of the over--stress function as

$$\langle \psi(F) \rangle = \begin{cases} F & \text{if } F > 0 \\ 0 & \text{if } F \le 0 \end{cases} \qquad (6)$$

being $F = F(\boldsymbol{\sigma}, \mathbf{q})$ a convex yield function which defines the limit of the elastic domain. Finally equation 5 represents the evolution law of the hardening/softening variables **q** by means of a suitable tensor function **H** of the state variables. In the continuous formulation, equations 1 to 5 are complemented by a consistency parameter $\dot{\lambda}$, see Ponthot (1995), defined as an increasing function of the over-stress

$$\dot\lambda = \frac{\langle \psi(F) \rangle}{\eta} \tag{7}$$

So that the evolution equations 2 and 5 take now the classical forms

$$\boldsymbol{\varepsilon}_{vp} = \dot\lambda \mathbf{m} \tag{8}$$

$$\dot{\mathbf{q}} = \dot\lambda \mathbf{H} : \mathbf{m} = \dot\lambda \mathbf{h} \tag{9}$$

being $\mathbf{h} = \mathbf{H}:\mathbf{m}$. Thus, from equations 2 and 8 follows

$$F = \psi^{-1}\left(\frac{\|\boldsymbol{\varepsilon}_{vp}\|}{\|\mathbf{m}\|} \eta \right) = \psi^{-1}(\dot\lambda \eta) \tag{10}$$

We may now define for the viscoplastic range, the new constraint condition

$$\bar F = F - \psi^{-1}(\dot\lambda \eta) = 0 \tag{11}$$

which represents a generalization of the inviscid yield condition *F=0* for rate-dependent Perzyna viscoplastic materials. The name *continuous formulation* is due to the fact that the condition η =0 (no viscosity effect) leads to the elastoplastic yield condition *F=0*. Moreover, from equation 7 follows that when $\eta \to 0$ the consistency parameter remains finite and positive since also the over-stress goes to zero. The other extreme case, $\eta \to \infty$ leads to the inequality $\bar F < 0$ for every possible stress state, indicating that only elastic response may be activated.
The constraint defined by equation 11 allows a generalization of the Kuhn-Tucker conditions which may be now written as

$$\dot\lambda \bar F = 0 \qquad \dot\lambda \geq 0 \qquad \bar F \leq 0 \tag{12}$$

Finally, the viscoplastic consistency condition expands into

$$\dot{\bar F} = \mathbf{n} : \dot{\boldsymbol\sigma} + \bar{\mathbf{r}} \dot{\mathbf{q}} - \frac{\partial \psi^{-1}(\dot\lambda\eta)}{\partial \lambda} \ddot\lambda = 0 \tag{13}$$

where

$$\bar{\mathbf{r}} = \frac{\partial \bar F}{\partial \mathbf{q}} = \frac{\partial F}{\partial \mathbf{q}} - \frac{\partial \psi^{-1}(\dot\lambda\eta)}{\partial \mathbf{q}} \tag{14}$$

Other recent and interesting approach to this problem is due to Wang (see Wang et al., 1997), which includes the strain rate as state variable into the flow and viscoplastic potential function, i.e.

$$F^{vp} = F^{vp}(\boldsymbol\sigma, \mathbf{q}, \boldsymbol\varepsilon) \tag{15}$$

this also leads to a rate dependent Kuhn-Tucker conditions as in case of the continuous Perzyna formulation.

2 CONSISTENT TANGENT STIFFNESS OPERATOR

The algorithmic tangent operator can be formulated from the linearization of the viscoplastic consistency condition, see equation 13 , for a finite increment *d*, quite similar to rate independent plasticity,

$$d\bar F = \mathbf{n} : d\boldsymbol\sigma + \bar{\mathbf{r}} : d\mathbf{q} - \frac{\partial \psi^{-1}(\dot\lambda\eta)}{\partial \lambda} d\dot\lambda = 0 \tag{16}$$

In order to avoid further complications, it is supposed here that $\dot\lambda$ is accurately approximated by $\dot\lambda = \Delta\lambda / \Delta t$, i.e. $\Delta\lambda = \Delta t \langle \psi(F) \rangle / \eta$, which leads to $d\dot\lambda = d\Delta\lambda / \Delta t$ .The consequences of this assumption are analyzed in other work of the authors (Carosio et al., 2000).
Proceeding in a similar form to the algebraic elastoplastic problem, i.e. substituting in equation 16 the differential changes of the stress tensor and of the state variables evaluated in a consistent form with the backward Euler scheme

$$d\boldsymbol\sigma = \mathbf{E}^m : (d\boldsymbol\varepsilon - d\Delta\lambda \mathbf{m}) \tag{17}$$

$$d\mathbf{q} = d\Delta\lambda \mathbf{h} + \Delta\lambda \mathbf{p} : \mathbf{E}^m (d\boldsymbol\varepsilon - d\Delta\lambda \mathbf{m}) \tag{18}$$

where

$$\left[\mathbf{E}^m \right]^{-1} = (\mathbf{E}^{-1} + \Delta\lambda \mathbf{M}) \tag{19}$$

$$\mathbf{p} = \frac{\partial \mathbf{h}}{\partial \boldsymbol\sigma} \tag{20}$$

we obtain the relations $d\boldsymbol\sigma = \left[\mathbf{E}^{Per}_{alg} \right]^{cont} : d\boldsymbol\varepsilon$, with the algorithmic operator

$$\left[\mathbf{E}^{Per}_{alg} \right]^{cont} = \mathbf{E}^m - \frac{\bar{\mathbf{m}} \otimes \bar{\mathbf{n}} + \Delta\lambda \bar{\mathbf{r}} : \bar{\mathbf{m}} \otimes \bar{\mathbf{p}}}{\bar E^m_m + \Delta\lambda E^m_p + E_i} \tag{21}$$

where $\bar{\mathbf{m}} = \mathbf{E}^m : \mathbf{m}$, $\bar{\mathbf{n}} = \mathbf{n} : \mathbf{E}^m$, $\bar{\mathbf{p}} = \mathbf{p} : \mathbf{E}^m$ and the scalar values $\bar{E}^m_m$, E^m_p and E_i defined as

$$\bar{E}^m_m = \mathbf{n} : \mathbf{E}^m : \mathbf{m} - \bar{\mathbf{r}} : \mathbf{h} \tag{22}$$

$$E^m_p = \bar{\mathbf{r}} : \mathbf{p} : \mathbf{E}^m : \mathbf{m} \tag{23}$$

$$E_i = \frac{1}{\Delta t}\frac{\partial \psi^{-1}}{\partial \dot{\lambda}} \tag{24}$$

The last three equations are similar to the elastoplastic case.

Note: equation 18 is valid for every possible order n of the tensor $\mathbf{q}$ of state variables. From equations 9 and 20 follow that the order of the tensor $\mathbf{h}$ is equal to that of $\mathbf{q}$, i.e. n, while the order of $\mathbf{p}$ is *n+2*.

3 TIME - DEPENDENT INTERFACE MODEL FORMULATION

In this section the rate-dependent extension of the interface model by Carol and Lopez (1999) is presented. The viscoplastic yield condition of the interface constitutive model can be expressed as

$$\bar{F} = \sigma_n^2 - (c - \tau tg\phi)^2 + (c - \chi tg\phi)^2 - (\dot{\lambda}\eta)^{1/N} \tag{25}$$

where σ_n and τ are the normal and tangential stress components to the interface with

$\chi =$ traction strength (vertex of hyperbola)
$c =$ cohesion or shear strength
$\phi =$ friction angle

The energy dissipated during the fracture process is defined as

$$dW^{vcr} = \sigma_n du_n^{vcr} + \tau du_t^{vcr} \quad \text{if } \sigma_n \geq 0 \tag{26}$$

$$dW^{vcr} = \tau du_t^{vcr}\left(1\text{-}\left|\frac{\sigma_n \text{tg}\phi}{\tau}\right|\right) \quad \text{if } \sigma < 0 \tag{27}$$

whereby u_n^{vcr} and u_t^{vcr} are the normal and tangential (critical) rate-dependent rupture displacements, respectively.

The viscoplastic flow is fully associated in tension while non-associated in compression, according to

$$\mathbf{m} = \mathbf{A} : \mathbf{n} \tag{28}$$

$$\mathbf{n} = \frac{\partial F}{\partial \boldsymbol{\sigma}} = \begin{bmatrix} \dfrac{\partial F}{\partial \sigma_n} \\ \dfrac{\partial F}{\partial \tau} \end{bmatrix} = \begin{bmatrix} 2tg\phi(c - \sigma tg\phi) \\ 2\tau \end{bmatrix} \tag{29}$$

$$\mathbf{m} = \frac{\partial Q}{\partial \boldsymbol{\sigma}} \qquad \mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ \text{if } \sigma > 0 \tag{30}$$

and

$$\mathbf{A} = \begin{bmatrix} f_\sigma^{dil} f_c^{dil} & 0 \\ 0 & 1 \end{bmatrix} \ \text{if } \sigma < 0 \tag{31}$$

being $\mathbf{A}$ a transformation matrix, $\mathbf{n}$ the gradient to the viscoplastic yield surface and $\mathbf{m}$ the gradient to the viscoplastic potential function. The factors f_c^{dil} and f_σ^{dil} account for the dilatancy effects in the compressive regime by means of a reduction of the interface normal component of the stress tensor, see Carol and Lopez (1999).

The continuum viscoplasticity form of the rate dependent interface constitutive model is defined by the following equations

$$\dot{\mathbf{u}} = \dot{\mathbf{u}}^e + \dot{\mathbf{u}}^{vcr} \qquad \dot{\mathbf{u}}^e = \left(\mathbf{K}^0\right)^{-1}\dot{\boldsymbol{\sigma}} \tag{32}$$

$$\dot{\boldsymbol{\sigma}} = \mathbf{K}^0(\mathbf{u} - \mathbf{u}^{vcr}) \tag{33}$$

where $\dot{\mathbf{u}}$ are the rate of total relative displacements which are decomposed into an elastic $\dot{\mathbf{u}}^e$ and a viscoplastic component $\dot{\mathbf{u}}^{vcr}$, ($\mathbf{K}^0$) is the elastic stiffness matrix which has a diagonal structure with non-zero terms equal to the constant assumed normal and shear stiffnesses $K_n^0 = K_t^0$.

The viscoplastic consistency condition, from which the algorithmic tangent operator of the rate-dependent interface model is obtained, takes now the form

$$\dot{\bar{F}} = \mathbf{n} \cdot \dot{\boldsymbol{\sigma}} - \bar{r}_i \dot{q}_i - \frac{1}{N}\eta(\eta\dot{\lambda})^{1/N-1} \tag{34}$$

$$\bar{r}_i = \frac{\partial F}{\partial q_i}\frac{\partial q_i}{\partial W^{vcr}} \tag{35}$$

$$\dot{q}_i = \frac{\partial W^{vcr}}{\partial \mathbf{u}^{vcr}}\mathbf{m}\dot{\lambda} \tag{36}$$

thereby, the parameters q_i of the yield surface, which evolve with hardening/softening, are three in this case: χ , c and ϕ .

4 NUMERICAL ANALYSIS

In this section the predictions of the proposed rate-dependent interface model of the tensile and shear tests are analyzed. Figure 1 illustrates the performance of the uniaxial tensile test with different relations Δt. For this test the mesh indicated in the Figure 1 was used which is composed by 2 *Q4* elements and one interface element in between them. This test was performed with the Perzyna exponent N =1 and the ratio E_c/E_j = 0.25 between the stiffness of the continuum and that of the joint or interface element. The results in Figure 1 demonstrate that with increasing viscosity the ductility of the post-peak regime increase and elastic solution is approached.

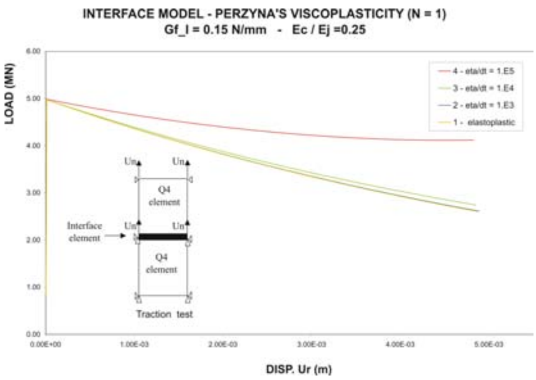

Figure 1. Uniaxial tensile test. Viscoplastic predictions for Perzyna exponent N = 1.0.

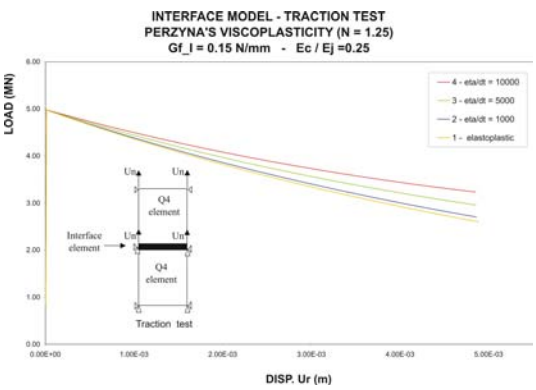

Figure 2. Uniaxial tensile test. Viscoplastic predictions for Perzyna exponent N = 1.25.

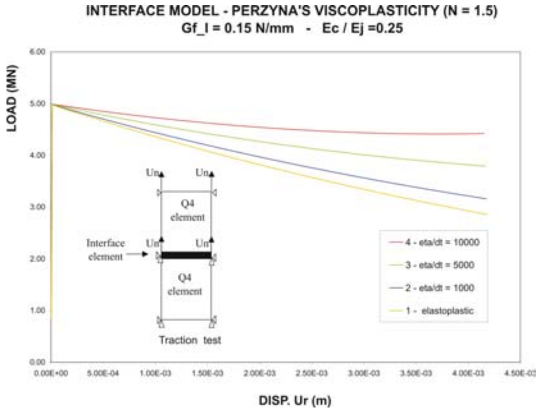

Figure 3. Uniaxial tensile test. Viscoplastic predictions for Perzyna exponent N = 1.5.

The relevant influence of the Perzyna exponent N can be observed from the comparison between the results in Figures 2, 3 and 4. There the predictions of the rate-dependent interface model for the tensile test with N = 1, N = 1.25 and N = 1.5 are illustrated for different relations η/Δt. With increasing values of N the influence of the viscosity (the relation η/Δt) in the model predictions becomes more relevant.

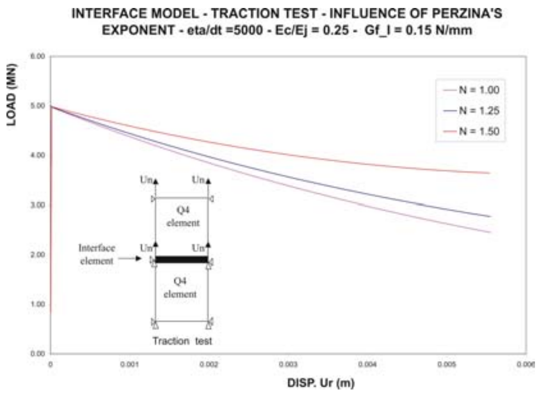

Figure 4. Uniaxial tensile test. Predictions for different Perzyna exponents .

Finally, in Figure 5 the predictions of the model for the shear test is indicated. This test was performed under a vertical confinement pressure of $2MPa$. The results illustrate the rate-dependency of the model and the increment of ductility with increasing rates η/Δt .

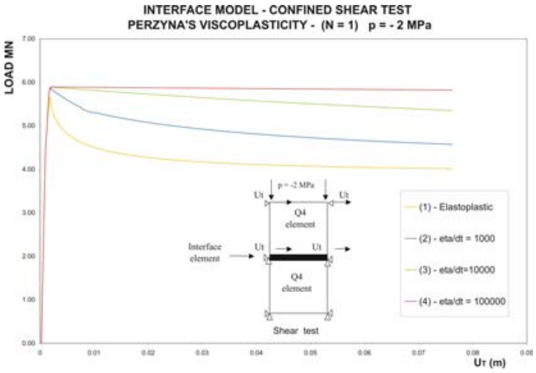

Figure 5. Confined shear test. Elastoplastic and viscoplastic predictions for different relations $\eta/\Delta t$.

5 CONCLUSIONS

In this work a continuum Perzyna extension of the interface model by Carol and Lopez (1999) was presented. The model was implemented in the framework of the numerical algorithms for continuum viscoplasticity by Carosio, Willam and Etse (2000) which include an algorithmic tangent operator to improve the convergence rate in the non-linear regime. The results in this work demonstrate the capabilities of the proposed model to reproduce the rate-dependency of interface behaviors which leads to ductility increments of post-peak responses for increasing rates $\eta/\Delta t$. In the next steps of this research the proposed rate-dependent interface model will be considered to analyzed failure behavior of concrete at the meso-mechanical level when different rates of displacements and forces are applied.

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