# Using post-peak unload cycles with the two-parameter data reduction method

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ABSTRACT: This study explores the use of information contained in post-peak unload-reload cycles to improve the accuracy of the two-parameter data reduction method. Laboratory and simulated experiments have been used to determine that a size-independent fracture toughness can not, in general, be identified from laboratory tests even with the additional information. Yet, the results of this study indicate that the fracture toughness value obtained might still be useful in predicting crack growth in unreinforced concrete members within twice the size of the test specimen.

Keywords: fracture toughness testing, two-parameter, experiments, numerical simulations, process zone

## 1 INTRODUCTION

Predicting crack propagation in materials such as glass has been done accurately for some time. In these materials the zone of damaged material at the crack tip, the process zone, is very small relative to the size of the crack and the geometry of the body. Such a crack in these materials is said to experience linear elastic fracture mechanics, LEFM, conditions. Crack propagation in such materials can occur when the stress intensity, $K_I$, at the crack tip reaches the limiting value for the material, $K_{Ic}$. That limiting value is called the fracture toughness and is size-independent by definition.

Cracks in concrete, however, typically develop a relatively large process zone (Fig. 1a). Therefore, cracks in concrete are said to experience non-linear fracture mechanics, NLFM, conditions. To predict crack growth under such conditions, other crack propagation criteria must be used. One such criteria, the two-parameter criteria, was proposed by Jenq & Shah (1985a).

Existing methods for obtaining the two parameters from laboratory tests, however, do not produce consistent results (Hanson 2000, Hanson & Ingraffea 2003). Therefore, the authors have investigated whether information contained in post-peak unload-reload cycles can improve the accuracy of the parameter values obtained from the tests. In order to explore the use of multiple unload-reload cycles, the authors have used laboratory and numerically simulated tests.

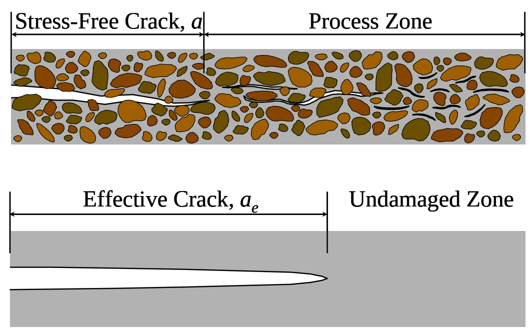


Figure 1. (a, top) Process zone ahead of stress-free crack in concrete. (b, bottom) Effective crack based on the two-parameter crack propagation model.

## 2 TWO-PARAMETER DATA REDUCTION METHOD

### 2.1 *Predicting crack propagation in concrete*

The concept of the two-parameter criteria for crack growth is that in a structure experiencing peak load the stress-free crack length, $a$, and process zone can be represented by a effectively longer LEFM crack of length $a_e$ (Fig. 1b). At peak load, the stress intensity of the effective crack is equal to the two-parameter fracture toughness, $K_{Ic}^{TP}$,

$$K_{Ic}^{TP} = f_1\left(P_{max}, a_e, geometry\right) \quad (1)$$

where $f_1(\ )$ is a function dependent upon geometry and boundary conditions and can be found in reference books for certain geometries (Murakami 1987, Tada et al. 1985) or calculated numerically. To predict the peak load of the structure, $K_{Ic}^{TP}$, $a_e$, and the geometry must be known. The two-parameter fracture toughness is presumed to be a material property and must be determined from tests on the concrete. However, the effective crack length, $a_e$, in a structure at peak load is not known in general. With two unknown quantities in Equation 1, a unique solution can not be obtained without another pertinent expression. Jenq & Shah (1985a) chose to use an expression for the elastic critical crack tip opening displacement, $CTOD_c$, to provide a unique solution for $P_{max}$ and $a_e$,

$$CTOD_c = f_2\left(P_{max}, a_e, geometry\right) \qquad (2)$$

where $f_2(\ )$ is also a function dependent upon geometry and boundary conditions but different from $f_1(\ )$. The elastic critical crack tip opening displacement is defined as the crack opening displacement, COD, of the effective LEFM crack when the structure experiences peak load and is measured at the location of the initial crack tip, $a_o$. The $CTOD_c$ is presumed to be a material property and must be known from tests on the concrete.

Equations 1 and 2 combine to provide a unique solution for $P_{max}$ and $a_e$. Therefore, if $K_{Ic}^{TP}$ and $CTOD_c$ are known material properties for a concrete, the peak load capacity of an unreinforced structure made of that concrete can be predicted using the two-parameter criteria for crack propagation.

Unfortunately, existing methods for determining the two-parameters $K_{Ic}^{TP}$ and $CTOD_c$ (Jenq & Shah 1985b, RILEM 1990) do not produce consistent results across specimen sizes for many concrete mixes (Hanson 2000, Hanson & Ingraffea 2003). Therefore, the existing methods are not, in general, producing material properties.

## 2.2 *Overview of the two-parameter data reduction method*

The two-parameter criteria for crack propagation can be applied to a test specimen to determine the limiting property values $K_{Ic}^{TP}$ and $CTOD_c$. That process is called the two-parameter data reduction method (Jenq & Shah 1985b, RILEM 1990).

The test specimen geometry and loading conditions typically used are the single edge specimen loaded in bending, SE(B) (Fig. 2). The functions $f_1(\ )$ and $f_2(\ )$ from Equations 1 and 2 have already been determined for this geometry (RILEM 1990). During the test, the specimen is loaded until the peak load, $P_{max}$, is identified. The only other information needed in order to calculate $K_{Ic}^{TP}$ and $CTOD_c$ from Equations 1 and 2 is the effective crack length, $a_e$, at peak load. At peak load, or soon after, the test specimen is unloaded to obtain the unloading compliance, $C_u$. The ratio of unloading compliance to initial compliance, $C_i$, is then used to determine $a_e$ from the following expression,

$$\frac{C_u}{C_i} = \frac{f_3\left(a_e, geometry\right)}{f_3\left(a_o, geometry\right)} \qquad (3)$$

where $f_3(\ )$ is a function dependent upon geometry and boundary conditions and has already been determined for the SE(B) specimen (RILEM 1990).

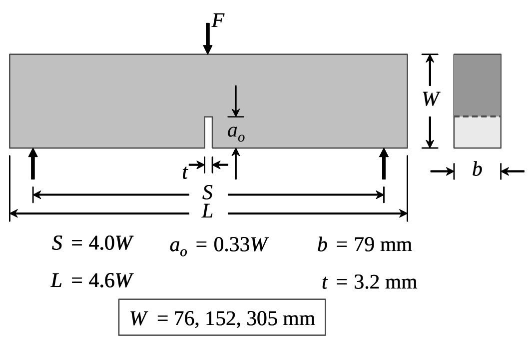


Figure 2. Single edge specimen used in the laboratory and simulated experiments.

## 2.3 *Theoretical basis for using multiple unload cycles*

The data reduction method prescribes use of the peak load and associated unloading compliance in order to determine the two parameters $K_{Ic}^{TP}$ and $CTOD_c$. However, if the process zone has grown to a steady-state size when the peak load is reached, the $K_{Ic}^{TP}$ value obtained should be a material property. Therefore, in such cases it should not matter whether the peak load and associated $a_e$ or a subsequent load and associated $a_e$ are used in Equations 1 and 3 to obtain the fracture toughness. The $K_{Ic}^{TP}$ values obtained from peak load unload-reload cycles and from subsequent unload-reload cycles should, in theory, be the same.

Jenq & Shah (1985a) found that they could use the $K_{Ic}^{TP}$ value from application of the data reduction method to peak load in order to reproduce the observed load versus CMOD response, including post-peak response, of laboratory specimens. Therefore, they demonstrated that for at least some concretes, $K_{Ic}^{TP}$ is a material property. Since they were able to use a single $K_{Ic}^{TP}$ value to reproduce the observed test data, it is reasonable to expect that they would obtain a single $K_{Ic}^{TP}$ value if the data reduction method was applied to post-peak unload-reload cycles.

Unfortunately, the $CTOD_c$ value can only be obtained by considering the peak load and associated $a_e$. By definition, $CTOD_c$ is the crack tip opening at peak load. As the test is continued beyond peak load, the effective crack length grows and the crack tip opening displacement grows. There does not appear to be a way to determine $CTOD_c$ from post-peak data.

## 3 LABORATORY TESTS

The authors performed analysis of laboratory test data in order to answer two questions. Are the $K_{Ic}^{TP}$ values from multiple unload-reload cycles from one test specimen consistent? If the $K_{Ic}^{TP}$ values are not consistent, do they asymptotically approach a value that is the same for different sizes of test specimen?

To answer these questions, laboratory tests on SE(B) specimens were performed according to the protocol presented by RILEM (1990) with the addition of several post-peak unload-reload cycles. The specimens varied in depth, maximum aggregate size, and water/cement ratio (Table 1). The $K_{Ic}^{TP}$ values obtained from each of the unload-reload cycles for each of the test specimens are plotted as a function of the normalized effective crack length, $a_e/d$, in Figures 3-7. Note that only one size of test specimen, 152 mm, was used to generate the results in Figures 3 and 4.

Table 1. Summary of concretes used in laboratory experiments.

| Concrete | Max. Aggr. Size (mm) | w/c | Avg. $f_t$ (MPa) | Avg. $f_c$ (MPa) |
|---|---|---|---|---|
| NM | 1 | 0.42 | 3.62 | - |
| NS | 13 | 0.42 | 3.13 | 44.5 |
| N | 20 | 0.58 | 3.07 | 41.3 |
| H | 20 | 0.26 | 4.46 | 52.3 |
| VH | 20 | 0.21* | 4.56 | 55.1 |

* Ratio is water/cementitious materials; silica fume was included.

The $K_{Ic}^{TP}$ values obtained from the tests on mortar (Fig. 3) and smaller aggregate concrete (Fig. 4) are reasonably consistent for all unload-reload cycles. These types of concrete will tend to have smaller relative process zone sizes compared to the larger aggregate concrete (El-Sayed et al. 1998, Mihashi et al. 1998); therefore, these concretes should be more likely to produce a size-independent value.

Not all of the test specimens exhibited consistent $K_{Ic}^{TP}$ values though. Figures 5 and 6 show increasing $K_{Ic}^{TP}$ values from post-peak unload-reload cycles for the small and middle size specimens. The large specimens appear to have produced converged results in both figures. The authors attempted to use several mathematical functions to fit the $K_{Ic}^{TP}$ values of the small and medium size specimens in order to predict the converged results from the large specimens. None produced consistent results. Therefore, it does not appear possible to predict the converged result of a larger specimen using the $K_{Ic}^{TP}$ values from multiple unload-reload cycles of smaller specimens. In fact, the $K_{Ic}^{TP}$ values for each specimen size appear to converge to different values in Figure 7.

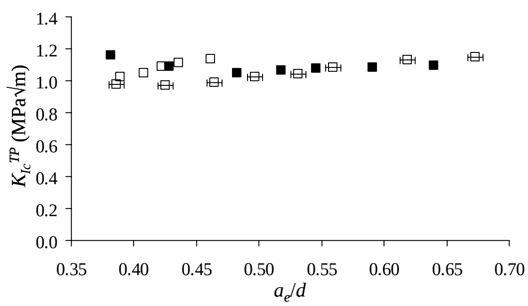


Figure 3. Results from laboratory series NM.

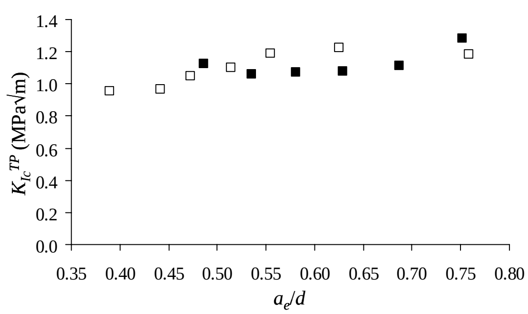


Figure 4. Results from laboratory series NS.

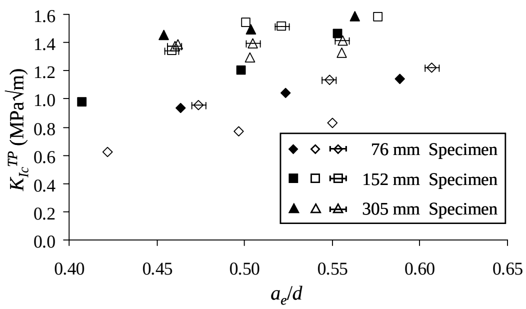


Figure 5. Results from laboratory series N.

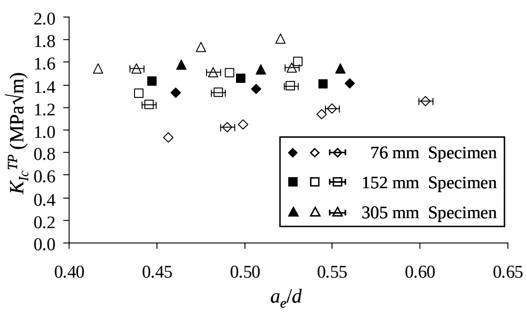


Figure 6. Results from laboratory series H.

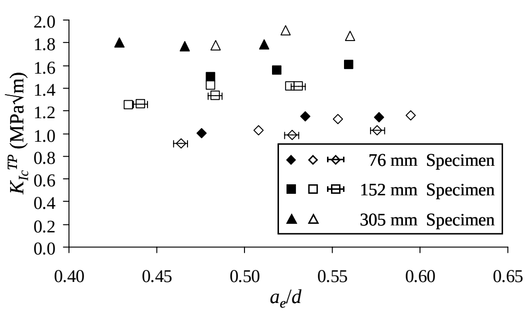


Figure 7. Results from laboratory series VH.

## 4 SIMULATED TESTS

### 4.1 *Basis for using simulated tests*

It can not be concluded from the laboratory tests whether any of the $K_{Ic}^{TP}$ values obtained are material properties. Nor can the process zone size be determined easily during a test. Therefore, the authors chose to perform simulated tests. The simulated tests have several advantages over laboratory tests: the size-independent fracture toughness is known, the process zone size can be measured directly, and a broad range of concretes can be investigated in a short time. The process used for simulating the tests is described in detail by Hanson & Ingraffea (2003). The simulations use a cohesive stress-crack opening displacement relationship to reproduce the process zone (Fig. 8).

The authors performed simulated tests with the goal of answering the following questions. If the $K_{Ic}^{TP}$ values from multiple unload-reload cycles on one test specimen are consistent, are they size independent and, therefore, a material property? Is there a relationship between process zone size and the $K_{Ic}^{TP}$ values from multiple unload-reload cycles?

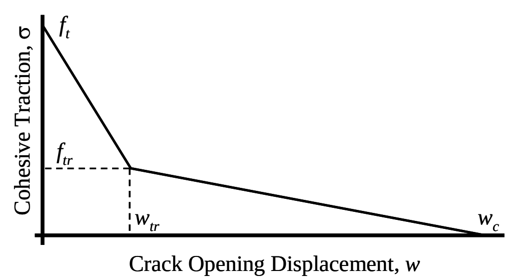


Figure 8. Cohesive model used in simulations.

### 4.2 *Summary of application to simulation data*

In order to explore the possible range for which the two-parameter data reduction method might produce a size-independent value of $K_{Ic}^{TP}$, the authors simulated concretes with a wide range of properties. The cohesive zone property values used are presented in Table 2 where the variables are defined in Figure 8. The responses of three sizes of SE(B) were generated for each simulated concrete. The dimensions of the simulated specimens are given in Figure 2.

Table 2. Cohesive zone property values for simulated tests.

| Series | $f_t$ (MPa) | $f_{tr}$ (MPa) | $w_{tr}$ (mm) | $w_c$ (mm) | $K_{Ic}$ (MPa√m) |
|---|---|---|---|---|---|
| A | 2.17 | 0.22 | 0.0134 | 0.2688 | 1.10 |
| D | 2.17 | - | - | 0.0403 | 1.10 |
| F | 2.17 | 1.09 | 0.0586 | 1.1729 | 4.40 |
| I | 9.31 | 0.93 | 0.0031 | 0.0628 | 1.10 |
| K | 9.31 | 0.93 | 0.0078 | 0.0157 | 1.10 |
| P | 9.31 | - | - | 0.1506 | 4.40 |

The $K_{Ic}^{TP}$ values obtained from each of the unload-reload cycles for each of the test specimens are plotted as a function of the normalized effective crack length, $a_e/d$, in Figures 9-14. The figures also contain the relative process zone lengths, $L_{Process\ Zone}/d$, as a function of normalized effective crack length. The legend in the first figure applies to the subsequent figures.

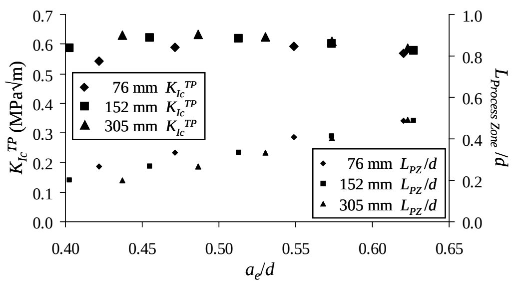


Figure 9. Results from simulated series A.

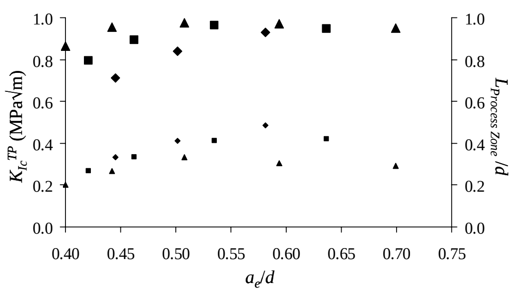


Figure 10. Results from simulated series D.

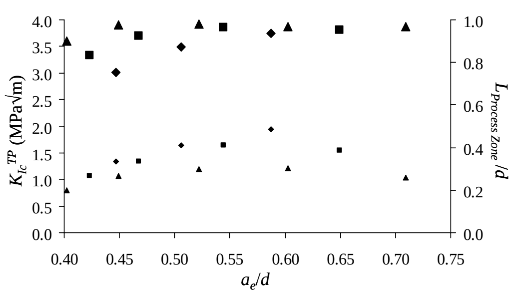


Figure 11. Results from simulated series P.

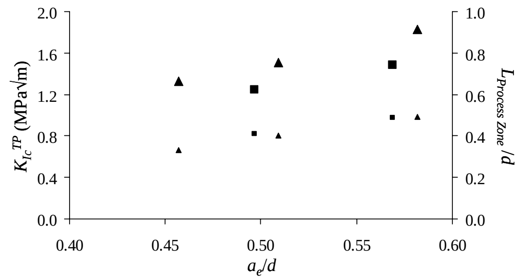


Figure 12. Results from simulated series F.

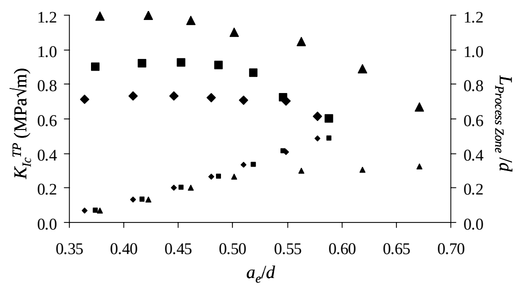


Figure 13. Results from simulated series I.

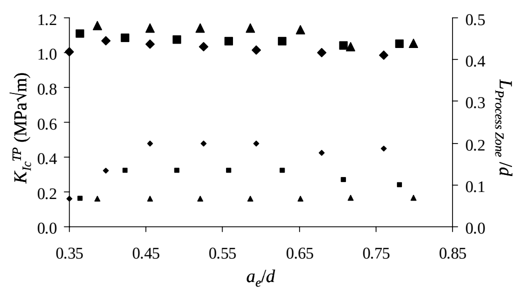


Figure 14. Results from simulated series K.

The $K_{Ic}^{TP}$ values from multiple unload-reload cycles for all three sizes of simulated test specimen in Figure 9 are consistently around 0.6 MPa√m. However, the size-independent fracture toughness is 1.1 MPa√m. Therefore, consistent $K_{Ic}^{TP}$ values do not necessarily indicate that the measured fracture toughness is a material property. Note that the process zone size did not reach steady-state for any of the three specimen sizes.

In contrast, the $K_{Ic}^{TP}$ values in Figure 10 converge to within 10% of the size-independent fracture toughness for all three specimen sizes. For this concrete, the process zone size does reach steady state and does so at the same effective crack length that the $K_{Ic}^{TP}$ values converge. Figure 11, a concrete with size-independent fracture toughness of 4.4 MPa√m, exhibits similar tendencies. The $K_{Ic}^{TP}$ values converge to a value close to the size-independent value at the same effective crack length when the process zone stops growing.

For a concrete with a relatively high size-independent fracture toughness, 4.4 MPa√m, and low tensile strength, 2.2 MPa (Fig. 12), the $K_{Ic}^{TP}$ values do not converge and are below half the size-independent value. Such a concrete will tend to have a large process zone. As expected, the process zone size does not reach steady-state.

Figures 13 and 14 present the results for two concretes with a relatively low size-independent fracture toughness 1.1 MPa√m, and high tensile strength, 9.3 MPa. Such concretes will tend to have small process zones. Interestingly, the data in Figure 13 is the only instance the authors observed where the $K_{Ic}^{TP}$ values decreased significantly with increased effective crack length. In this case, the process zone sizes were increasing until the last few unload-reload cycles for the large test specimen. The data in Figure 14, however, is consistent with the size-independent fracture toughness value, and the process zone sizes have reached steady state for all three sizes of test specimen.

## 5 CONCLUSIONS

The goal of this study was to determine whether the information contained in multiple unload-reload cycles could be used with the two-parameter data reduction method to provide more accurate estimates of the size-independent fracture toughness of concrete. The fracture toughness value must be specimen size independent in order to be a material property. Based on the results of laboratory and simulated tests, the answer is no.

For some combinations of concrete mixes and specimen size, multiple unload-reload cycles produce consistent fracture toughness values, but that value is not always the size-independent fracture toughness. In these cases, the authors have been unable to identify, from the test data, whether the value is the size-independent fracture toughness. However, the simulations indicate that if the process zone size has reached steady state, the measured fracture toughness value is often the size-independent value and is, therefore, a material property. Unfortunately, direct measurement of the process zone size during the test is not a practical addition to the test.

For some combinations of concrete mixes and specimen size, multiple unload-reload cycles produce increasing fracture toughness values. The authors have explored several mathematical functions for predicting the asymptotic value of fracture toughness that such specimens are approaching. None of these functions consistently predict an asymptotic fracture toughness value near the size-independent value. Therefore, it does not appear possible to extrapolate to the size-independent fracture toughness given the fracture toughness values from multiple unload-reload cycles.

Fortunately, the data from the two-parameter data reduction method might still be useful for predicting crack growth in unreinforced concrete structures. The objective of this study was to obtain a material property. The results indicate, however, that the peak fracture toughness value obtained from a test will often be consistent for specimens up to twice the depth of the specimen tested. Therefore, the peak fracture toughness value obtained using multiple unload-reload cycles might still lead to accurate predictions of crack growth in unreinforced concrete members within twice the size of the specimen tested.

## 6 ACKNOWLEDGEMENTS

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## 7 REFERENCES

El-Sayed, K. M., Guinea, G. V., Rocco, C., Planas, J., & Elices, M. 1998. Influence of aggregate shape on the fracture behavior of concrete. In H. Mihashi & K. Rokugo (eds), *Fracture Mechanics of Concrete Structures; Proc. Intern. Conf., Gifu, 12-16 October 1998*. Vol 1:171-180. Frieburg: Aedificatio.

Hanson, J. H. 2000. An experimental-computational evaluation of the accuracy of fracture toughness tests on concrete. Ph.D. Dissertation: Cornell University, Ithaca, NY.

Hanson, J. H. & Ingraffea, A. R. 2003. Using numerical simulations to compare the fracture toughness values for concrete from the size-effect, two-parameter and fictitious crack models. *Engineering Fracture Mechanics* 70(7-8): 1015-1027.

Jenq, Y. S. & Shah, S. P. 1985a. A fracture toughness criterion for concrete. *Engineering Fracture Mechanics* 21(5): 1055-1069.

Jenq, Y. S. & Shah, S. P. 1985b. Two parameter fracture model for concrete. *Journal of Engineering Mechanics* 111(10): 1227-1241.

Mihashi, H., Kirikoshi, K., Nomura, N., Otsuka, K., & Kaneko, Y. 1998. Microcracking behavior and softening properties of concrete. In H. Mihashi & K. Rokugo (eds), *Fracture*

*Mechanics of Concrete Structures Proc. Intern. Conf., Gifu, 12-16 October 1998*. Vol 1:181-192. Frieburg: Aedificatio.
Murakami, Y. (ed), 1987. *Stress Intensity Factors Handbook*. Pergamon: Oxford.
RILEM, 1990. Determination of fracture parameters ($K_{Ic}^{s}$ and $CTOD_c$) of plain concrete using three-point bend tests. *Materials and Structures* 23: 457-460.
Tada, H., Paris, P. C., & Irwin, G. R. 1985. *The Stress Analysis of Cracks Handbook*. Paris Productions: St. Louis.