

# 3D fatigue analysis of RC bridge slabs and slab repairs by fiber cementitious materials

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**ABSTRACT:** The present paper considers the application of fracture mechanics to develop an analytical tool for predicting fatigue responses of RC bridge slabs. The bridging stress degradation of concrete cracks under fatigue loading is considered as a primary mechanism for the propagations of cracks that induce the slab failure. 3D FEM analysis of slabs under a fatigue moving load is proposed in order to reproduce the failure mechanism of slabs under traffic load and to predict fatigue responses. Concrete cracks are allowed to initiate in three directions in smeared crack elements according to the principal stress cracking criteria. The hysteresis bridging stress relation with the cyclic-dependent degradation is assigned for each crack alignment. The analysis of underlay and overlay repaired RC slabs are analyzed with normal concrete and Engineered Cementitious Composites (ECC). The comparison between fatigue life and midspan deflection characteristics are investigated among several repair cases.

**Keywords:** fatigue analysis, RC slab, S-N relationship, moving load, bridging stress degradation

## 1 INTRODUCTION

Reinforced concrete (RC) bridge slabs, one of fatigue intensive structural members, are typically subjected to a large number of load repetitions from heavy traffic. They are reported to deteriorate significantly due to the continuous increase in traffic amount and insufficient slab thickness in the past design. The development of effective repair methods as well as repair design methods becomes an urgent task in the near future.

One promising and cost-effective repair method is the application of fiber cementitious materials (Shah & Rangan 1971, Li 2002). The existence of fibers leads to the improvement of tensile and fatigue performances. Several kinds of fiber cementitious materials, such as FRC and ECC, have been investigated for introducing to RC slab repairs (Kanda et al. 2001, Suthiwarapirak et al. 2002, Matsumoto et al. 2002a). It was shown that

ECC and FRC show much more improvement in flexural fatigue properties than normal concrete.

The design guidelines of RC slab repair by cementitious materials and fiber cementitious materials were proposed based on static performances and empirical knowledge from experiment results (Committee of highway investigation 1995, Schrader & Munch 1976). Although these empirical design guidelines are simple in the design procedure, they are not intended for the design optimization of each damaged slab case because it lacks the consideration of fatigue degradation characteristics of RC slab after repair. An analysis method that considers governing failure mechanisms is an effective tool for the development of repair design optimization.

The analysis method for predicting structural performances of cementitious materials has been proposed based on the concept of crack propagation under fatigue loading (Zhang 1998, Suthiwarapirak & Matsumoto 2001). The fatigue analysis of RC beams without shear reinforcement

was proposed by Suthiwarapirak & Matsumoto (2001), and it was shown that the fatigue crack propagation concept is applicable to the prediction of fatigue flexural performances of RC beams.

In this study, an analysis model for predicting fatigue performances of RC slabs is proposed based on the fatigue crack propagation concept. The bridging stress degradation of concrete cracks is considered as a primary mechanism for the propagations of both flexural cracks and shear cracks that induce the failure of RC slabs. Three-dimensional FEM model considering those governing failure mechanisms are developed.

Using this FEM model, the application of ECC in RC slab repairs is investigated by both underlay and overlay methods. The analysis of original slab and repaired slabs by ECC and normal concrete are performed. The improvement in fatigue life and deflection capacities of repaired slabs are compared and discussed.

## 2 FATIGUE ANALYSIS SCHEME

### 2.1 Two Components of Fatigue Analysis

The development of fatigue analysis of RC bridge slabs is discussed in this section. The analysis scheme for predicting fatigue performances of RC structures consists of two components: an analysis model and material models as shown in Figure 1. An FEM model is proposed based on the fatigue failure mechanisms of RC slabs. The propagation

of fatigue cracks due to crack bridging degradation is considered as a primary mechanism. Material models of concrete and a reinforcing bar are established from material tests. With these input material properties, the structural performances of RC slab under fatigue loading, such as S-N relationship, deformation behavior, and failure mode, can be analyzed with given geometries and boundary conditions.

#### 2.1.1 Analysis Model

The analysis model is developed based on the failure mechanisms of RC slabs observed from experimental research (Matsui 1997). The propagation of both shear and flexural cracks under fatigue loading is considered as a main cause of RC slab failure. The propagation of cracks is caused by the degradation of crack bridging stress due to fatigue loading.

In a flexural member, a crack initiates with a crack length of  $a$  and a crack width of  $w$  as shown in Figure 2. When the crack experiences a repetitive loading, transferred stress across the crack reduces due to the progressive deterioration of material constituents on the crack plane. With the increase in loading cycles, existing cracks increase in length, or new cracks initiate. This progressive crack propagation results in the final failure that is usually a punching shear failure.

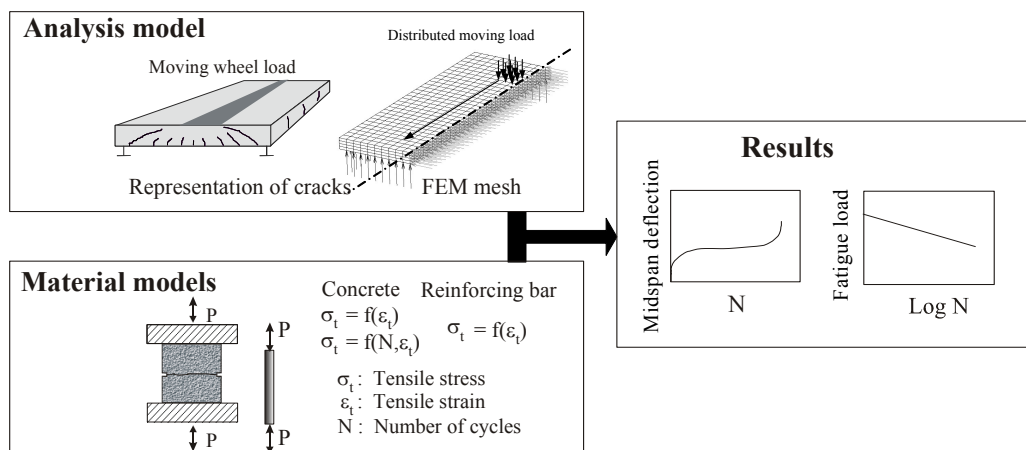


Figure 1. Two components of fatigue analysis

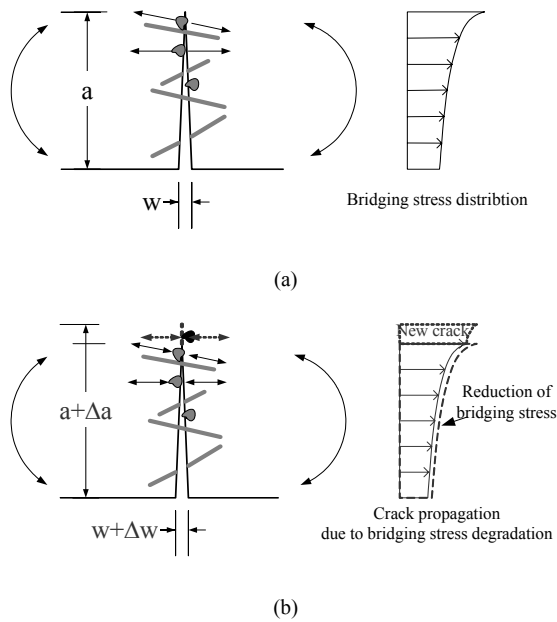


Figure 2. Crack propagation process (a) Crack initiation under first loading cycle (b) Crack propagation due to cyclic fatigue

The concept of smeared crack element is introduced for representing the distributed crack propagation process. The proposed analysis model is a three-dimensional FEM consisting of smeared crack element which represents cementitious materials and rod element which represents a reinforcing bar. Concrete cracks are allowed to initiate in three perpendicular directions in each crack element according to the principal stress cracking criteria.

## 2.2 Material Models

Materials treated in the analysis are cementitious materials and a reinforcing bar. For cementitious materials such as normal concrete and ECC, the bridging stress relation and the stress degradation relation under fatigue loading are introduced.

For concrete, bridging stress is mainly due to aggregate bridging. As shown in Figure 2, bridging stress of fiber cementitious materials is much improved due to fiber bridging. The existence of fibers slows down the propagation of fatigue crack.

The bridging stress degradation relation describes the stress degradation characteristics of fatigue

crack under fatigue loading. The degradation law of concrete and ECC is simply defined as a function of cracking tensile strain,  $\varepsilon_i$ , and number of cycles,  $N$  (Matsumoto et al. 2002b), and it can be expressed as

$$\frac{\sigma_N}{\sigma_1} = f(N, \varepsilon_i) \quad (1)$$

where  $f(N, \varepsilon_i)$  is a bridging stress degradation ratio, which is less than 1.  $\sigma_N$  and  $\sigma_1$  are the bridging stress at  $N$ -th cycle and first cycle, respectively.

The considerations of shear stress component and the degradation of shear stress are neglected.

Bilinear stress-strain relationship is assigned for a reinforcing bar. No fatigue fracture of a rebar is considered in the analysis.

## 3 ANALYSIS OF REPAIRED RC SLABS

Three-dimensional FEM with smeared crack elements is introduced in the analysis of repaired RC slab. The analysis of overlay and underlay RC slab repaired with normal concrete and ECC are conducted, and the results are compared with the original slab.

### 3.1 Dimension of Original Slab and Repaired Slabs

The analysis of original RC slabs under moving load is first conducted. A slab dimension of 2.5x3.8x0.18m with 1.3% and 0.5% reinforcement ratio in longitudinal and transverse direction is adopted (Okada et al 1978).

For repaired slabs, the thickness of repair layer for both underlay and overlay is assumed to be 50 mm. The dimension of repaired slab is shown in Figure 3. Repair layer is attached to existing RC slab in two cases, overlay and underlay repair cases. It is noted that there is no additional steel reinforcement in repair layers. The cross sections of overlay repaired and underlay repaired RC slabs are shown in Figure 4.

### 3.2 Boundary Condition, Loading Condition, and Mesh

The simple support is assigned for all sides of slabs as shown by the dotted line in Figure 5. For loading condition, the wheel load from traffic is simulated

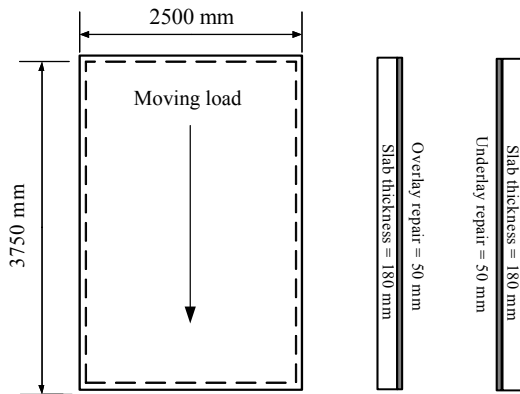


Figure 3. Dimension of overlay and underlay RC slabs

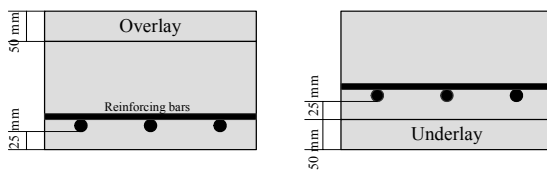


Figure 4. Reinforcement position of (a) overlay repaired slab (b) underlay repaired slab

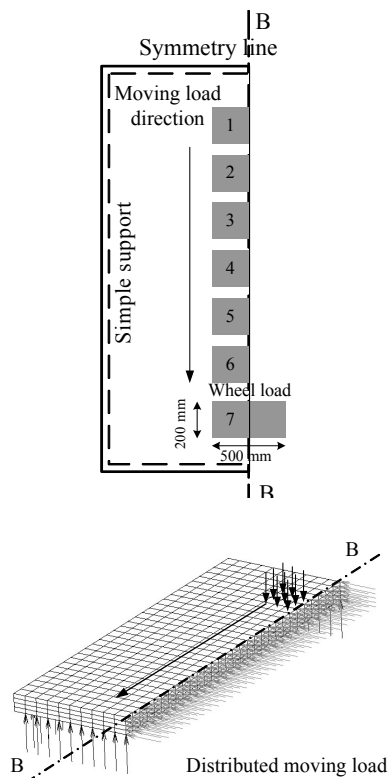


Figure 5. Loading condition, boundary condition and mesh of RC slabs under moving load

by moving a distributed load patch of the size of a contact wheel load area along the longitudinal direction of the slab as shown in the figure.

The FEM mesh for the slab analysis is also shown in Figure 5. By taking advantage of symmetry with respect to the centerline, half of a model is analyzed.

### 3.3 Material Properties for Fatigue Analysis

The basic properties of cementitious materials and reinforcing bar in the analysis are shown in Table 1.

Table 1 Basic properties of material in the analysis

Materials	Normal concrete	PVA-ECC	Steel
Young's modulus (GPa)	30	20	200
Poisson's ratio	0.15	0.17	0.2
Tensile strength (MPa)	2.5	4.3	452.4

For the original slab, the basic properties of normal concrete are used. The materials of repair layers are normal concrete and ECC reinforced with polyvinyl alcohol fibers (PVA-ECC).

#### 3.3.1 Constitutive relations of materials

The constitutive relations or the tensile stress-strain relationships of normal concrete and PVA-ECC are shown in Figure 6. The stress-strain relationship of PVA-ECC is adopted from the uniaxial fatigue test (Kanda et al 2001). It is noticed that PVA-ECC exhibits much more tensile strain capacity and ultimate tensile strength than normal concrete. For a reinforcing bar, the bilinear tensile stress-strain relationship is introduced as shown in Figure 7.

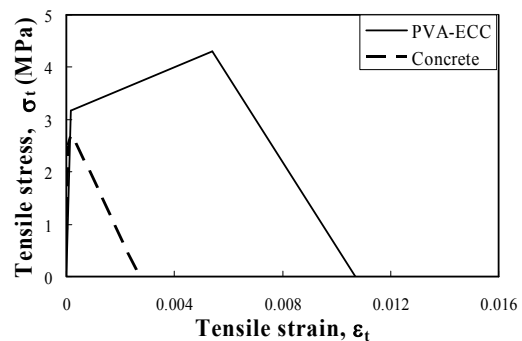


Figure 6. Tensile stress-strain relationships of cementitious materials

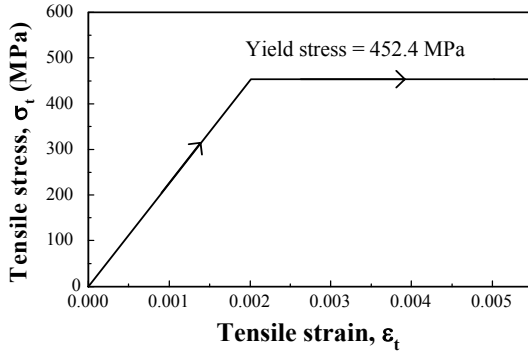


Figure 7. Tensile stress-strain relationship of rebar

### 3.3.2 Bridging stress degradation relations of cementitious materials

The bridging stress degradation relation of concrete proposed based on uniaxial fatigue test results by Zhang (Zhang 1998) and the relation of PVA-ECC proposed based on flexural fatigue test results by Suthiwarapirak & Matsumoto (2003) are adopted in this analysis and they are presented as below.

Normal concrete:

$$\frac{\sigma_N}{\sigma_1} = 1 - (0.08 + 4 \times \delta_{\max}) \text{Log}_{10}(N) \quad (2)$$

PVA-ECC:

$$\frac{\sigma_N}{\sigma_1} = 1 - (0.025 + 1.5 \times \delta_{\max}) \text{Log}_{10}(N) \quad (3)$$

where  $\delta_{\max}$  is the maximum crack width. It is noticed that the bridging stress degradation ratio is less than one and it decreases when the number of cycles or the maximum crack width increases. Moreover, the bridging stress of PVA-ECC reduces at slower rate in comparison with that of normal concrete.

In order to introduce these bridging stress degradation relations to smeared crack elements, the maximum crack width,  $\delta_{\max}$ , is rewritten in the form of tensile strain,  $\epsilon_t$ , by considering unit element length ( $\delta_{\max} = \epsilon_t \times 1$ ).

The hysteresis bridging stress relation with the cyclic-dependent degradation of normal concrete is illustrated in Figure 8. It is assumed that no plastic deformation occurred and the reloading path is the same as unloading path.

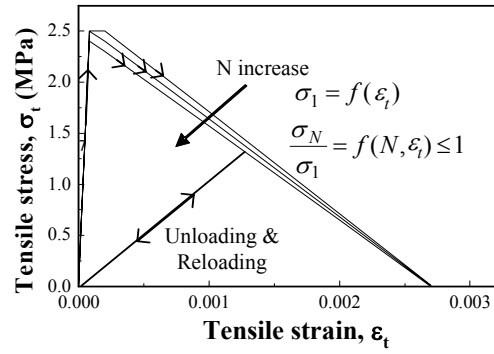


Figure 8. Stress-strain relationship of normal concrete under fatigue

## 4 FATIGUE ANALYSIS RESULTS

The analysis results of the original RC slab and RC slab repaired by ECC and normal concrete by both overlay and underlay methods are presented and compared in this section.

In the case of overlay repaired slabs, the results of prolonged fatigue life of RC slab repaired by ECC and normal concrete are shown in comparison with that of original slab in Figure 9. NC and ECC in the figure refer to the RC slabs repaired by normal concrete and ECC, respectively. It is obvious that both repair cases exhibit much longer fatigue life than the original RC slab at the same fatigue loading level. Moreover, RC slab overlay repaired by ECC exhibits more prolonged fatigue life than that by normal concrete.

Figure 10a and Figure 10b present the deflection evolution of ECC overlay repaired slabs and normal concrete overlay repaired slabs, respectively. The deflection evolution of the original slab at fatigue load of 432 kN is also plotted for comparison. It is shown that the deflection of both cases of repaired slab reduces significantly when compared with the original RC slab. It is also noticed that slabs repaired by ECC overlay showed larger midspan deflection than those repaired by normal concrete because the Young's modulus of ECC is lower.

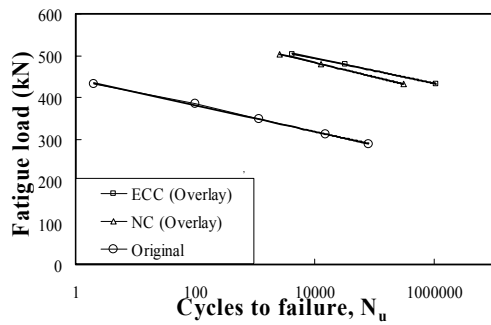


Figure 9. Extended fatigue life of overlay repaired RC slabs

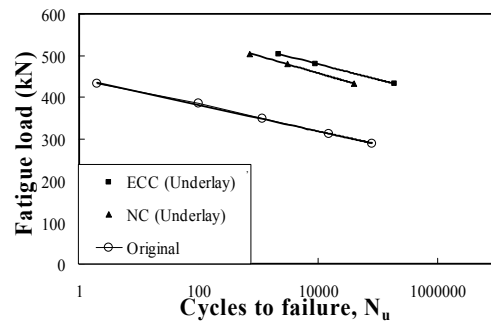


Figure 11. Extended fatigue life of underlay repaired RC slabs

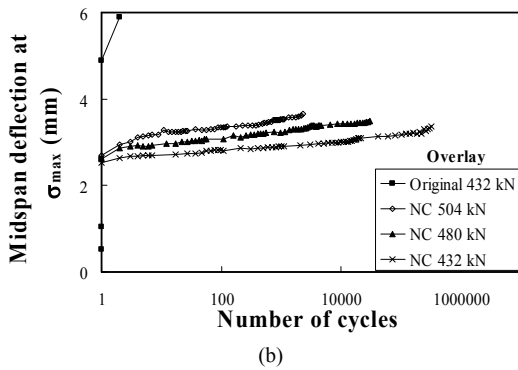
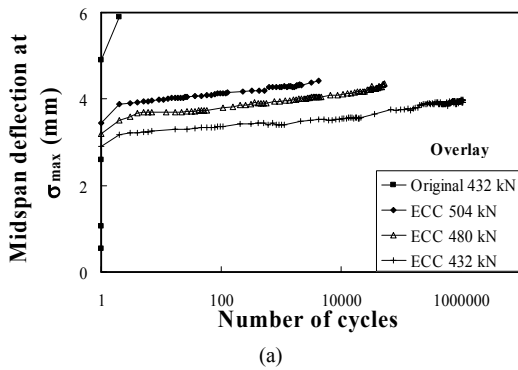


Figure 10. Midspan deflection evolution of (a) ECC overlay repaired slabs (b) normal concrete overlay repaired slabs

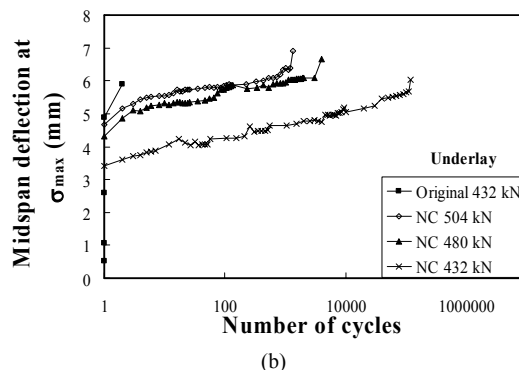
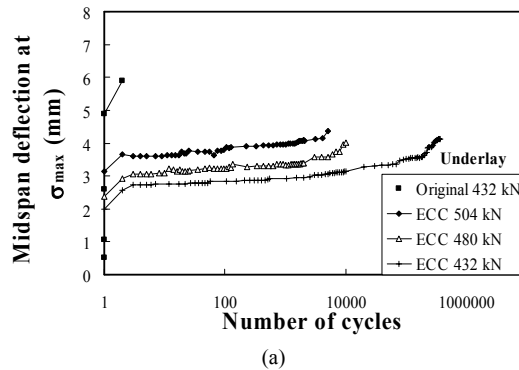


Figure 12. Midspan deflection evolution of (a) ECC underlay repaired slabs (b) normal concrete underlay repaired slabs

In the case of underlay repaired RC slab, the prolonged fatigue life as compared with the original RC slab is illustrated in Figure 11. The extended fatigue life can be demonstrated by the fatigue analysis. Similar to the case of overlay repair, the RC slab repaired by ECC shows longer fatigue life than the slab repaired by normal concrete.

For midspan deflection evolution, it is shown that there is a large difference in the deflection characteristic between the two repair material cases as shown in Figure 12a and Figure 12b. The RC slabs repaired by ECC exhibit significantly improved deflection characteristics when compared with those of normal concrete. The deflection significantly decreases in the case of ECC repair.

It is noted that there is no additional reinforcement in the underlay repair layers so the severe damage due to fatigue crack occurs in the underlay layer. Especially, in the normal concrete underlay case, concrete with low tensile resistance show large opening cracks; therefore, the deflection in the normal concrete underlay case is very large as compared with the case of ECC underlay.

## 5 DISCUSSIONS

The comparison of the fatigue life between all repaired RC slabs is shown in Figure 13. It is found that the overlay repair cases exhibit longer fatigue life than the underlay repair cases. This is because the effective depth in the case of overlay becomes larger and it leads to the increase in flexural load capacity of the overlay repaired RC slabs.

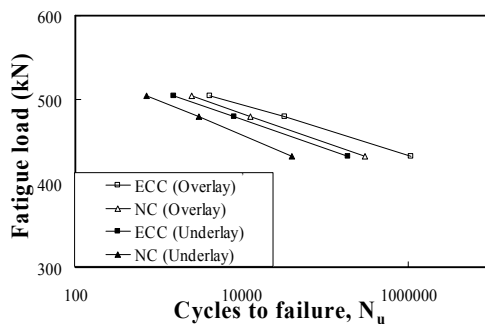


Figure 13. Comparison between fatigue life of all repaired slab cases

The comparison of midspan deflection evolutions between all repaired slab cases at fatigue load of 480 kN are presented in Figure 14. It is noticed that in the case of overlay repaired slabs, the slab repaired by ECC shows larger deflection than that repaired by normal concrete, while in the case of underlay repaired slabs, the reverse occurs.

In the case of overlay repaired slabs, the repair layer is subjected to compression. Since ECC has a lower Young's modulus than normal concrete, the deformation of the ECC repair layer is larger. This leads to the slab repaired by ECC exhibiting higher deflection than the slab repaired by normal concrete.

On the other hand, for underlay repaired slab, the repair layer is subjected to fatigue tension. ECC shows much superior tensile resistance as

compared with normal concrete; therefore, the deflection capacity of slab repaired by ECC is much improved than that repaired by normal concrete.

It is also noticed that there are differences in deflection evolution characteristics between underlay and overlay repair cases. For underlay repaired slab case, it is easy to specify the damage point from each deflection result because the rapid increase of displacement before failure can be noticed readily, while for overlay repaired slab case, only small displacement jump occurs at failure. This is because of the condition for crack localization. As shown in Figure 4, the depth of concrete below reinforcing bar is 75 mm in the case of underlay repaired slab comparing with 25 mm in the case of overlay repaired slabs. Therefore, crack localization occurs easier in the case of underlay repaired slab.

Although the analysis results showed that overlay repair slab exhibits better improvement than underlay repaired slab, in real application, steel mesh reinforcement is placed before spraying or covering with fiber cementitious materials. The existence of steel reinforcement in repair layers should be also considered in the further analysis.

It is found from the analysis results that the ECC underlay repaired slab without steel reinforcement exhibits improved fatigue life and deflection capacity, nearly to the case of normal concrete overlay repaired slab. This probably implies that the repair layer by normal concrete with steel reinforcement can be replaced by ECC with no steel reinforcement

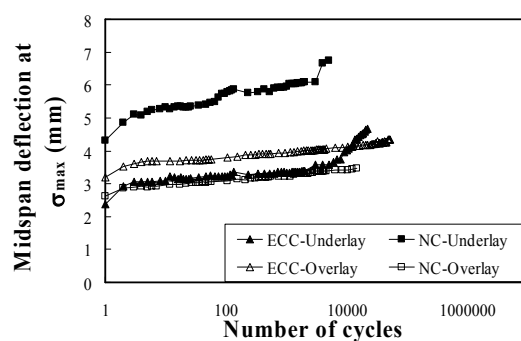


Figure 14. Comparison between midspan deflection of all repaired slab cases at fatigue load 480 kN

## 6 CONCLUSIONS

The fatigue analysis of RC bridge slabs is proposed based on the concept of fatigue crack propagation in cementitious materials. The degradation of crack bridging stress is considered as the primary cause of the development of cracks that induces slab failure. Three-dimensional FEM of RC slabs consists of smeared crack elements which represents cementitious materials and rod elements which represents a reinforcing bar. Traffic load is simulated by moving a distributed load patch of the size of a contact wheel load area along the longitudinal direction of the slab.

The analysis of repaired RC slabs by normal concrete or ECC is performed for both underlay and overlay repair cases. The improvement in fatigue life and deflection capacity of repaired RC slabs can be demonstrated by the proposed analysis. All repaired slabs exhibit a large extension of fatigue life and a reduction of midspan deflection when compared with the original slab.

The comparison between overlay and underlay repaired slabs showed that the overlay repair cases exhibit longer fatigue life than the underlay cases. This is because the effective depth of reinforcing bar in the cases of overlay is larger than that of underlay cases, and it leads to the increase in flexural load capacity of the overlay repaired RC slabs.

It is also noticed that slabs repaired by ECC exhibit more improvement in fatigue performances than slabs repaired by normal concrete. Especially, in the case of underlay repair in which the repair layer is introduced under tensile fatigue, the improvement of fatigue life and deflection capacity of ECC repaired slabs is significant. Since ECC exhibits much superior tensile fatigue resistance as compared with normal concrete, the midspan deflection characteristic of slab repaired by ECC is much improved than that repaired by normal concrete.

The fatigue performances of repaired slabs predicted by the proposed analysis are applicable to the development of design guideline in the future. For further development, the damage of existing RC slabs and the bond degradation between existing layer and repair layer should be considered in order to represent real failure mechanisms of repaired RC slabs.

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