The effect of tensile fracture energy on the size effect for shear strength of reinforced concrete beam members utilizing high strength concrete

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ABSTRACT: The authors confirmed the effectiveness of applying fracture mechanics to the size effect in shear strength for RC beams of high strength concrete without shear reinforcements. However, very few studies have been conducted for the fracture mechanics performance of high strength concrete. Therefore Fracture energy tests were performed on concrete with compressive strength in the range 35 N/mm^2 to 145 N/mm^2. Next, a program for estimating a tension softening curve using poly-linear approximation analysis was applied. The analysis method faithfully reproduced the test results, even for high strength concrete. The relationship between fracture energy and compressive strength in the test results showed different trends above and below 80 N/mm^2. Characteristic length displayed a high degree of correlation with compressive strength, and the regression equation was obtained. The regression equation was then utilized to examine the size effect in relation to shear strength.

Keywords: high strength concrete, fracture energy, characteristic length, shear strength, size effect

1. INTRODUCTION

Previously, the authors performed an experimental study using reinforced concrete beams without shear reinforcements, with a compressive strength of 36 - 100 N/mm^2, in order to identify the size effect in the nominal shear stress of high strength concrete reinforced concrete beams during the occurrence of diagonal cracking (hereafter "shear strength") (Fujita et al 2002). The authors also confirmed the effectiveness of applying fracture mechanics to the size effect in shear strength. (Fujita et al 2003) Nevertheless, in Standard Specification for Concrete Structures-2002 [Structural Performance Verification] (hereafter "JSCE 2002a") and the CEB-FIP 1990, the standards used in Japan and overseas countries, equations are proposed for estimating fracture energy with respect to tension softening performance. However, the scope of these equations is concrete with a compressive strength of 80 N/mm^2 or less, and very few studies have been conducted for the fracture mechanics performance of high strength concrete with a compressive strength of more than 80 N/mm^2, using standardized test methods or tension softening curve estimation methods.

For this reason, to study the size effect in the shear strength of high strength concrete with a compressive strength exceeding 80 N/mm^2, a determination of the material properties of the concrete is indispensable. Accordingly, fracture energy tests were conducted for concrete with a compressive strength of 35 - 145 N/mm^2, in accordance with the "Test method for fracture energy of plain concrete (draft)" (JCI 1993) (hereafter "Proposed test method") of the Japan Concrete Institute (hereafter JCI). Next, a program (Kitsutaka 2002) for estimating a tension softening curve using poly-linear approximation analysis was applied, and the results were compared with the test results and the standards, and the relationship between compressive strength and characteristic length was studied using the fracture energy test results and estimates. In addition, the size effect in shear strength was re-examined for concrete ranging from normal strength through high strength.

Table 1. test revel and test results

	CASE	f'_c (N/mm ^{2})	f_t (N/mm ^{2})	E_c (kN/mm ^{2})	G_f (N/mm)	l_{ck} (mm)	G_f^{cal} (N/mm)	l_{ch}^{cal} (mm)	σ_0 (N/mm ^{2})	E_c^{cal} (N/mm ^{2})	w_{cr} (N/mm ^{2})
A	L	35.1	2.87	29.0	0.197	692.3	0.201	707.7	4.92	26.4	0.710
	L-S	43.0	3.90	30.0	0.197	388.6	0.200	394.1	5.37	27.0	0.644
	M	50.2	4.19	32.1	0.201	368.2	0.202	369.0	6.27	32.5	0.330
	M-S	88.0	5.09	31.1	0.202	242.9	0.202	242.8	6.47	30.0	0.451
	U	85.6	5.47	37.2	0.192	208.5	0.216	268.6	7.31	31.4	0.350
	U-S	101.0	7.06	40.9	0.180	147.5	0.172	141.2	7.05	31.4	0.231
B	H-80	91.9	5.61	39.6	0.211	265.5	0.200	251.0	7.99	29.6	0.192
	H-100	102.4	6.68	40.7	0.170	154.7	0.158	144.4	11.13	34.0	0.122
	H-120	127.8	7.07	42.9	0.185	158.5	0.164	141.0	11.67	33.7	0.109
	H-140	138.9	8.52	47.1	0.172	111.5	0.159	103.3	13.67	36.0	0.080
	H-160	145.8	7.13	47.0	0.175	162.0	0.155	143.3	15.12	38.0	0.060

f'_c : compressive strength, f_t : tensil strength, E_c : Young's modulus, G_f : fracture energy, l_{ch} : charactaristic length,
G_f^{cal} : Estimated fracture energy, l_{ch}^{cal} : Characteristic length calculated using, σ_0 : Early binding stress level,
E_c^{cal} : Estimated Young's modulus, w_{cr} : Estimated critical opening displacement

2. OUTLINE OF TESTS

Fig. 1 shows the loading system and measurement positions. In accordance with the proposed test method (JCI 1993), the fracture energy test was conducted by means of three-point bending tests on a pre-notched beam, and the load - crack mouth opening displacement (CMOD) was measured. The size of the test model was 100 mm x 100 mm x 400 mm. The specific fracture energy testing and fracture energy calculation methods were as noted in the References (JCI 1993).

Table 1 shows the test level and test results. This test was conducted for concrete with a compressive strength of 35 - 145 N/mm ^{2} and fracture energy tests, a compressive strength test, a split-cylinder test and an elastic modulus test. The tests were conducted in two series. Series A tests were conducted for concrete with a compressive strength of 36(L), 60(M) and 100(U) N/mm ^{2}. The letter "S" refers to water-cured test samples. Series B tests were conducted for higher strength concrete of 80 - 145 N/mm ^{2}.

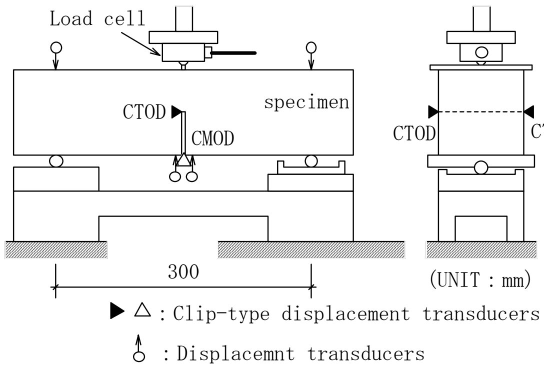

Fig.1 Loading system

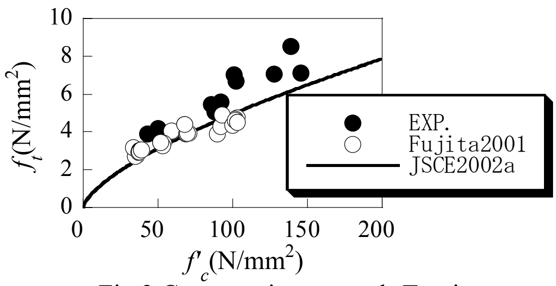

Fig.2 Compressive strength-Tension strength relation

3. TEST RESULTS AND DISCUSSION

3.1 *Tensile strength test*

Fig. 2 shows the relationship between compressive strength and tensile strength. The figure shows both existing test results (Fujita et al 2001) and the results from the JSCE equation (JSCE 2002a). The JSCE notes that the equation is applicable even to high strength concrete with a tensile strength of 80 N/mm ^{2}. According to Fig. 2, although tensile strength tends to virtually peak for concrete with a compressive strength of more than 100 N/mm ^{2}, values could generally be evaluated with the Standard Specification equation even for concrete with a compressive strength of 145 N/mm ^{2} or less.

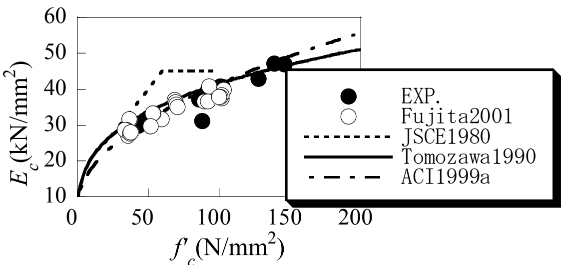

Fig.3 Compressive strength-Young's modulus relation

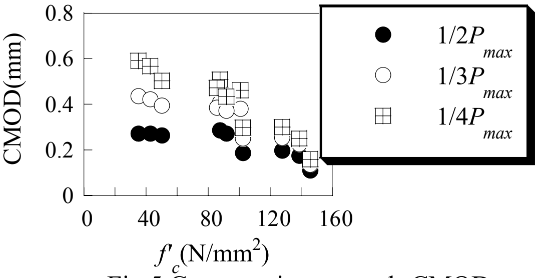

Fig.5 Compressive strength-CMOD relation

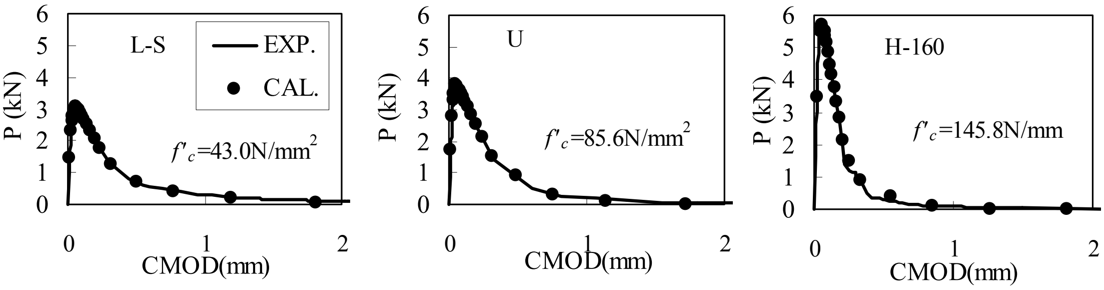

Fig.4 Load-CMOD relation

3.2 *Young's modulus*

Fig. 3 shows the relationship between compressive strength and Young's modulus. The figure shows the standard values for JSCE1980 as well as the values derived by Tomozawa et al 1990 and the values in ACI 1999a.

From Fig. 3, it can be seen that, within the scope of this study, the standard in the High Strength Guidelines evaluated the Young's modulus excessively high for compressive strength values exceeding 40 N/mm^2, while the equations proposed by Tomozawa and in ACI-363 were able to estimate the Young's modulus with comparative accuracy for concrete ranging from normal strength to high strength.

From the above, it can be confirmed that, within the scope of limited materials and data, the material properties of high strength concrete can be evaluated through appropriate selection of existing standard equations and proposed equations.

3.3 *Tension softening curve*

Fig. 4 shows the actual measurements for a load - CMOD curve and the estimated load - CMOD values, calculated through the process of analysis, for some representative test specimens.

From Fig. 4, it was confirmed that the slope of the load reduction after the maximum load is steeper with higher compressive strengths. To evaluate this in quantitative terms, Fig. 5 shows the relationship between compressive strength and CMOD. The average values for each case are shown for the CMOD at 1/2, 1/3 and 1/4 maximum load in the load reduction zone.

The results show that the higher the compressive strength is, the lower the CMOD at each load level becomes, and the smaller the difference in CMOD between load levels becomes. Observations of the test specimens' fracture surfaces after failure showed that in Case L, fracture of rough aggregate was not usually seen, but fracture of the mortar matrix was conspicuous. Conversely, observations showed that higher compressive strengths brought increasingly greater likelihood of rough aggregate fracture. In particular, in virtually all cases with compressive strength of H-120 or higher, fracture of the rough aggregate could be seen in the fracture surfaces, with cracking developing linearly. In other words, it appears that the higher the compressive strength, the greater the likelihood of brittle fracture.

There was close agreement between the estimated values and measured values at all compressive strength levels. This shows that a program using the poly-linear approximation analysis method

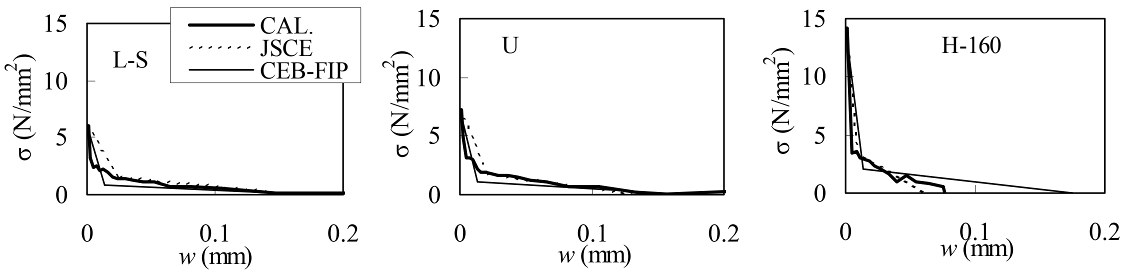

Fig.6 Tension softening curve

can be applied to even high strength concrete. Nevertheless, for the analyzable test specimens, the higher the compressive strength was, the greater the proportion of test specimens for which suitable results could not be obtained tended to be. It is thought that poly-linear approximation analysis method is not capable of accurate analysis in cases for which there are variations in the initial slope of the load - CMOD curve (JCI 1993). In the analysis, smoothing was conducted for load - CMOD for actual measurements, but for the higher compressive strength cases, there was a tendency for the initial slope of the load - CMOD curve to form a downward arch. This is thought to be the reason that accurate analysis results could not be obtained. Moreover, the overall estimates for Young's modulus were evaluated lower than the material test results; this tendency became more pronounced as the compressive strength increased, and the difference between the material test values and estimates became greater.

Fig. 6 shows the tension softening curve. For purposes of comparison, the JSCE and CEB-FIP tension softening curves are also shown. For the initial slope, the results are similar to the CEB-FIP equation. For high strength concrete, particularly for cases with a compressive strength of 100 N/mm^2 or more, the results were similar to the JSCE equation.

3.4 *Comparison of fracture energy*

Fig. 7 shows the relationship between compressive strength and fracture energy. G_f indicates the fracture energy in accordance with the proposed test method; G_f^{cal} indicates the estimated fracture energy derived with the poly-linear approximation analysis method.

For normal strength concrete, the values were approximately the same. For high strength concrete with a compressive strength exceeding 80 N/mm^2, however, differences were noted between the two.

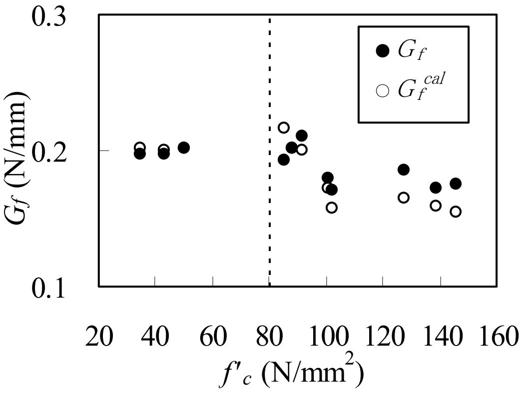

Fig.7 Compressive strength-
Fracture energy relation

The higher the compressive strength, the lower G_f^{cal} was as compared to G_f. In terms of the Young's modulus as well, as the compressive strength increased, there was a noticeable tendency for the estimated Young's modulus E_c^{cal} to be evaluated too low, and it is possible that this affected the difference between G_f and G_f^{cal} as well. The variation coefficient for both fracture energy G_f and G_f^{cal} in this paper was 20% or below. Existing research has confirmed a variation coefficient of around 20% (JCI 1993), and it is thought to be obtained with comparative accuracy for both G_f and G_f^{cal}, for concrete ranging from normal strength through high strength.

The relationship between fracture energy and compressive strength in the test results showed different trends above and below 80 N/mm^2. At or below a compressive strength of 80 N/mm^2, the fracture energy tended to increase as the compressive strength increased. In contrast, in the region over 80 N/mm^2, the fracture energy decreased as the compressive strength increased.

3.5 *Study of characteristic length*

Fig. 8 shows the relationship between compressive strength and characteristic length (Gustafsson & Hillerborg 1988). The characteristic length is derived the following : $l_{ch}=E_c \cdot G_f / f_t^2$. Where, E_c = Young's modulus, f_t = tensile strength.

The broken line in the figure represents the regression equation for l_{ch} (correlation coefficient (R)=0.96); the dotted line represents the regression equation for l_{ch}^{cal} (R=0.95). The two regression equations were in general agreement, and in both cases the compressive strength was proportional to the root of approximately -1.1. Assuming that the difference between G_f and G_f^{cal} is about the degree noted in this study, the difference between l_{ch} and l_{ch}^{cal} will be slight, and they will have the same compressive strength correlation. For this study, it will be assumed that characteristic length is proportional to the compressive strength to the -1 power; the solid line in the figure represents the following simple equation derived using the method of least squares.

$$l_{ch} = 20000 f'_c{}^{-1.0} \quad (1)$$

With the simple equation, there is an extremely good correlation between l_{ch} and l_{ch}^{cal}, and the correlation coefficient for both is 0.94. Equation (1) is an extremely simple equation, but it is capable of assessing characteristic length with sufficient accuracy.

4. APPROACH TO SIZE EFFECT USING FRACTURE MECHNICS

4.1 *Comparison with standard equations*

Table 2 shows the standard equations used in various countries. Fig. 9 shows a comparison with the results of shear tests for reinforced concrete

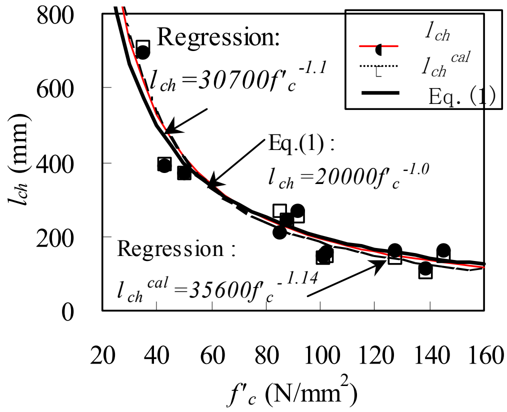

Fig.8 Dependence of compressive strength on characteristics

beams without shear reinforcements conducted by the authors.

As can be seen, the Standard Specification equation gives an evaluation on the danger side for high strength concrete with a value of d = 1000 mm. The authors proposed a shear strength equation, based on the JSCE2002a equation but covering domains outside its area of applicability, for the high strength concrete domain of compressive strength 80 N/mm^2 or above. As a result, it was demonstrated that the size effect was proportional to d/l_{ch} to the -1/2 power, not d/l_{ch} to the -1/4 power as previously thought. Further, the JSCE2002a equations are applicable up to 80 N/mm^2, and the tendency to peak at 60 N/mm^2 or higher can be countered by establishing an upper limit at $0.2 f'_c{}^{1/3}$. This means that there is still an issue with the area from 60N/mm^2 to 80N/mm^2. Accordingly, in this paper the authors have focused on the evaluation of size effect for different strengths, dividing compressive

Table 2. Formulas about shear strength for RC beams without shear reinforcement

	Existing formulas		Range of application
JSCE 2002a	$f_v = 0.2 \cdot \sqrt[3]{f'_c} \cdot \sqrt[3]{100 p_w} \cdot \sqrt[4]{1000/d}$	(2)	$0.2 \cdot \sqrt[3]{f'_c} \le 0.72$ N/mm^2 $f'_c \le 80$ N/mm^2
ACI 1999	$f_v = 1.9 \cdot \sqrt{f'_c} + 2500 p_w \cdot V_u \cdot d / M_u$ d : (in), f'_c : (psi) M_u : factored moment occurring simultaneously V_u : factored shear forced at section considered	(3)	f_v :($\le 3.5 \sqrt{f'_c}$) (psi) $\sqrt{f'_c} \le 100$ psi
CEB-FIP 1993	$f_v = 0.12 \cdot \sqrt[3]{f'_c} \cdot \sqrt[3]{100 p_w} \cdot (1+\sqrt{200/d})$	(4)	$f'_c \le 50$ N/mm^2

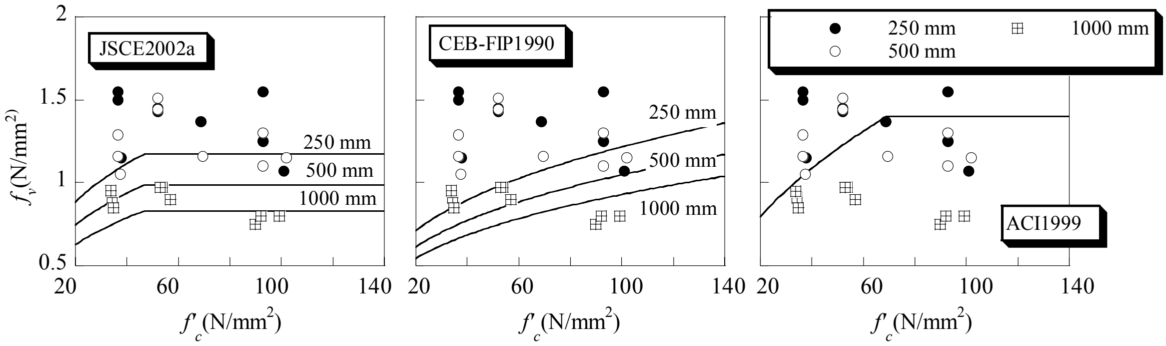

Fig.9 Compression of design code formulas with test results

strength into three domains, up to 60N/mm^2, from 60N/mm^2 up to 80N/mm^2, and over 80N/mm^2.

4.2 *Size effect in different compressive strength domains*

Table 3 shows the results of shear tests conducted by the authors (27 items of data) and existing experimental data (107 items of data). The shear span ratio a/d is 3 in all cases. Based on these data, a study of the size effect in shear strength in the domains ranging from normal strength concrete through high strength concrete was conducted, using the characteristic length derived in the previous section. In studying the size effect, the standard JSCE 2002a equation, Niwa's equation (Niwa et al 1986), was used.

Table 3. Outline of data used

Data	Number	d(mm)	P_w (%)	f'_c (N/mm^2)
Fujita et al.2002	27	250-1000	1.53	33.7-103
PWRI 1996	6	300-950	1.22-1.35	20.6-27.3
PWRI 1996	6	350-950	0.55-1.19	55.1-87.2
Matsui et al 1995	16	150-300	2.55-2.65	32.4-127.5
Abe et al 1999	6	150-650	0.54-1.27	90.6-107.7
Niwa et al 1986	3	1000-2000	0.14-0.28	25.4-28.0
Iguro et al 1980	3	1000-3000	0.04	21.9-28.5
JSCE 2002b	11	400-690	0.67-2.32	73.5-102.8
Tsuchiya et al 2002	7	260-1300	1.42-1.47	29.4-82.5
Hara 2001	5	350	1.84	27.8-55.2
Collins 1999	4	225,450	0.81,0.89	37.2,98.8
Kim et al 1994	12	267-915	1.01-3.35	53.7
Kani 1967	1	1092	2.72	27.0
Bhal 1968	6	600-1200	0.60-1.28	24.3-29.1
Taylor 1972	5	465-930	1.35	24.9-32.1
Walraven	2	420,720	0.74,0.79	27.4,27.8
Lonhardt 1962	3	300-600	1.33	37.7
Chana 1981	6	356	1.73	33.3-49.3
Rajagopalan 1968	6	356	1.69	41.6-49.3

$$f_v = 0.2 f_c'^{1/3} \rho_w^{1/3} (1000/d)^{1/4} (0.75 + 1.4d/a) \quad (5)$$

$$f_v^* = f_v / \rho_w^{1/3} / (0.75 + 1.4d/a) \quad (6)$$

where f_v = shear strength (N/mm^2), a = shear span length (mm).

In this paper, as shown in Equation (6), the offset value f_v^* for shear strength, which took into account the tension reinforcement ratio and the shear span ratio, was used. In addition, Gustafsson & Hillerborg 1988 reported that, with regard to the size effect in the shear strength of normal strength concrete, f_v/f_t was proportional to d/l_{ch} to the -1/4 power. Previously, the authors have also studied the size effect based on this relational expression; in this study, a re-examination was conducted based on the additional data. On difference is that in order to take into account the effect of shear strength on compressive strength, although Gustafsson & Hillerborg 1988 used f_t for normalizing f_v, the authors here used $f_c^{1/3}$ for normalizing, following the JSCE2002a equation and CEB-FIP1990 equations.

Fig. 10 shows the results of a study of size effect. Regression analysis was conducted on the data for compressive strength of 60 N/mm^2 or lower, using least squares fitting to obtain equation (7). Since the JSCE 2002a formula is said to be capable of evaluating up to the effective depth to the -1/4 power, the authors followed that lead for the regression analysis in this study. As can be seen in equation (8), the result was a regression formula in which the correlation coefficient is hardly changed.

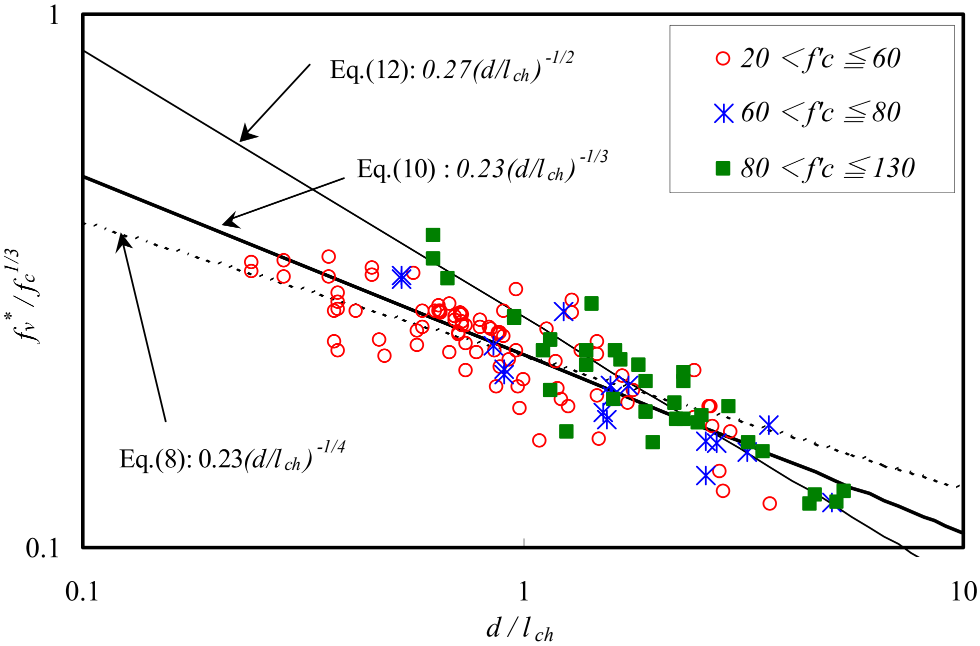

Fig.10 Study of size effect fracture mechanics with existing tests data

For this reason, it was concluded that the same characteristics as the JSCE 2002a formula were shown by the 60 N/mm^2 or lower data used in this study.

$$f_v^* / f_c^{1/3} = 0.23 d / l_{ch}^{-0.296} \quad \text{(R=0.81)} \quad (7)$$

$$f_v^* / f_c^{1/3} = 0.23 d / l_{ch}^{-1/4} \quad \text{(R= 0.80)} \quad (8)$$

Next, for the data for compressive strength from 60 N/mm^2 up to 80 N/mm^2, regression analysis produced equation (9). Equation (9) could then be simplified to obtain Equation (10), with the correlation coefficient virtually unchanged.

$$f_v^* / f'^{1/3}_c = 0.23 d / l_{ch}^{-0.388} \quad \text{(R=0.91)} \quad (9)$$

$$f_v^* / f'^{1/3}_c = 0.23 d / l_{ch}^{-1/3} \quad \text{(R=0.90)} \quad (10)$$

Also, for the data for compressive strength exceeding 80 N/mm^2, regression analysis produced equation (11). Equation (11) could then be simplified to obtain Equation (12), with the correlation coefficient virtually unchanged.

$$f_v^* / f'^{1/3}_c = 0.26 d / l_{ch}^{-0.46} \quad \text{(R=0.91)} \quad (11)$$

$$f_v^* / f'^{1/3}_c = 0.27 d / l_{ch}^{-1/2} \quad \text{(R=0.90)} \quad (12)$$

From these results, the application of fracture mechanics has shown that the size effect for shear strength can be evaluated with d/l_{ch} to the -1/4 power for concrete up to 60 N/mm^2, with d/l_{ch} to the -1/3 power for concrete from 60 N/mm^2 up to 80 N/mm^2, and with d/l_{ch} to the -1/2 power for concrete over 80 N/mm^2, indicating that the size effect changes in steps from domain to domain. It was surmised that this is related to the change in failure mode to a brittle mode due to the location of fracture surface cracks shifting from the mortar matrix at the aggregate boundary to the aggregate itself in conjunction with the increase in concrete strength. Future plans call for this relational expression for size effect to be used to study the assessment equation for shear strength for the domain ranging from normal strength through high strength concrete,

5. Conclusions

The knowledge obtained within the scope of this study can be summarized as follows.

1. Tensile strength could be evaluated for concrete ranging from normal strength through high strength, using the JSCE equation. Young's modulus could be evaluated using the equation proposed by Tomozawa et al and the equation proposed in ACI.

2. In the compressive strength range of 80 N/mm^2 or less, fracture energy increases as compressive strength increases. However, in compressive strength ranges exceeding 80 N/mm^2, fracture energy appears to decrease as compressive strength increases.
3. Since the two fracture energy values G_f and G_f^{cal} were generally the same for concrete ranging from normal strength through high strength, the load - crack mouth opening displacement (CMOD) curve obtained from reverse analysis using the poly-linear approximation analysis method faithfully reproduced the test results, even for high strength concrete with a compression strength exceeding 80 N/mm^2.
4. The tension softening curve for high strength concrete demonstrated a shape close to that of the 1/4 model.
5. Characteristic length decreased as compressive strength increased, for concrete ranging from normal strength through high strength.
6. the size effect in the shear strength of reinforced concrete beams without shear reinforcements can be approximately evaluated with with d/l_{ch} to the -1/4 power for concrete up to 60 N/mm^2, with d/l_{ch} to the -1/3 power for concrete from 60 N/mm^2 up to 80 N/mm^2, and with d/l_{ch} to the -1/2 power for concrete over 80 N/mm^2

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